KFKI-1984-47

K. SZLACHÁNYI

NON-LOCAL CHARGED FIELDS AND THE PHASES OF THE Z(2) HIGGS MODEL

Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

KFKI-1984-47

NON-LOCAL CHARGED FIELDS AND THE PHASES OF THE Z(2) HIGGS MODEL

KORNÉL SZLACHÁNYI

Central Research Institute for Physics
H-1525 Budapest 114, P.O.B. 49, Hungary

HU ISSN 0368 5330
ISBN 963 372 222 5

ABSTRACT

Non-local order parameters are constructed to test confinement, deconfinement and broken charge symmetry phases of lattice gauge theories with matter fields. The relation of these classical fields to the structure of the quantum theory is analyzed. In the case of the Z(2) Higgs model we explicitely construct a charged sector in the free-charge phase. The confinement-sceening phase turns out to be split into two phases from the standpoint of non-local fields. This phase transition can be interpreted as the breaking of the global gauge symmetry.

АННОТАЦИЯ

Введены нелокальные поля для различия между замыкающей, незамыкающей фазами и фазой нарушенной зарядовой симметрии в калибровочных теориях с материальными полями на решетке. Проанализирована связь этих классических полей со структурой квантовой теории. В случае хиггсовской Z(2) модели дано явное построение заряженного сектора в фазе незамкнутого заряда. Фаза с экранированным замыканием оказывается разделенной на две фазы с точки зрения нелокальных полей. Соответствующий фазовый переход может быть интерпретирован как нарушение глобальной калибровочной симметрии.

KIVONAT

Nemlokális rendparamétereket konstruáltunk a bezáró, szabad és a töltés szimmetria sértő fázisok megkülönböztetésére rács mértékelméletekben. Megvizsgáltuk ezen klasszikus térmennyiségek kapcsolatát a kvantumelmélettel. A Z(2) Higgs modell esetén explicit konstrukciót adtunk a szabad fázis töltött szektorára. A bezáró-Higgs fázist egy csak a nemlokális rendparaméterrel érzékelhető fázisátalakulás osztja ketté, amelyet a globális gauge szimmetria sérüléseként interpretáltunk.

I. INTRODUCTION

The problem of confinement in lattice gauge theories with matter fields can be studied on two levels:

1/ Searching for a classical quantity which serves as an order parameter in the classical statistical system.

2/ Finding out a representation of the algebra of gauge invariant quantum fields as operators acting on a Hilbert space $\mathcal{H}$ such that the vectors in $\mathcal{H}$ have a non-trivial charge (compared to that of the vacuum). Naturally, the suitability of an order parameter can be decided only on the 2^{nd} level. Order parameters to distinguish between confining, deconfining and Higgs phases were proposed by Mack and Meyer [1] and by Bricmont and Fröhlich [2]. An approach on the 2^{nd} level was presented by Fredenhagen and Marcu in the case of the Z(2) Higgs model [3]. They analyzed the possibility of charged representations in the C^*-algebraic framework and constructed a translation covariant charged sector of the model.

In this paper we report on an other approach which makes the connection between the two levels to be more transparent. This is a generalization of the construction in which one builds the Hilbert space directly from classical fields. This method was applied to lattice gauge theories at first by Osterwalder and Seiler [4]. Its generalization lies in the use of non-local fields besides the local ones. That the .-local fields may have importance was argued in Ref. [5]. Here I wish to g. : a brief account of the exact results obtained in the Z(2) Higgs model. The more detailed discussion including the proofs will be published elsewhere.

