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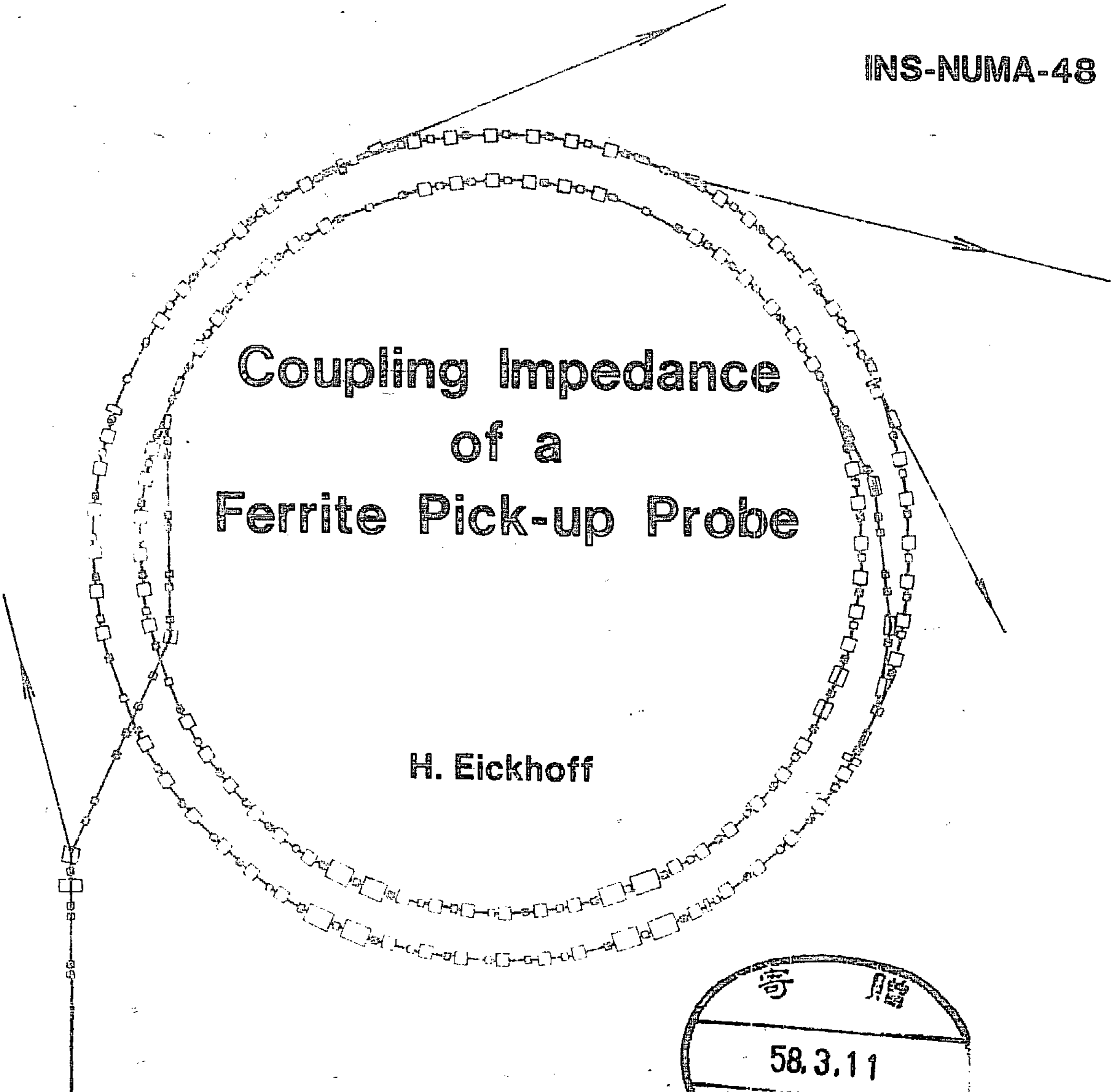

Coupling Impedance of a Ferrite Pick-up Probe

H. Eickhoff

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STUDY GROUP OF NUMATRON AND
HIGH-ENERGY HEAVY-ION PHYSICS
INSTITUTE FOR NUCLEAR STUDY
UNIVERSITY OF TOKYO
Midori-Cho 3-2-1, Tanashi-Shi,
Tokyo 188, Japan

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H. Eickhoff*

Institute for Nuclear Study, University of Tokyo

3-2-1 Midoricho, Tanashi-shi, Tokyo, Japan

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The Coupling Impedance of a Ferrite pick-up probe

I) Introduction

For TARN two different pick-up monitors for a stochastic cooling system are considered: a helix-type and a ferrite-type. For the helix-type probe the calculations of the coupling impedance Z_c, which is defined by the ratio of the voltage that is induced in the probe and the beam current, have already been performed.

