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ABSTRACT

The identification of gravitation with the massless spin 2 gauge field (the gauge group is the group of translations) requires to restrict the solutions of Einstein's equations to the class of topologically trivial manifolds. It is shown that the validity of this restriction in nature is supported by the present-day empirical facts. The identification has a drastic impact on cosmology, because the fulfilment of the cosmological principle is claimed to be improbable.

АННОТАЦИЯ

Идентичность гравитации и безмассового поля со спином два (калибровочной группой является группа трансляции) требует ограничить решения уравнения Эйнштейна классом топологически тривиальных простанств. Показано, что сбывание этого ограничения в природе в настоящее время подтверждается эмпирическими фактами. Эта идентичность имеет драстическое влияние на космологию, потому что сбывание космологического принципа является невероятным.

KIVONAT

A gravitáció és a kettes spinü tömegnélküli mező (a mértékcsoport az eltolási csoport) azonositása megköveteli, hogy az Einstein egyenletek megoldásait a topológiailag triviális sokaságok osztályára korlátozzuk. Rámutatunk arra, hogy e korlátozás teljesülését a természetben a mai empirikus tapasztalatok alátámasztják. Az azonositás hatása a kozmológiára nézve drasztikus, mivel a kozmológiai elv teljesülése ezekután valószinütlen.

INTRODUCTION

Einstein's theory of gravitation may be obtained in various ways (see, e.g., [1], Box.17.2). One of the possible derivation is the field-theoretic route, which is in essence based on the identification of the massless spin 2 field with the gravitation. Numerous papers discuss this identification (see [2]-[23] and the references of [21] and [22]). The situation is reviewed in [22], where the results of the field-theoretic route to the Einstein's theory of gravitation are summerized, too. These results are the following: a) the approach immediately leads to Einstein's theory, where one necessarily has to take a zero cosmological constant, if one identifies the gravitation with the massless spin 2 field; b) the Riemannian structure of space-time is explained; c) the route predicts that the gravitational constant is positive; d) the route uses only the standard notions of field theory.

General relativity allows non-zero cosmological constant, but the identification of the gravitation with the massless spin 2 field needs a zero constant. Thus this identification leads to a restriction in general relativity, which can be empirically tested. Naturally the question arises: does this identification lead to further restrictions in general relativity?

The first purpose of this paper is to search for an answer to this question. It is shown that beside the zero cosmological term there is also a second restriction in general relativity, according to which: if the gravitation is identical to the massless spin 2 field, then the solutions of Einstein's equations are restricted to topologically trivial manifolds. The validity of this restriction must be decided empirically; the second purpose of this paper is to show that this restriction is indeed supported by the observations of the investigated part of the Universe. Thus it seems that one may doubtlessly identify the gravitation with the massless spin 2 field. The drastic impact of this identification on cosmology is also shortly discussed.

We shall proceed as follows: In Section 1. we build the theory of the massless spin 2 field. It is shown that from this theory the second restriction-mentioned above-follows, and it is studied in Section 2. Finally the results of paper are summerized.

We use the system $\hbar=c=1$.

1. THE THEORY OF THE MASSLESS SPIN 2 FIELD

Here we build up theory of the massless spin 2 field. We remain in the classical limit. The purpose is to search for the differences between Einstein's theory and the theory of massless spin 2 field.

We choose a procedure which seems to be the most logical one. We assume that we have never heard of gravitation. Thus we imagine that the mathematical theory of curved spaces is well-known, but we have never heard of curved space-times used in physics. In the flat space-time one may use the inertial and non-inertial frames, as it is the case (c.f., e.g., [24], Chapt.I.,III.). In constructing the theory of the massless spin 2 field we use three things: the assumption that the theory is similar to electrodynamics as fas as possible; the Ockam's razor (nature likes things as simple as possible); the standard theory of the gauge and matter fields. No empirical or theoretical facts about gravitation are used. Our heuristic route to Einstein's gravity essentially differs from the previous routes in two aspects: first, in our approach the immediate appointment of the gauge group is essential; second, no facts about gravitation are used (compare, e.g., with the procedures of [6],[9],[10],[16] or [20]). The advantage of our route consists in the fact that the Yang-Mills character of the massless spin 2 field is especially raised. (Note that several attempts were done to give a gauge formulation of the Einstein's gravity; c.f., [25]-[35] and the references therein; nevertheless from these papers the massless spin 2 field character of gravitation is not so obvious, and our approach is to be considered as proper one to solve this.)