II. NON-LOCAL FIELDS

If one wants to write a field down which is invariant under the local but not under the global gauge transformations one necessarily meets with non-locality. A prototype of such a field is Dirac's gauge invariant electron field in continuum QED [6]:

$$\Psi^{inv}(x^o,\underline{x}) = \Psi(x^o,\underline{x})\exp\left\{i\int d^3\underline{y}\Delta^{-1}(\underline{x}-\underline{y})\,\mathrm{div}\underline{A}(x^o,\underline{y})\right\} \quad . \tag{1}$$

An other example is the semi-infinite string in lattice gauge theories:

$$\sigma_x^{inv} = \sigma_x U_{xx_1} U_{x_1x_2} \ldots U_{x_nx_{n+1}} \ldots \quad |x_n - x_{n+1}| = 1 \ , \quad \lim_{n\to\infty} |x_n| = \infty \ . \qquad (2)$$

$\sigma(U)$ denotes the matter (gauge) field residing on sites (links). The non-locality of (1) and (2) amounts to much more than that these fields merely have infinite support in spacetime. Their basic properties can be summarized rather in the following two observations:

I. The fields (1) and (2) considered as functions of the gauge configurations A and U respectively are nowhere continuous. I.e. even if A and U are allowed to vary only in arbitrary far away regions of spacetime the values of (1) and (2) can suffer drastic changes.

II. If somebody wanted to put expressions (1) or (2) into the (infinite volume) functional integral he would find a serious difficulty: These functionals are defined only on a zero-measure set of configurations. Actually the exponent of (1) exists only if A vanishes faster then 1/r at spacelike infinity. Or transforming it to be an O(4) invariant condition: (1) exists for those A's which have finite Euclidean action,

$$\frac{1}{4}\int d^4x F_{\mu\nu}^2 < \infty \quad .$$

Similarly (2) exists only for U's which are equal to unity on all but a finite set of links.

Property I is a consequence of Gauss' law. It is the sensitivity of the field for the values of U of far-away regions what makes it possible to measure the charge by an observer being outside any large sphere. So, despite being a mathematical flaw, property I is physically acceptable. On the contrary property II at first glance seems to be disastrous for the non-local fields. Fortunately, the resolution of this problem is possible and we will describe it for purely bosonic theories.

Let φ denote a configuration which is a pair (σ,U) made of the matter and gauge configuations on an infinite lattice. We have to give up the usual expectation that the infinite volume statistical average ρ of a classical quantity $f(\varphi)$ should be written in the form

$$\rho(f) = \int d\mu(\varphi) f(\varphi) \qquad (3)$$

with some Lebesgue measure $d\mu$ on the configurations space $\mathcal{C}$. Such a measure naturally exists if f is local. However if f is non-local only the finite volume averages are integrals:

$$\rho_\Lambda(f) = \sum_{i=1}^{N} c_i \int_{\mathcal{C}_\Lambda^i} d\varphi \frac{e^{-S_\Lambda(\varphi)}}{Z_\Lambda} f(\varphi) \quad , \qquad (4)$$

where

$$\mathcal{C}^i_\Lambda = \{\varphi \in \mathcal{C} | \varphi = \bar{\varphi}^i \text{ outside of the finite volume } \Lambda\}$$

and $d\varphi$ is the a priori measure on $\mathcal{C}$. The $\bar{\varphi}^i$'s ($i = 1,\ldots,N$) are translation invariant minima of the action. (In certain models there is a continuous family of minima and the sum on i in (4) must be replaced by an integral.) The choice of the numbers c_i fixes the boundary condition. The thermodynamical limit of (4)

$$\rho(f) = \lim_{\Lambda\to\infty} \rho_\Lambda(f) \tag{5}$$

may well exist even if it cannot be written as a functional integral (3). For local f, of course, (5) reduces to (3). Therefore the (in reality evident) definition (4), (5) gives an extension of the functional integral formula to non-local fields. Let us observe that for the calculation of (5) by means of (4) we have to know $f(\varphi)$ only for those φ's which equal to one of the $\bar{\varphi}^i$'s on all sites and links except a finite set of them. So property II doesn't arise as a problem in this way.