The coupling impedance of a ferrite-type probe may be derived in a similar way, the formulas, however, are more complicated. Because the calculations are very time-consuming a computer-program has been developed, which also includes the calculations for the helix-type pick-up. It is possible to calculate Z_c for different boundary conditions and various geometrical or material parameters.

II) Theory

The experimental set-up is shown in Fig. 1a: the particle beam, which is surrounded by a ferrite core, produces a magnetic flux in this core. The time-variation of this flux induces a voltage U in the pick up coil. The coupling impedance Z_c is defined as $Z_c = U/I$ (I = beam current).

II a) The electric field for a small ferrite (metal wall-matching)

The starting point to calculate the voltage U is the calculation of the radial dependence of the longitudinal electric field component $E_z(r)$ that is produced by the beam (it is assumed that the geometry is symmetrical concerning the azimuthal component).

By solving Maxwell's equations one gets two solutions, one for the region inside the beam (current $\neq 0$) and one (current = 0) for the region outside the beam (Fig. 1b):

$$E_{ZI}(r) = A * J_o(\gamma_S * r) \qquad 0 \leq r \leq a \qquad (1)$$

$$E_{ZII}(r) = B * I_o(\gamma_o * r) + D * K_o(\gamma_o * r) \qquad r > a \qquad (2)$$

(J_0, I_0, K_0 = Bessel functions, A is the field value for r = 0, γ_S, γ_O are the propagation constants inside and outside the beam)

The constant γ_o is given by the relation:

$$\gamma_o^2 = \omega^2 \left(\frac{1}{v^2} - \frac{1}{c^2}\right) = \frac{\omega^2}{c^2}\left(\frac{1}{\beta^2} - 1\right) \qquad (3)$$

(v = beam velocity, c = velocity of light, $\beta = v/c$)

The values of the coefficients B and D and the propagation constant γ_S, however, have to be calculated by introducing special matching conditions. The first condition is that the electric field at the beam edge (r = a) should be continuous:

$$E_{ZI}(a) = E_{ZII}(a)$$
$$\frac{\partial}{\partial r} E_{ZI}(a) = \frac{\partial}{\partial r} E_{ZII}(a) \qquad (4)$$

The second matching condition reflects the influence of the surrounding material. In the case of well conducting metal plates like for electrostatic or helix-type monitors this condition is that the electric field at the position of this metal plate (i.g. r=d) should vanish:

$$E_{ZII}(d) = 0 \qquad (5)$$

The conditions (4) and (5) are also valid for a ferrite pick up probe that does not influence the field distribution (for example a small core that is not very close to the beam). In this case the matching conditions may be written as:

$$A \cdot J_0(\gamma_s a) = B \cdot I_0(\gamma_0 a) + D \cdot K_0(\gamma_0 a) \qquad (6a)$$

$$-\gamma_s \cdot A \cdot J_1(\gamma_s a) = \gamma_0 (B \cdot I_1(\gamma_0 a) - D \cdot K_1(\gamma_0 a)) \qquad (6b)$$

$$0 = B \cdot I_0(\gamma_0 d) + D \cdot K_0(\gamma_0 d) \qquad (6c)$$

(d = the radial position of the vacuum chamber wall, Fig. 1b)
From these conditions the following equation for γ_s can be derived:

$$-\frac{(\gamma_s a) J_1(\gamma_s a)}{J_0(\gamma_s a)} = (\gamma_0 a) \frac{I_1(\gamma_0 a) - D/B\, K_1(\gamma_0 a)}{I_0(\gamma_0 a) + D/B\, K_0(\gamma_0 a)} \qquad (7a)$$

$$= (\gamma_0 a) \frac{I_1(\gamma_0 a) K_0(\gamma_0 d) + K_1(\gamma_0 a) I_0(\gamma_0 d)}{I_0(\gamma_0 a) K_0(\gamma_0 d) - K_0(\gamma_0 a) I_0(\gamma_0 d)} \qquad (7b)$$

This equation can be solved either graphically or numerically for different frequencies f. The graphical solution is shown in Fig. 2 (the solutions for $\gamma_s a$ are given by the crossing points of LHS and RHS) The solution is not unique, because the left hand side of equation (7) is a periodic function. The calculations, however, take only the first branch of this function into account. After having determined γ_s the calculation of the coefficients B and D is possible by using the relations (6). The shape of the resulting electric field $E_z(r)$ is shown in Fig. 4 $(A=E_{zI}(r=0)=1)$.