We begin to work in an inertial frame (x), in which the Minkowski tensor has the form $\eta^{ij} = \eta_{ij} \equiv \mathrm{diag}(1,-1,-1,-1)$. Along the paper the coordinate system (x) denotes a frame of axes $-\infty<x^i<\infty$; Latin indices take the values 0,1,2,3; dependence on x denotes the dependence on the four coordinates.) The massless spin 2 free field equations are [6] $(U^{ij}(x)=U^{ji}(x))$

$$\Box U^{ij}(x) - U^{k(i}(x)^{,j)}{}_{k} + U(x)^{,ij} + \eta^{ij}(U^{km}(x)_{,km} - \Box U(x)) = 0, \qquad (1.1)$$

where an index after a comma denotes partial derivative $U^i_i(x) \equiv U(x)$ and ( ) denotes symmetrization without the factor 1/2 (i.e. $U^{k(i}(x)^{,j)}{}_{k} = U^{ki}(x)^{,j}{}_{k} + U^{kj}(x)^{,i}{}_{k}$). Equations (1.1) are the analogue of Maxwell's equations:

$$\Box A^i(x) - A^j(x)^{,i}{}_{j} = 0, \qquad (1.2)$$

where $A^i(x)$ is the vector-potential. Requiring the maximum similarity to electrodynamics the massless spin 2 field should also be a gauge field, and thus we have to determine the corresponding gauge group. We know that the global symmetry transformations of U(1) group leads, by means of the first Noether theorem, to the conservation of charge. This charge may be obtained from the four-current $J^i(x)$ by integration over a Cauchy surface. The source $J^i(x)$ emerges on the right-hand-side of (1.2). Similarly to electrodynamics the source of the massless spin 2 field has to emerge on the right-hand-side of (1.1). This must be a symmetric tensor $t^{ij}(x)=t^{ji}(x)$ with property $t^{ij}(x)_{,j}=0$ as it is obvious from the left-hand-side of (1.1). Thus by integration over a Cauchy surface one has to obtain a conserved four-vector P^i. The global symmetry transformations of the gauge group G have to lead, via the first Noether theorem, to this P^i, too. Therefore, the gauge group G is a four-parameter symmetry group, whose global symmetry transformations lead to P^i while the local symmetry transformations has to require the introduction of the massless spin 2 field. As far as it is known today, there is only one symmetry with the required properties: the symmetry of flat space-time under the four-translations. Thus, if the massless spin 2 field is highly similar to the massless spin 1 field, then the relevant gauge group G is the external symmetry group of translations.

We attempt to introduce the massless spin 2 field by a standard way of Yang-Mills fields. The general gauge transformations are the local translations

$$\bar{x}^i(x) = x^i + a^i(x), \qquad (1.3)$$

where $a^i(x)$'s are supposed to be C^3 functions, and $-\infty<x^i<\infty$, $-\infty<\bar{x}^i<\infty$ hold. If the metric tensor

$$\bar{b}^{ij}(\bar{x}) = \frac{\partial\bar{x}^i(x)}{\partial x^m}\frac{\partial\bar{x}^j(x)}{\partial x^n}\eta^{mn} = \eta^{ij} + a^{(i}(x)^{,j)} + a^i(x)_{,m}\, a^j(x)_{,n}\,\eta^{mn} \qquad (1.4)$$

in the system $(\bar{x})$ is not of the form η^{ij}, then $(\bar{x})$ is already a non-inertial frame. Thus the gauge transformations (1.3) are in fact the coordinate transformations of the flat space-time among inertial and non-inertial reference frames (one may write $f^i(x)\equiv x^i + a^i(x)$). Nevertheless, the coordinate systems are restricted by the condition $-\infty<x^i<\infty$, $-\infty<\bar{x}^i<\infty$ (for example, spherical coordinates cannot be used). In the followings only the coordinate systems of flat space-time connected by (1.3) are considered.