The real question now is whether the definition (5) is a non-trivial extension of (3) to include non-local charged fields. The answer depends very much on the dynamics, i.e. on the special form of the action. The field of type (2) of course has no relevance in the thermodynamical limit because it is associated with an infinitely long string of electric flux which has infinite energy even in a deconfining phase. In a charged field favoured by the dynamics the string must be smeared over in an appropriate way. We can prove that such a field really exist in the free-charge phase (in the terminology of Fradkin and Shenker [7]) of the Z(2) Higgs model.

III. RESULTS IN THE 3-DIMENSIONAL Z(2) HIGGS MODEL

In this case both σ and U take their values in the set {1, -1}. The action is

$$S_\Lambda = -\alpha \sum_{\ell\in\Lambda} U_\ell \sigma(\partial\ell) - \beta \sum_{p\in\Lambda} U(\partial p) \quad . \tag{6}$$

We have now two translation invariant minima (N = 2 in (4)): $\bar{\varphi}^1 = (\sigma^1, U^1) = (1,1)$ and $\bar{\varphi}^2 = (\sigma^2, U^2) = (-1,1)$. Λ in (6) is chosen to be an open box in the 3-dimensional cubic lattice, i.e. it doesn't contain its boundary $\partial\Lambda$. Although $\partial\Lambda \cap \Lambda = \emptyset$, for the calculation of S_Λ we need the values of (σ, U) on $\partial\Lambda$ too, what is frozen to be $\bar{\varphi}^1$ or $\bar{\varphi}^2$. In order to have a statistical average ρ_Λ invariant under the charge (= global gauge) transformation

$$f \to Cf \qquad Cf(\sigma,U) = f(-\sigma,U) \tag{7}$$

we set $c_1 = c_2 = \frac{1}{2}$ in (4).

The algebra $\mathfrak{B}$ of gauge invariant non-local fields can be obtained from the algebra spanned by the gauge invariant functionals (Q is a finite set of sites, F is bounded)

$$f_Q(\sigma,U) = \prod_{x\in Q} \sigma_x F(U) \tag{8}$$

by taking series of the form

$$f = \sum_Q f_Q$$

which are norm-convergent with respect to the norm

$$||f|| = \sup_{(\sigma,U)\in \mathfrak{C}'} |f(\sigma,U)| \tag{9}$$

$\mathfrak{C}'$ is the restricted configuration space on which the bounded functions of $\mathfrak{B}$ are defined,

$$\mathfrak{C}' = \{(\sigma,U)\in \mathfrak{C} | \sigma \text{ is arbitrary but } \sum_\ell \frac{1-U_\ell}{2} < \infty\} \quad .$$

(We have no motivation to introduce non-localities in σ.) With the norm (9) and the usual complex conjugation $\mathfrak{B}$ turns out to be a commutative C^*-algebra. For a fixed pair (α,β) of coupling constants let us denote by $\mathcal{A}$ the largest sub-C^*-algebra of $\mathfrak{B}$ for which the thermodynamical limit (5) exists and is translation invariant. An other subalgebra $\mathcal{A}^{loc}$ of $\mathfrak{B}$ consisting of continuous functionals on $\mathfrak{C}'$ can be shown to be isomorphic to the usual C^*-algebra of local fields, the latter being the norm closure of the algebra of finitely supported fields on $\mathfrak{C}$. The existence of the thermodynamical limit for local quantities, what can be proven e.g. by using GKS inequalities, shows that $\mathcal{A}^{loc}$ is always contained in $\mathcal{A}$. For $f \in \mathcal{A}^{loc}$ $\rho(f)$ has the form (3) but for general $f \in \mathcal{A}$ ρ is 'only' a continuous linear functional which is positive ($\rho(f^*f) \geq 0$), has norm 1 ($\rho(1) = 1$) and if S_Λ is given by (6) it satisfies reflection positivity: If $f \in \mathcal{A}_+$ then $\rho(\Theta(f)f) \geq 0$. Here Θ is the reflection to the $x^0 = 0$ plane combined with complex conjugation and

$$\mathcal{A}_+ = \{f \in \mathcal{A} | \text{Suppf is in the } x^0 > 0 \text{ half space}\} \quad .$$

After this preparation we can summarize our results in the following three theorems.