II B) The magnetic field and the pick-up voltage

The electric field is surrounded by an azimuthal magnetic field $H_\theta(r)$ (Fig. 4):

$$H_\theta(r) = \frac{j\omega\varepsilon_0}{\gamma_0^2} \frac{\partial E_z}{\partial r} \qquad (8a)$$

$$= \frac{j\omega\varepsilon_o}{\gamma_o} (B \cdot I_1(\gamma_o r) - D \cdot K_1(\gamma_o r)) \tag{8b}$$

This magnetic field produces the magnetic flux Φ inside the ferrite core:

$$\Phi = \mu_o \mu \cdot z_o \cdot \int_b^c H_\theta(r) dr \tag{9}$$

The induced voltage U_c inside the pick-up coil of the length l_c is given by the time derivative of Φ :

$$|U_c| = \oint_{\ell c} E_c \cdot dl \approx l_c \cdot E_c = \frac{\partial\Phi}{\partial t} = \omega \cdot \Phi$$
$$= \omega \cdot \mu_o \cdot \mu \cdot z_o \cdot \int_b^c H_\theta(r) dr \tag{10}$$

In order to determine the coupling impedance Z_c it is necessary to calculate the beam current I.

II c) The beam current and the coupling impedance

From the Maxwell's equations and the continuity equation the following relation for the a.c.-part of the beam current density i_1 may be derived (the small signal assumption is used):

$$i_1 = - \rho_o \cdot u_1 + \rho_1 u_o$$
$$= - j\omega\varepsilon_o \frac{\omega p^2}{(\omega - ku_o)^2} E_z$$

(u_o = beam velocity; ω_p = plasma frequency; $k = \omega/c$)

The propagation constant γ_s may also be written in terms of the plasma frequency:

$$\gamma_s{}^2 = \gamma_o{}^2 (\frac{\omega_p{}^2}{(\omega - ku_o)^2} - 1)$$

Thus the current density i_1 may be expressed in the following way:

$$i_1 = -j\omega\varepsilon_o \frac{\gamma s^2 + \gamma o^2}{\gamma_o{}^2} E_z$$

As the electric field inside the beam is $E_z(r) = J_o(\gamma_s r)$ $(E_z(0) \equiv 1)$ the beam current I, which is the integral of i_1 over the beam cross section comes out as:

$$I = 2\pi \int_0^a r i_1(r) dr$$
$$= -j2\pi\omega\varepsilon \frac{\gamma s^2 + \gamma o^2}{\gamma_o{}^2 \cdot \gamma_s{}^2} \cdot \int_0^{\gamma_s a} (\gamma_s r) \cdot J_o(\gamma_s r) d(\gamma_s r)$$

$$= -j \frac{\gamma_s^2+\gamma_o^2}{\gamma_o^2} \cdot 2\pi\cdot\omega\cdot\varepsilon_o\cdot \frac{a}{\gamma_s} J_1(\gamma_s a) \qquad (11)$$

With the equations (11) and (10) the coupling impedance Z_c may be written as:

$$|Z_c| = |\frac{U_c}{I}|$$
$$= \frac{\mu\cdot\mu_o\cdot z_o\cdot\gamma_o^2\cdot\gamma_s}{2\pi a\cdot\varepsilon_o\cdot(\gamma_s^2+\gamma_o^2)\cdot J_1(\gamma_s\cdot a)} \cdot \int_b^c H_\theta(r)dr \qquad (12)$$