Considers now the neutral spin 1 field ("Proca's field") as a matter field. The Lagrangian in an arbitrary (x) frame (here (x) may be inertial or non-inertial frame too) is given by ([36], Chapt. 3.3)

$$L_1^o = -\frac{1}{2}\, b^{im}(x)\, b^{jn}(x)\, w_i(x)_{;j}\,(w_m(x)_{;n} - w_n(x)_{;m}) + \frac{m^2}{2}\, b^{ij}(x)\, w_i(x) w_j(x), \qquad (1.5)$$

where $w_i(x)$ is the vector-potential, m is the mass of field, $b^{ij}(x)$ is the metric tensor in the system (x) and semicolon denotes covariant derivative with respect to the the metric tensor $b^{ij}(x)$. (It is not prohibited to write down the physical laws in flat space-time for the non-inertial frames too; this question is illustrated in detail in, e.g., [24], Chapt.III.1.) In accordance with the standard pattern of the Yang-Mills fields L_1^o is not yet gauge invariant (i.e. it has different forms in the various coordinate systems connected by (1.3)), and it will be gauge invariant after the introducing the massless spin 2 gauge field. (1.5) is already invariant under the (1.3) gauge transformations, except for the form of metric tensor (i.e. the metric tensor is η^{ij} in an inertial frame and $b^{ij}(x) \neq \eta^{ij}$ in a non-inertial frame). The introduction of the massless spin 2 gauge field has to remove also the various form of the metric tensor in (1.5), and in the systems connected by the gauge transformations (1.3) the metric tensor has to have the same form. But, of course, the removal of the various forms of the metric tensor of flat space--time is not possible, if the space-time remains flat. The metric tensor in the flat space-time is defined once for all, and if the space-time remains flat, the metric tensor remains unchanged. The difference of the forms of the metric tensor of the flat space-time is a basic property and is connected to the frames in flat space-time. Thus introducing the massless spin 2 gauge field the space-time cannot remain flat.

Indeed, there must be a metric tensor in the presence of the massless spin 2 field too (at least in classical limit). It is physically reasonable to require that it would be possible to define time, distances, geodetical motion, etc... in the presence of the massless spin 2 gauge field too, i.e. that it would be possible to define a metric tensor. Thus introducing this gauge field the metric tensor of the flat space-time necessarily changes into a metric tensor of a non-flat space-time. What is the structure of this non--flat space-time? (Our considerations lead to a non-flat space-time, but this space-time need not necessarily be a curved Riemannian manifold.) To answer this question we refer to Thirring's papers [6], which gave arguments in favour of Riemannian structure. In [22] this choice is also suggested. Therefore we accept the assumption that the non-flat space-time is a Riemannian curved manifold.

Now we introduce the massless spin 2 gauge field. First, similarly to any physical field, we must define the $U^{ij}(x) = U^{ji}(x)$ potentials in an arbitratily chosen reference frame (x) on the flat space-time background. Similarly to any physical fields $U^{ij}(x) \to 0$ holds in the asymptotic regions of the Cauchy surfaces. (A Cauchy surface is for example the x^0 = const. hypersurface, and the asymptotic regions are given by $|(x^1)^2 + (x^2)^2 + (x^3)^2| \to \infty$). This condition is necessary for any reasonable field theory, because it is required by the existence of the dynamical invariants and by the Fourier decomposition of potentials. Second, we suppose that the ten $U^{ij}(x)$'s change the metric tensor of the flat space-time background into a metric tensor of a curved Riemannian space-time. In accordance with Ockam's razor we suppose that in an arbitrarily chosen (x) reference frame the contravariant metric tensor of the curved space-time $g^{ij}(x)$ has the form