Theorem 1: In region I of *Fig. 1*, i.e. for sufficiently small α and β, if a field $f \in \mathfrak{B}$ has charge -1, $Cf = -f$ then $\rho(\Theta(f)f) = 0$.

Theorem 2: In region II, i.e. for sufficiently large α, there exists an $f^3 \in \mathcal{A}_+$ such that

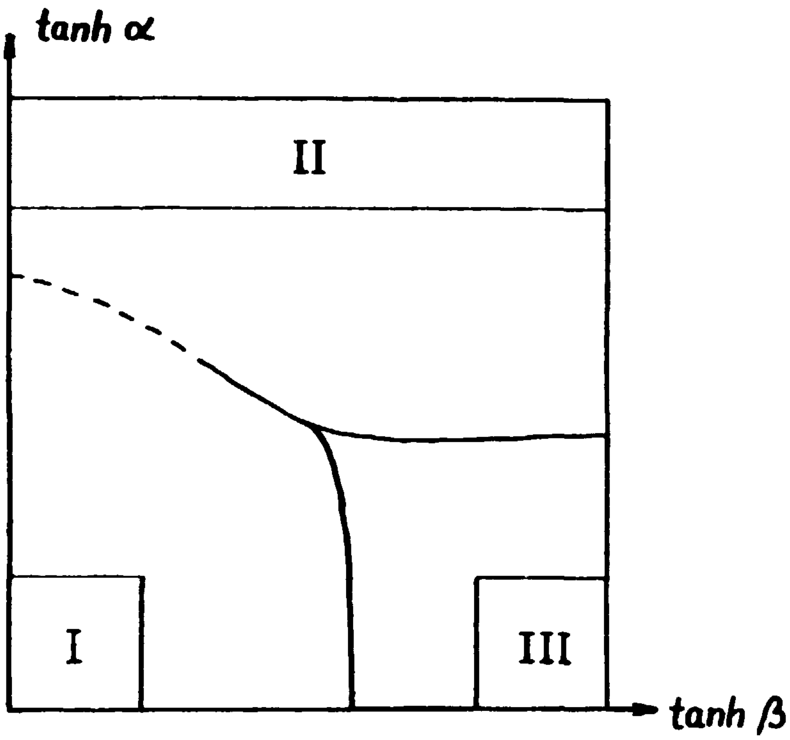

Fig. 1

The phase diagram of the Z(2) Higgs model and the regions I, II and III of cluster expansions. Solid lines: thermodynamic transitions. Dashed line: the simplest possible place of the "non-thermodynamical" transition.

(i) $Cf^S = -f^S$;

(ii) $\rho(\Theta(f^S)\alpha_x(f^S))$ is an analytic function of $e^{-2\alpha}$ and $e^{-2\beta}$;

(iii) $\lim_{n\to\infty}[\rho(\Theta(f^S)\alpha_{nx}(f^S)) - \rho(\Theta(f^S))\rho(f^S)] = B^2 > 0$,

where x is any lattice vector with $x^0 \geq 0$ and α_x is the translation by x. This theorem states that ρ is not a clustering state on $\mathcal{A}$, therefore it can not be a pure state. Indeed, $\rho = \frac{1}{2}(\rho^+ + \rho^-)$, where $\rho^\pm$ are defined as the thermodynamical limit of (4) with $c_1 = 1$, $c_2 = 0$ and $c_1 = 0$, $c_2 = 1$, respectively. $\rho^+ \neq \rho^-$ because $\rho^+(f^S) = -\rho^-(f^S) = B \neq 0$. Since ρ^+ and ρ^- are not symmetric under C, instead $\rho^+(Cf) = \rho^-(f)$, we see that in region II a pure state ρ_{pure} on $\mathcal{A}$, which occurs in the decomposition of ρ into pure states, can not be symmetric under C (charge symmetry breaking in ρ).