II d) Field matching at the ferrite core

If the ferrite core has a strong influence on the field distribution the matching conditions have to be modified. While the first condition (4) still remains valid the second one (5) has to be replaced by the condition that the electric and magnetic fields of region II and III have the same value at the boundary r=b (Fig. 1b). Inside the ferrite the propagation constant γ_1 may be expressed as:

$$\gamma_1^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c'^2} = \frac{\omega^2}{c^2}\left(\frac{1}{\beta^2} - \varepsilon\mu\right) \qquad (13)$$

($c' = c/\sqrt{\varepsilon\mu}$ = light velocity in the ferrite)

The equation for the electric field (current =0) is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) \pm |\gamma_1^2|\, E_z = 0 \qquad (14)$$

$$\left(+ \text{ for } \frac{1}{\beta^2} < \varepsilon\mu \right.$$
$$\left. - \text{ for } \frac{1}{\beta^2} > \varepsilon\mu\right)$$

Because of the frequency dependence of μ the "+"-sign has to be chosen for a certain lower frequency range ($\omega < \omega_o$), where the value of γ_1^2 is negative; the "-"-sign has to be chosen for an upper range ($\omega > \omega_o$). The solutions of equation (14) are:

$$E_{zIII}(r) = M\cdot J_o(\gamma_1 r) + NY_o(\gamma_1 r) \qquad (\omega < \omega_o \;;\; \gamma_1^2 < 0) \qquad (15a)$$

$$E_{zIII}(r) = M\cdot I_o(\gamma_1 r) + NK_o(\gamma_1 r) \qquad (\omega > \omega_o \;;\; \gamma_1^2 > 0) \qquad (15b)$$

For $\omega < \omega_o$ the magnetic fields in region II and III are given by the relations:

$$-j\cdot H_{\theta II}(r) = \frac{\omega\varepsilon_o}{\gamma_o^2}\frac{\partial E_{zII}}{\partial r} = \frac{\omega\varepsilon_o}{\gamma_o}\left(BI_1(\gamma_o r) - DK_1(\gamma_o r)\right) \qquad (16)$$

$$-j\cdot H_{\theta III}(r) = \frac{\omega\varepsilon\varepsilon_o}{\gamma_1^2}\frac{\partial E_{zIII}}{\partial r} = \frac{-\omega\varepsilon\varepsilon_o}{\gamma_1}(MJ_1(\gamma_1 r) + NY_1(\gamma_1 r)) \qquad (17)$$

The matching conditions for the ferrite may be expressed by the following formulas ($\omega < \omega_o$):

$$E_{zII}(r=b) = E_{zIII}\ (r=b)$$

$$B\cdot I_o(\gamma_o b) + D\cdot K_o(\gamma_o b) = MJ_o(\gamma_1 b) + NY_o(\gamma_1 b) \qquad (18a)$$

$$H_{\theta II}\ (r=b) = H_{\theta III}\ (r=b)$$

$$\frac{\varepsilon_o}{\gamma_o}(BI_1(\gamma_o b) - D\cdot K_1(\gamma_o b)) = -\frac{\varepsilon\varepsilon_o}{\gamma_1}(MJ_1(\gamma_1 b) + NY_1(\gamma_1 b) \qquad (18b)$$

$$E_{zIII}(r=c) = 0$$

$$M\cdot J_o(\gamma_1 c) + N\cdot Y_o(\gamma_1 c) = 0 \qquad (18c)$$

The condition (18c) assumes, that there is a metallic wall at the outer border of the ferrite core.
To calculate γ_s the relation (7a) is still valid; the value of the ratio D/B, however, changes. This ratio may be calculated by using the formulas (18a–c). The result is:

$$D/B = \frac{P\cdot I_1(\gamma_o b) - QI_o(\gamma_o b)}{P\cdot K_1(\gamma_o b) + QK_o(\gamma_o b)} \qquad (19)$$

with $P = 1/Y_o(\gamma_1 c)*(J_o(\gamma_1 b)\cdot Y_o(\gamma_1 c) - J_o(\gamma_1 c)\cdot Y_o(\gamma_1 b))$ $\quad(\omega<\omega_o)$

$(P = 1/K_o(\gamma_1 c)*(I_o(\gamma_1 b)\cdot K_o(\gamma_1 c) - I_o(\gamma_1 c)\cdot K_o(\gamma_1 b))$ $\quad(\omega>\omega_o)$

$Q = -\varepsilon\gamma_o/(\gamma_1\cdot Y_o(\gamma_1 c))*(J_1(\gamma_1 b)\cdot Y_o(\gamma_1 c) - J_o(\gamma_1 c)\cdot Y_1(\gamma_1 b))$ $\quad(\omega<\omega_o)$