$$g^{ij}(x) = \eta^{ij} + f\, U^{ij}(x), \quad f \neq 0, \qquad (1.6)$$

where f is a constant of dimension of length, because $g^{ij}(x)$ is dimensionless and $U^{ij}(x)$'s have the dimension $(\text{length})^{-1}$ similarly to any Bose field. It is obvious that a much simple change of the metric tensor is not possible. The decomposition (1.6) of the contravariant metric tensor is clearly Lorentz covariant and gauge invariant. The transformation formulae of the potentials $U^{ij}(x)$ under the (1.3) gauge transformations are the following:

$$\bar{U}^{ij}(\bar{x}) = \frac{1}{f}(a^{(i}(x)^{,j)} + \eta^{mn}\, a^i(x)_{,m}\, a^j(x)_{,n}) + U^{mn}(x)(\delta^i_m + a^i(x)_{,m})(\delta^j_n + a^j(x)_{,n}). \tag{1.7}$$

These equations immediately follow from the transformation formulae of the contravariant metric tensor. In contrast to $g^{ij}(x)$ the potentials $U^{ij}(x)$ are not the components of a tensor defined in the curved space-time.

In the infinite space-like regions $\bar{U}^{ij}(\bar{x})$'s and $U^{ij}(x)$'s go to zero and therefore in these regions either $a^i(x)$'s are infinitesimal or (1.3) define the global symmetry transformations of the Poincaré group. Obviously in (1.7) $\bar{U}^{ij}(\bar{x})$'s and $U^{ij}(x)$'s are infinitesimal, if and only if $a^i(x)$'s are infinitesimal or (1.3) determines an inhomogeneous global Lorentz transformation.

Having defined the contravariant metric tensor by (1.6) one needs its covariant form too. One has

$$g_{ij}(x) = \frac{\text{minor}\ g^{ij}(x)}{\det\ g^{ij}(x)}\ , \tag{1.8}$$

and therefore $g_{ij}(x)$ as functions of $U^{ij}(x)$'s are given by infinite series.

The curved Riemannian manifold R determined by the metric tensor (1.6) and (1.8) has the same topological and global properties as the flat Minkowski space-time M. This follows from the fact that on the flat background M are defined the potentials $U^{ij}(x)$. R and M are covered by the same coordinate chart $-\infty < x^i < \infty$, and thus there is an one-one map between the points of R and M. In addition to, R is asymptotically flat - i.e. $g^{ij}(x) \to \eta^{ij}$ holds in the asymptotic regions of any Cauchy surface - and thus R goes into M in the infinite space-like regions.

Having defined the metric tensor in the presence of the massless spin 2 gauge field the Lagrangian (1.5) changes into

$$L_1 = -\frac{1}{2}\, g^{im}(x)\, g^{jn}(x)\, w_i(x)_{;j}\, (w_m(x)_{;n} - w_n(x)_{;m}) + \frac{m^2}{2}\, g^{ij}(x)\, w_i(x)\, w_j(x) =$$

$$= L_1^o + L^{int}(U^{ij}(x),\, w^i(x)), \tag{1,9}$$

where the indices are moved and the covariant derivatives are defined by the metric tensor (1.6) and (1.8). (L_1^o is given by (1.5) if $b^{ij}(x) = \eta^{ij}$ hold). The potentials $w^i(x)$ give a vector in the curved space-time R.