Theorem 3: In region III the field f^S appearing in Theorem 2 satisfies the following bounds: there exist positive numbers K_1, K_2, m_1, m_2 such that

$$K_1 e^{-m_1|x|} < \rho(\Theta(f^S)\alpha_x(f^S)) < K_2 e^{-m_2|x|} .$$

Furthermore the correlation function $\rho(\Theta(f^S)\alpha_x(f^S))$ is analytic in α and $e^{-2\beta}$.
These theorems were proven by using cluster expansion technique (see e.g. [8]).

The non-local charged field f^S taking part in Theorem 2 and 3 has the form

$$f_x^S \equiv \alpha_x(f^S) = \sigma_x \Omega_x^S(U) \quad , \tag{10}$$

where $\Omega_x^S(U)$ is the value of the gauge transformation $\Omega^S(U)$ at x which transforms the configuration U into a specific gauge. This gauge is characterized by a mapping S from the set of connected closed curves C of the dual lattice to the set of connected surfaces made from dual plaquettes. The mapping S is such that the boundary $\partial S(C)$ of $S(C)$ is just C. The gauge configuration V satisfying the gauge condition and being gauge equivalent to U is this:

$$V_\ell = \prod_{i=1}^{n} (-1)^{\delta_{\ell^* \in S(C_i)}} \quad . \tag{11}$$

ℓ^* denotes the plaquette dual to ℓ and the set $\{C_1,\ldots,C_n\}$ of connected closed curves (C_i is disconnected from C_j if $i \neq j$) consists exactly of those links which are dual to the flipped plaquettes, where $U(\partial p) = -1$. The latter set of plaquettes is finite because $(\sigma,U) \in \mathfrak{C}'$. With the aid of (11) we can write that

$$f_x^S = \sigma_x \Big(\prod_{\ell \in J} U_\ell\Big)\Big(\prod_{\ell \in J} V_\ell\Big) \quad , \tag{12}$$

where J is a semi-infinite string starting from the point x. In reality (12) is independent of the special choice of J.

In the course of proving the above theorems we made a restriction on the mapping S, namely that the surface $|S(C)|$ of $S(C)$ must be controlled by the perimeter $|C|$ of C. More precisely there exists a number λ such that

$$|S(C)| \le \lambda |C|^2 \qquad \text{for all } C. \tag{13}$$

Now we turn to the problem of what the above theorems imply for the quantized system. To quantize the theory we follow the strategy of Ref. [4]. The Hilbert space $\mathcal{H}$ is the norm closure of the factor space $\mathcal{A}_+/\mathcal{N}$ with respect to the norm

$$\Phi_f = f + \mathcal{N} \in \mathcal{A}_+/\mathcal{N} \longrightarrow \|\Phi_f\| = (\Phi_f,\Phi_f)^{1/2} = [\rho(\Theta(f)f)]^{1/2} \quad ,$$

where $\mathcal{N} = \{f \in \mathcal{A}_+ | \rho(\Theta(f)f) = 0\}$. The positive definite transfer matrix $\hat{T}$ on $\mathcal{H}$ is defined by $\hat{T}\Phi_f = \Phi_{Tf}$ where $T = \alpha_{(2,0,0)}$. This $\hat{T}$ satisfies $\hat{T} \le 1$ too. Hence Φ_1 is always a ground state: $\hat{T}\Phi_1 = \Phi_1$.