$(Q = \varepsilon\gamma_o/(\gamma_1\cdot K_o(\gamma_1 c))*(I_1(\gamma_1 b)\cdot K_o(\gamma_1 c) + I_o(\gamma_1 c)\cdot K_1(\gamma_1 b))$ $\quad(\omega>\omega_o)$

After having determined D/B and γ_s with the relations (7a) and (19) the coefficients B and D for the electric and magnetic fields in region II may be calculated by using equation (6a):

$$B = \frac{A\cdot J_o(\gamma_s a)}{I_o(\gamma_o a) + D/B\ K_o(\gamma_o a)}$$

$$D = \frac{1}{K_o(\gamma_o a)}(A\cdot J_o(\gamma_s a) - B\cdot I_o(\gamma_o a)) \qquad (20)$$

With the values of the parameters B,D and γ_s the electric and magnetic fields in region I and II can be calculated by using the relation (1), (2) and (8).
The coefficients M and N for the fields inside the ferrite (region III) may be calculated with the equations (18a) and (18c). The resultis:

$$M = \frac{1}{p} \cdot (B \cdot I_o(\gamma_o b) + D \cdot K_o(\gamma_o b))$$

$$N = - M * \frac{J_o(\gamma_1 c)}{Y_o(\gamma_1 c)} \qquad (\omega < \omega_o)$$

$$(N = - M * \frac{I_o(\gamma_1 c)}{K_o(\gamma_1 c)} \qquad (\omega > \omega_o))$$

With these parameters the calculation of $H_\theta(r)$ in the ferrite and of the coupling impedance Z_c by using equation (12) is possible. It can be shown that with those coefficients the induced voltage in the pick up coil is mainly a function of E_{zII} (b):

$$|U_c| = \frac{z_o \cdot |E_{zII}(b)|}{\left|\frac{1}{\beta^2 \varepsilon\mu} - 1\right|}$$

II e) Discussion of the matching condition

The general matching condition (7a) contains a term D/B, which for a non-metallic wall is a function of the admittance $Y = -H_\theta(r)/E_z(r)$.
If this admittance is a real or a complex value the matching condition becomes very complicated, because the right hand side of equation (7a) becomes complex and one has to calculate Bessel functions J_o and J_1 with complex arguments. A solution for such general boundary conditions has been given by Birdsall and Whinnery [2].
In the case of a pure inductive admittance with no losses ($\mu'' = \varepsilon'' = 0$) the value of Y is purely imaginary. Thus one has to solve the equation (7a) only for real arguments.

III Results

The results of the calculations (electric-,magnetic fields and coupling impedances) are shown in the following figures.
Apparently there is a drastic difference in the coupling impedance between a ferrite with a fast decreasing μ (Fig. 3a, 8b) and one with a slowly decreasing μ (Fig. 3b, 8c, 9a). In the first case the coupling impedance has a maximum at a rather low frequency and a low, almost constant value for frequencies above 50 MHz. In the second case Z_c % f shows a resonance peak at defined frequency. This behaviour can be explained as a phenomena of a pure inductive admittance wall [2].
Fig. 7, 8d, 9b show the calculation of the coupling impedance for the same parameters shown in Fig. 2 except that the ferrite core is closer

to the beam. The result is that the peak is shifted to a higher frequency and that the lower frequency-side parts of Z_c are enhanced. In Fig. 12 a comparison of Z_c % f for different μ-dependences is given; all other parameters are kept constant in these calculations. Fig. 13b shows the coupling impedance of a helix type monitor (length = 0.6 m) with the helix at the radial position b and metal-plates in position d (the transfer-function has been taken into account, see Appendix). A comparison between this helix type monitor and a ferrite pick-up probe of the same length is shown in Fig. 13c (for the ferrite monitor the μ-dependence of curve 3 in Fig. 12 has been used). The conclusion of these calculation are: in order to achieve a high coupling impedance by use of a ferrite pick up probe

- the ferrite core should be as close to the beam as possible;
- it is desirable to choose a material with a smooth decrease of μ versus frequency.

Under these conditions an improved coupling impedance compared to that of the helix monitor is achievable.