We have considered the Proca's field as matter field. But this choice was not essential. One may consider arbitrary set of matter fields (denoted formally by v) in the flat space-time background determined by a Lagrangian $L^o(v)$. This Lagrangian is obviously invariant under the gauge transformations (1.3), except for the form of metric tensor. Introducing the massless spin 2 field and thus the metric tensor of the curved space-time R by (1.6) and (1.8) the Lagrangian of the interacting matter and massless spin 2 gauge field owes the form

$$L = L^o(v) + L^{int}(U^{ij},v). \tag{1.10}$$

Following this pattern we strive to obtain $L_2^o(U^{ij})$, which is determined by the massless spin 2 field itself, and which must be added to (1.10) as it is required by the standard pattern of the Yang-Mills theory. Obviously in a reference frame (x) in the flat space-time equations (1.1) follows from the Lagrangian

$$L_2^o(U^{ij}) = \frac{1}{2}(U^{ij}(x)^{;k}\, U_{ij}(x)_{;k} - 2U^{ij}(x)U^{ij;k}U_{ik}(x)_{;j} + \\ + 2U(x)^{;k}\, U_k^i(x)_{;i} - U(x)^{;i}\, U(x)_{;i}), \tag{1.11}$$

where the indices are moved and the covariant derivatives are defined by the metric tensor $b^{ij}(x)$ of the flat space-time. Now, it seems, one may identify $b^{ij}(x)$ with $g^{ij}(x)$ given by (1.6), and express $U^{ij}(x)$ by (1.6), too. Unfortunately, this pattern does not work here, because $U^{ij}(x) = (g^{ij}(x) - \eta^{ij})/f$ is not a tensor in curved space-time, and therefore it is not clear, how to define its covariant derivatives.

In order to solve this problem let us first consider a special case of the massless spin 2 gauge field. Let $fU^{ij}(x)$'s be infinitesimal on the whole Minkowskian background. Then we can omit the products containing three or more terms $fU^{ij}(x)$ in L_2^o. In this special case the indices are moved by η^{ij} and η_{ij}, the covariant derivatives may be subrtituted by partial derivatives and therefore L_2^o has the form (we substitute $U^{ij}(x) = (g^{ij}(x) - \eta^{ij})/f$)

$$L_2^o = \frac{1}{2f^2}(g^{ij}(x)^{,k}\, g_{ij}(x)_{,k} - 2\, g^{ij}(x)^{,k}\, g_{ik}(x)_{,j} + \\ + 2\, g^{ij}(x)_{,j}\, \eta^{mn}\, g_{mn}(x)_{,i} - g^{ij}(x)^{,k}\, g_{mn}(x)_{,k}\, \eta_{ij}\, \eta^{mn}). \tag{1.12}$$

Now we have to generalize L_2^o to the general case of the massless spin 2 field, i.e. for $fU^{ij}(x)$'s that are not infinitesimal. In (1.12) the partial derivatives cannot be substituted by covariant derivatives. If one made it, L_2^o would become identically zero. Thus one ad hoc has to assume that L_2^o is already given by (1.12) in the general case too, and only η^{ij} (η_{ij}) is substituted by (1.6) (by (1.8)). The argument for this assumption follows from the Ockam's razor: One can hardly have a simpler not identically zero L_2^o. One has

$$L_2^o(U^{ij}) = \frac{1}{2f^2}(g^{ij}(x)^{,k}(g_{ij}(x)_{,k} - 2\, g_{ik}(x)_{,j}) + g^{mn}(x)\, g_{mn}(x)_{,i}\, (2g^{ij}(x)_{,j} -$$

$$- g_{jk}(x)\, g^{jk}(x)^{,i})) = -\frac{2}{f^2}\, g^{ik}(x)(\Gamma^m{}_{ij}\, \Gamma^j{}_{mk} - \Gamma^j{}_{ik}\, \Gamma^m{}_{mj}), \quad (1.13)$$

where $\Gamma^i{}_{jk}$ are the Christoffel symbols corresponding to the metric tensor (1.6) and (1.8). $L_2^o(U^{ij})$, as a function of the $U^{ij}(x)$'s, becomes an infinite series. Thus the massless spin 2 gauge field is self-interacting. This is nothing new in the theory of Yang-Mills fields. The complete Lagrangian of the matter fields and of the massless spin 2 gauge field is following:

$$L = L^o(v) + L^{int}(v,U^{ij}) + L_2^o(U^{ij}). \quad (1.14)$$

We have heuristically constructed a theory of the massless spin 2 gauge field. Of course, we did not prove that there were no other possible theories of the massless spin 2 gauge field. We have used the Ockam's razor and the assumption that the massless spin 2 field was highly similar to electrodynamics. In principle, there may be a theory of the massless spin 2 field, which is for example not so similar to electrodynamics.