In view of this Hilbert space construction the content of Theorem 1 is that in region I it is impossible to have a vector of $\mathcal{H}$ what possesses a non-zero norm and charge -1 at the same time. In other words, the charge operator defined by $\hat{C}\Phi_f = \Phi_{Cf}$ acts trivially on the whole $\mathcal{H}$: $\hat{C} = \mathbb{1}$. In this sense region I is in the confinement phase. On the other hand in region II and III $\mathcal{H}$ is split into two eigenspaces $\mathcal{H}^+$ and $\mathcal{H}^-$ corresponding respectively to the eigenvalues +1 and -1 of $\hat{C}$. While in region II the vector

$$\Phi_1' = \lim_{n\to\infty} \hat{T}^n \Phi_{f^s} \in \mathcal{H}^-$$

is degenerate with the vacuum $\Phi_1 \in \mathcal{H}^+$, in region III $\Phi_{f^s} \in \mathcal{H}^-$ has positive but finite energy. So the interpretation of Theorem 3 is that region III is a phase where free charges exist. It can be shown as well that under the condition (13) the sector determined by f^s, i.e. the closure of the set

$$\{\Phi_{f^s A} \mid A \in \mathcal{A}_+^{loc}\}$$

does not depend on S. Unfortunately the interpretation of Theorem 2 as a charge symmetry breaking on the quantum level is not established yet. This is because we were unable to construct non-local operators analogous to the classical field f^s what would favourize certain linear combinations of Φ_1 and Φ_1'. As a matter of fact, local operators feel the states Φ_1, Φ_1' and any linear combinations of them as entirely equivalent ones. So, the interpretation of Theorem 2 at this moment extends so far as to say that in region II the symmetry C is broken in the classical statistical system. (See the remark after Theorem 2.) The existence of a phase transition between regions I and II (dashed line on *Fig. 1*) is not in contradiction with the results of Ref. [4] and [7] because it is a "non-thermodynamical" transition: it can be revealed only by means of non-local fields.

Until now we have used a notion of charge (C and its quantum counterpart $\hat{C}$) which can be traced back to a global algebraic property of our classical fields ((7)). It is interesting to see whether this charge coinsides with the charge what an observer could find if he makes local measurements only. Let $Q_{\underline{\Lambda}}$ be the operator of the charge of the finite space volume $\underline{\Lambda}$. What the observer could define as the charge of a state Φ compared to that of the vacuum is [3]

$$Q[\Phi] = \lim_{\underline{\Lambda}\to\infty} \frac{(\Phi, Q_{\underline{\Lambda}}\Phi)}{(\Phi_1, Q_{\underline{\Lambda}}\Phi_1)} \quad . \tag{14}$$

Cluster expansion yields the following results: In regions I and III the two notions of charge $\hat{C}$ and Q coincide, $Q[\Phi] = (\Phi, \hat{C}\Phi)$. While in region II $Q[\Phi]$ is always +1. More precisely we can prove that

$$Q[\Phi_{f^s}] = \begin{cases} +1 & \text{in regions I and II} \\ -1 & \text{in region III} \end{cases}$$

$$Q[\Phi_f] = +1 \qquad \text{for all } f \in \mathcal{A}_+^{loc}$$

The physical picture behind the discrepancy between $\hat{C}$ and Q in region II can be that the observable charge Q escapes to infinity during the process in which we prepare the state Φ_{f^s} starting from the vacuum Φ_1 in the remote past and acting with a time ordered operator corresponding to f^s.

IV. MORE GENERAL GAUGE MODELS

We conclude with an outlook towards the application of the non-local fields in gauge theories with more complicated gauge groups than Z(2). The direct generalization of (12) to continuous gauge groups does not seem to be realizable. The following ansatz, however, yields a gauge invariant charged field for arbitrary gauge groups. Let we have a d-dimensional gauge theory with gauge group $\mathcal{G}$ and

$$f^M_{x\alpha} = M_x^{\alpha\beta}(U^{(x^0)})\sigma_x^\beta \qquad (\sigma,U) \in \mathcal{E}'$$

be the charged field. $M_x^{\alpha\beta}(U^{(x^0)})$ denotes the magnetization in the (d-1)-dimensional $\mathcal{G}$-spin model with boundary condition $g = 1$ in the presence of frustrations $U^{(x^0)}$. $U^{(x^0)}$ is the restriction of the gauge configuration U onto the links of the x^0-hyperplane.