2. THE SPIN 2 CHARACTER OF GRAVITATION

In this section we compare the theory of Section 1. with the theories of gravitation.

It is obvious that the theory of the massless spin 2 field is essentially Einstein's theory of gravitation, if $f^2 = 32\pi G$, where G is the gravitational constant. (Thus we have proved again, that the gravitational constant is positive, if the gravitation is a massless spin 2 field.) Nevertheless, there are differences between the theory of massless spin 2 field and the usual Einstein's theory. These differences are:

a. In the theory of the massless spin 2 field the cosmological constant does not emerge, whereas usual Einstein's theory allows for a non-zero cosmological term too.

b. The potentials $U^{ij}(x)$'s were introduced on the flat background, and therefore the massless spin 2 field is defined on an unobservable flat background. In the presence of the massless spin 2 field the curved space-time has same topological and global properties as Minkowski space-time. On the other hand, Einstein's theory allows also the existence of space-times with complicated topology (for example the closed Friemann Universe-model).

c. In the presence of the massless spin 2 field the curved space-time is asymptotically flat, and the usual dynamical invariants of field theory are defined. On the other hand, in Einstein's theory the asymptotic flatness is not necessary (for example the Friedmann Universe-models are not asymptotically flat).

Point a. is not new, and shall not discussed here (interesting discussions on this question are given, e.g., in [10] or [22]). We remark only that, as it is well-known, there is no empirical evidence for a non-zero cosmological term.

Both b. and c. are new. They follow immediately from the route of Section 1., and in fact are highly reasonable. For example the decomposition (1.6) has a meaning only in the case of curved manifolds for which the unobservable Minkowskian background may be introduced. Nevertheless, the decomposition (1.6) was essential to our theory, and actually in the Lorentz covariant quantum gravity it plays a central role, too [20]. One can hardly interpret a curved manifold, for which the unobservable flat background cannot be introduced, as a system of the matter and gauge field. The asymptotic flatness is a necessary condition for the Fourier decomposition of the potentials and for the existence of dynamical invariants, and thus without this restriction one hardly can construct a reasonable field theory. The asymptotically flat curved manifolds, for which the unobservable Minkowkian background may be introduced, are called as "curved manifold with trivial topology" or "topologically trivial manifolds".

From b. and c. it follows: the gravitation is a massless spin 2 g field, if the solutions of Einstein's equations are restricted in nature to the topologically trivial manifolds. The requirement of the zero cosmological term need not be emphasized, because a non-zero cosmological term does not allow the existence of topologically trivial manifolds.

Indeed, it was conjectured: if the gravitation is a massless spin 2 field, then the gravitation must be defined on a flat background. E.g. in [32] it is shown that the simplest gauge theory of gravitation is the Einstein's one defined on a flat background. Various other papers speak about the gravitation on a flat background (e.g., [6],[10],[20],[22]). Nevertheless, as far as it is known, it is never claimed that the occurence of a curved manifold with non--trivial topology contradicts to the identification of gravitation with the massless spin 2 field.

In Einstein's theory the topologically trivial manifolds determine a very special class of the solutions of Einstein's equations. Obviously the question of the existence of a topologically non-trivial manifold in nature must be answered empirically. The search for a topologically non-trivial space-time may decide whether gravitation is a massless spin 2 field or not. Indeed, this search can reject this identification (if a non-trivial topology is detected), but the identification can never be confirmed with certainty.

Of course, there are other possibilities too, to make this decision. The best confirmation would be the direct observation of the massless spin 2 particles, i.e. of the gravitons. The empirical verification of Einstein's theory is in fact a further test of this identification. In this paper these other possibilities of the empirical verifications of the identification of gravitation with the massless spin 2 field are not discussed. We remark only that the present-day empirical facts about gravitation do not contradict to the Einstein's theory with zero cosmological constant; for a survey of this question see, e.g., [37] o. [38]; from the more recent works we mention [39] (about the gravitational radiation of binary pulsar), [40] (instruments for the detection of gravitational waves), [41] (test of the Newton's law) and [42] (about the gravitational lenses).

In the followings we shall consider the empirical facts and shall study the question: is there any indication in the observed part of nature for a non-trivial topology?

It is obvious that from the scales of experimental particle physics to the scales of the Solar system - these scales are already studied experimentally - there is no evidence for a non-trivial topology. In these scales the linearized Einstein's theory is a good approximation, and this theory considers only topologically trivial manifolds too. Note here that there are known some speculations [43]-[44] suggesting non-trivial topology on scales comparable with Planck's radius. Nevertheless, these speculations are very far from the possibility of empirical confirmation, and thus have no relevance for our purpose.

In the search for a non-trivial topology we have to consider astronomical observations. First we consider the scales much smaller than the Hubble radius. Any object with larger sizes than the relevant gravitational radius may roughly by described by a primitive "central body - vacuum exterior" model. In other words, these astronomical bodies may roughly be taken as bodies immersed in the Minkowskian vacuum. Therefore, for these objects the occurence of a non--trivial topology is excluded. The only objects, that can lead to a non-trivial topology, are the collapsed bodies, because here the occurence of the extended Schwarzschild or Kerr metric (see [45], p.149-168) is in principle allowed. On the other hand, for a collapsed body the "central body - vacuum exterior" model is allowed too. The observational investigation of black holes is still in fact at the beginning (see, e.g., [46]), and for our purpose it is essential that there is no observational evidence for the occurence of any non-trivial topology.

We consider the scales comparable with the Hubble radius now. It is usually believed that the complete physical Universe is in first approximation described by a Friedmann universe model. If this were true, then there would be a non-trivial topology. Nevertheless, the usual Friedmann model is cosmology is only a model of the observable part of the Universe, i.e. of the metagalaxy. This obvious fact is often overlooked, nevertheless in the precise cosmological considerations it is particularly stressed (see, e.g., [45], p.135 or [47], p.274). Under the notion "Universe" of cosmology one has to understand the metagalaxy, i.e. a limited part of the complete physical Universe. The metagalaxy is in principle limited by the present particle horizon. Therefore, the fulfilment of the cosmological principle is not so obvious as it is usually believed. The cosmological principle (the formulation see in [48], p.13) having a great philosophical significance is in fact an assumption that the metagalaxy is homogeneous and isotropic on the scales larger than ~100 Mpc, and that these properties of the metagalaxy may be extrapolated beyond the particle horizon to the complete physical Universe. Nevertheless, the extrapolating of the properties of metagalaxy beyond the particle horizon is a fully open question which cannot be studied by direct observations. It is not necessary that the properties of the metagalaxy be valid in the complete physical Universe, because the existence of an edge of matter beyond the present particle horizon is not prohibited (see [49], p.293). In other words, the cosmological principle may be - but need not be - fulfilled beyond the particle horizon. This fact alone is enough to conclude that the present-day cosmology does not prove the existence of a non-trivial topology in nature. One cannot exclude the possibility that the metagalaxy is an interior of a giant "island" having an edge beyond the particle horizon ([47],p.297).