$$M_x^{\alpha\beta}(U) = \lim_{\underline{\Lambda}\to\infty} \frac{1}{Z_{\underline{\Lambda}}(U)} \prod_{\underline{y}\in\underline{\Lambda}} \int_{\mathcal{G}} dg_{\underline{y}} \exp\left\{\gamma \sum_{<\underline{z},\underline{y}>\subset\underline{\Lambda}} \chi(g_{\underline{z}}^{-1}U_{\underline{zy}}g_{\underline{y}})\right\} D^{\alpha\beta}(g_{\underline{x}}) \Bigg|_{g_{\underline{y}}=1 \text{ if } \underline{y}\in\partial\underline{\Lambda}} ,$$

$$Z_{\underline{\Lambda}}(U) = \prod_{\underline{y}\in\underline{\Lambda}} \int_{\mathcal{G}} dg_{\underline{y}} \exp\left\{\gamma \sum_{<\underline{z},\underline{y}>\subset\underline{\Lambda}} \chi(g_{\underline{z}}^{-1}U_{\underline{zy}}g_{\underline{y}})\right\} \Bigg|_{g_{\underline{y}}=1 \text{ if } \underline{y}\in\partial\underline{\Lambda}} . \qquad (16)$$

D is the representation of the group $\mathcal{G}$ under which the matter field σ transforms. If the inverse temperature γ of the spin model is sufficiently large and the dimension (d-1) is greater than a critical dimension of the $\mathcal{G}$-spin model this system is in the ordered phase and $M_x \neq 0$. (The frustrations cause only local disturbances according to our definition of $\mathcal{E}'$.)

In case of $\mathcal{G} = Z(2)$ and $d = 3$ $M_x(U)$ is the magnetization of the frustrated Ising model discussed in [9]. If we consider non-compact electrodynamics $\mathcal{G} = \mathbb{R}$ (additive group of real numbers),

$$D(g_{\underline{x}}) = e^{ig_{\underline{x}}}$$

and we chose

$$\chi(g_{\underline{x}} - g_{\underline{y}} - A_{\underline{xy}}) = -\frac{1}{2}(g_{\underline{x}} - g_{\underline{y}} - A_{\underline{xy}})^2$$

then f^M_x reduces to Dirac's ansatz (1).

ACKNOWLEDGEMENTS

The author wishes to thank to P. Vecsernyés for valuable discussions and to A. Frenkel for his interest in this work.

REFERENCES

[1] G. Mack, H. Meyer, Nucl. Phys. B, 200 (FS 4) (1982) 249

[2] J. Bricmont, J. Fröhlich, Phys. Lett., 122B (1983) 73

[3] K. Fredenhagen, M. Marcu, Com. Math. Phys., 92 (1983) 81

[4] K. Osterwalder, E. Seiler, Ann. Phys., 110 (1978) 440

[5] K. Szlachányi, in: Proceedings of the international Europhysics Conference on High Energy Physics (Brighton, July 1983)

[6] P.A.M. Dirac, Can. J. Phys., 33 (1955) 650

[7] E. Fradkin, S.H. Shenker, Phys. Rev. D, 19 (1979) 3682

[8] E. Seiler, "Gauge Theories as a Problem of Cosntructive Quantum Field Theory and Statistical Mechanics", Lecture Notes in Physics (Springer-Verlag, 1982) p. 159

[9] E. Fradkin, B.A. Huberman, S.H. Shenker, Phys. Rev. B, 18 (1978) 4789

Kiadja a Központi Fizikai Kutató Intézet
Felelős kiadó: Szegő Károly
Szakmai lektor: Hraskó Péter
Nyelvi lektor: Frenkel Andor
Gépelte: Polgár Julianna
Példányszám: 210 Törzsszám: 84-241
Készült a KFKI sokszorosító üzemében
Felelős vezető: Nagy Károly
Budapest, 1984. április hó