Moreover, some new cosmological studies support a more strict claim. The paper [49] calls for a non-friedmannian model of the metagalaxy itself, because one can hardly explain some anisotropies of the cosmic background radiation in the standard Friedmann model of the metalagaxy. In [50] it is shown that the anticollapse-island model of the metagalaxy seems to be better than the standard Friedmann one. The anticollapse-island model of the metagalaxy in essence supposes that the metagalaxy is the interior of a giant anticollapsing body - of an anticollapsing "island" -, and in the exterior of this body there is a vacuum. (The anticollapse is a gravitational collapse with reversed sense of time.) In other words, the existence of the edge of the matter beyond the particle horizon seems to be probable. The papers [47],[49] and [50] suggest that the cosmological principle, i.e. the assumption of the homogeneity and isotropy, is hardly fulfilled already even in the metagalaxy. (We note here that the observational evidence of homogeneity in the metagalaxy was never satisfactory. As far as Shapley [51] querried the uniform density of galaxy distribution; the history and the survey of this question may be found in [48]. In any case the emergence of an inhomogeneous model of metagalaxy is not fully unexpected.) We may claim: "some cosmological considerations support the existence of an edge beyond the particle horizon", which is more strict than "the existence of an edge beyond the particle horizon is not prohibited". The rough "central body - vacuum exterior" model seems to be plausible for the scales larger than the Hubble radius.

Summarizing one may claim that there is no empirical evidence for the occurence of any non-trivial topology. The restriction to the class of topologically trivial manifolds is supported on the scales of nature that we empirically study. Of course, it is not proved yet that a non-trivial topology does not exist. Nevertheless, it is remarkable that the most primitive "central body - vacuum exterior" model is highly satisfactory from the regions of experimental particle physics to the scales larger than the Hubble radius.

It seems that the empirically supported non-existence of the non-trivial topology is not a chance, but as a matter of fact there is a prohibition of the occurence of any non-trivial topology, and this prohibition is the consequence of the identity of gravitation with a massless spin 2 field.

This identity has a drastic impact on cosmology. The existence of an edge of the matter beyond the particle horizon is then required and thus the metagalaxy necessarily is an interior of an "island". Therefore the symmetry properties of the metagalaxy cannot be extrapolated to the complete physical Universe, because in the exterior of the "island" there is a vacuum. Thus the cosmological principle breaks down in some regions beyond the particle horizon. In principle the homogeneous mean matter density may still exist in the metagalaxy, nevertheless this possibility is improbable because of the following reason: the uniform density in the interior of any astronomical body is in principle allowed, but from the physical point of view is improbable (see [52], Chapt.11.5). Therefore the mean density is hardly homogeneous in the metagalaxy, and in it the fulfilment of the assumption of the homogeneous density is improbable. Thus the fulfilment of the cosmological principle is anywhere maximally questionable. As it was mentioned in the connection with the papers [47],[49] and [50], this possibility is suggested by the present-day cosmological observations,too.

CONCLUSIONS

It was known that the identification of gravitation with the massless spin 2 field required a zero cosmological constant in Einstein's theory of gravitation. Thus the Einstein's theory and the theory of the massless spin 2 field are not precisely identical, because in general relativity the non-zero cosmological term is allowed too. We attempt to search for other differences between the Einstein's theory and the theory of the massless spin 2 field. For this end we again rederive Einstein's theory by a field-theoretic route. Our approach differs from the previous routes. The first result of this paper immediately follows from this rederivation: the symmetry group of translations seems to be the gauge group of the massless spin 2 gauge field. The second result follows from our approach too: it is shown that the identification of gravitation with a massless spin 2 gauge field requires the restriction of the curved manifolds to the class of topologically trivial space-times. It is obvious that the validity of this restriction may be decided empirically only. Therefore we survey the present-day empirical facts (mainly from cosmology). It is shown - and this is the third and maybe the main result of paper - that there is no empirical evidence for a non-trivial topology in nature. Thus the identification of the gravitation with a massless spin 2 field is empirically supported, and this identification may doubtlessly be done. The fourth result follows from this identification: the fulfilment of the cosmological principle is proclaimed to be improbable.

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