Literature

[1] Beck, "Space-charge waves", Int. Series of Monographs on Electronics and Instrumentation, 1958

[2] Birdsall, Whinnery: "Waves in an Electron Stream with General Admittance Walls", Journal of Applied Physics, 24, No.3, 1953

Acknowledgment

I would like to express my thanks for the Support of Mr. H. Yonehara and Dr. N. Tokuda and for the consructive discussions that I had with Dr. T. Katayama.

Appendix

Relations for the coupling impedance of a helix-type pick up.

The field equations and matching conditions for a helix-type-monitor are described in Chap. II a. For a helix at a radial position b the longitudinal electric field may be expressed by:

$$E_z(b) = E_z(0) * J_0(\gamma_s a) \cdot \frac{K_0(\gamma_0 d) I_0(\gamma_0 b) - I_0(\gamma_0 d) K_0(\gamma_0 b)}{K_0(\gamma_0 d) I_0(\gamma_0 a) - I_0(\gamma_0 d) K_0(\gamma_0 a)}$$

The ratio $Z = |E_z(b)/I|$ may be calculated with formula (11) of chap. II c. In order to get the coupling impedance Z_c, e.g. the pick up voltage produced by the beam current I, the transfer function F and the length ℓ of the monitor has to be taken into account:

$$Z_c = \ell \cdot Z \cdot F$$

The function F may be expressed in the following way:

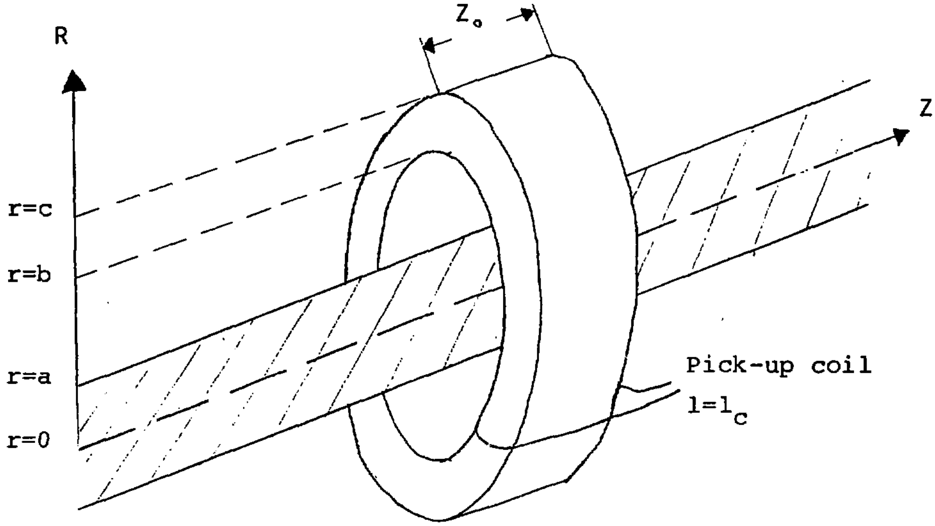
$$F = \frac{2\ell}{A} \cdot \sin\left(\frac{A}{2}\right)$$

$$\text{with } A = 2 \cdot \frac{\ell}{\ell_c} \cdot \arcsin\left(\frac{\omega}{\omega_c}\right) - \frac{\omega\ell}{\beta c}$$

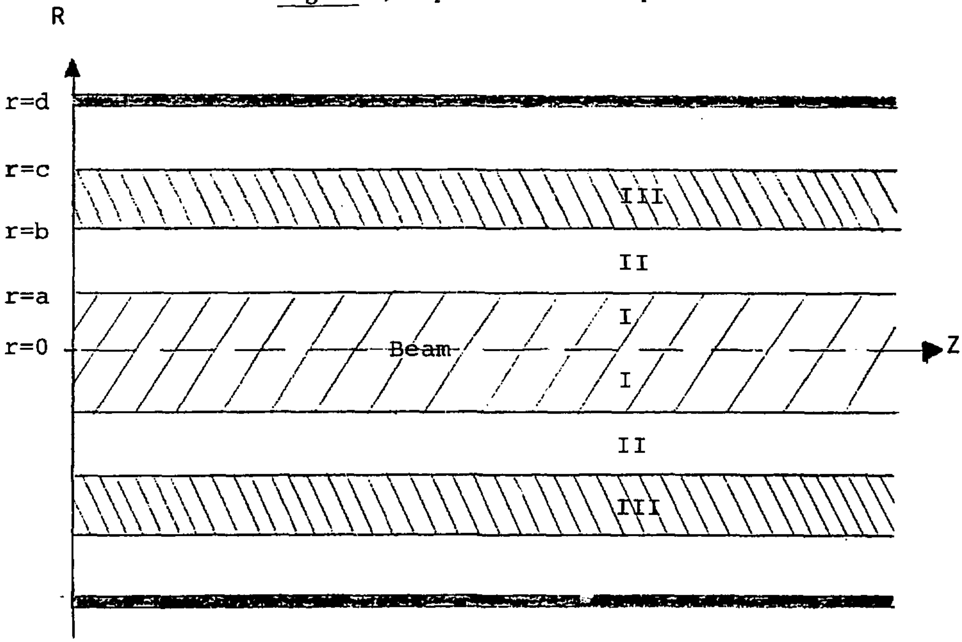
ℓ_c is the length of one helix cell (1 turn): ω_c is the cut-off-frequency:

$$\ell_c = 2\pi \cdot b \cdot \beta\gamma \qquad (\gamma = (1-\beta^2)^{-1/2})$$

$$\omega_c = \frac{2c}{\pi \cdot b \cdot (\eta/\ln(\frac{d}{b}))^{1/2}} \qquad \begin{array}{l}(c = \text{Speed of light} \\ \ \eta = \text{Nogoka coefficient})\end{array}$$



<u>Fig. 1</u> a) Experimental set-up



b) Definition of geometrical parameters

Parameter-list

velocity-parameter	v/c	=	0.12	
beamradius	a	=	4.8	cm
inn. core radius	b	=	13	cm
out. core radius	c	=	19	cm
radius of vac. ch.	d	=	1[illegible]	cm
core length	$z_{ö}$	=	6	cm
Dielectricity const.		=	10	
numb. of turns (pick-up-coil)		=	1	

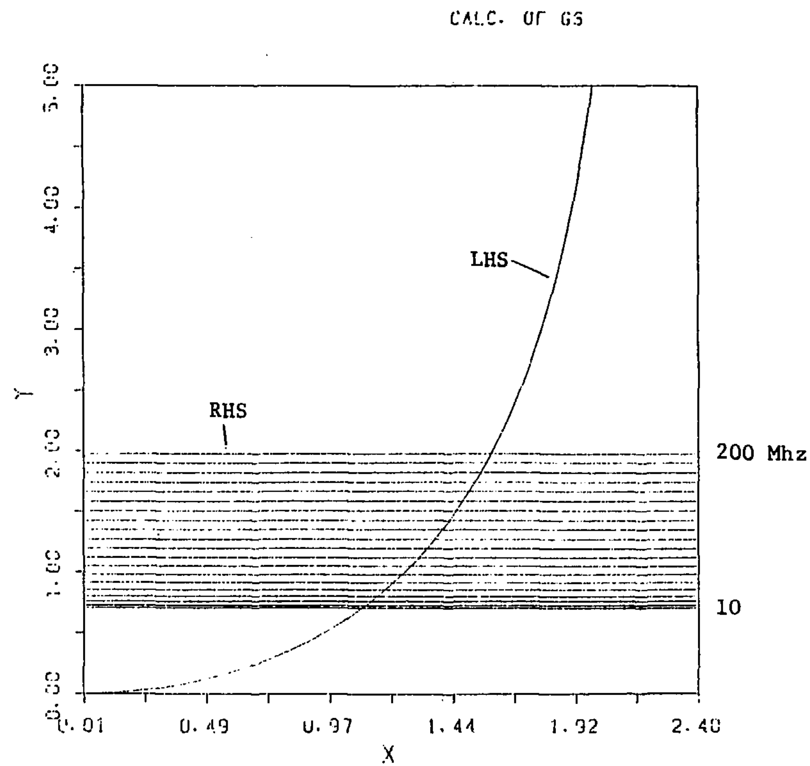


Fig. 2) Parameterlist; graphical solution of $\underline{Y}_s$

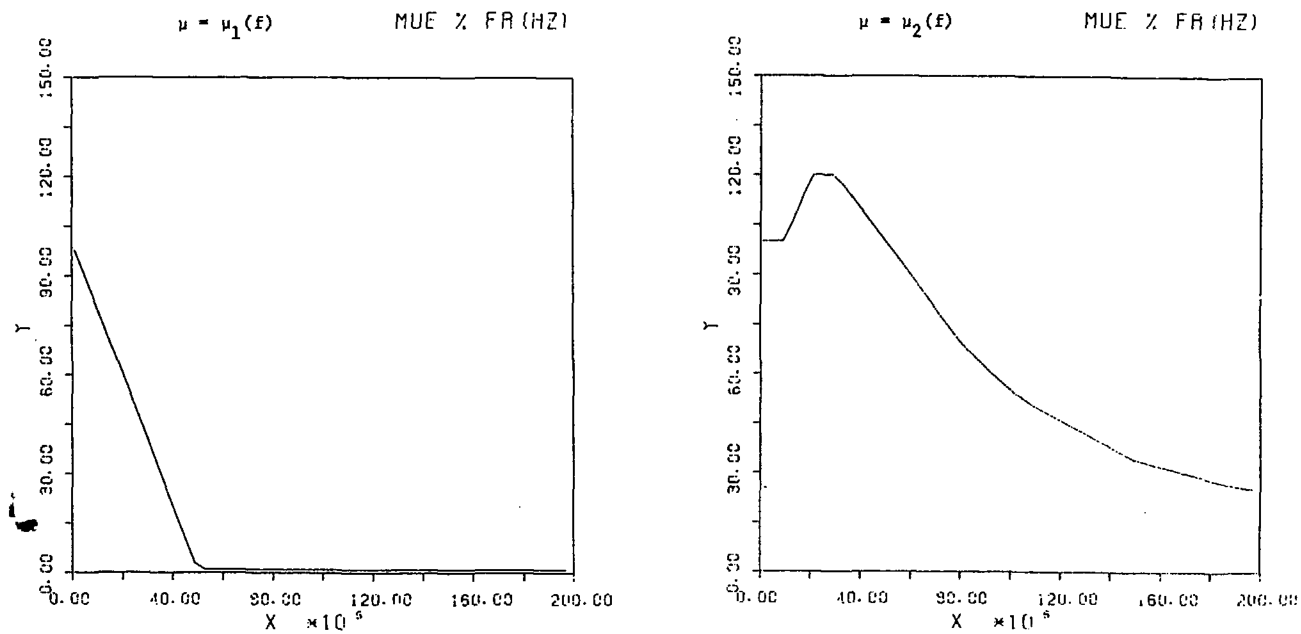


Fig. 3) Frequency-dependences of μ used in the calculations

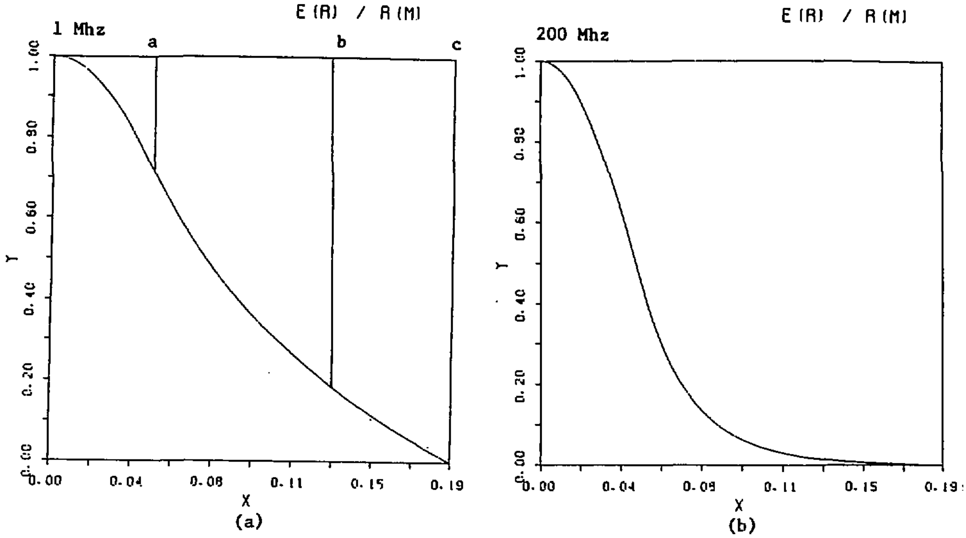
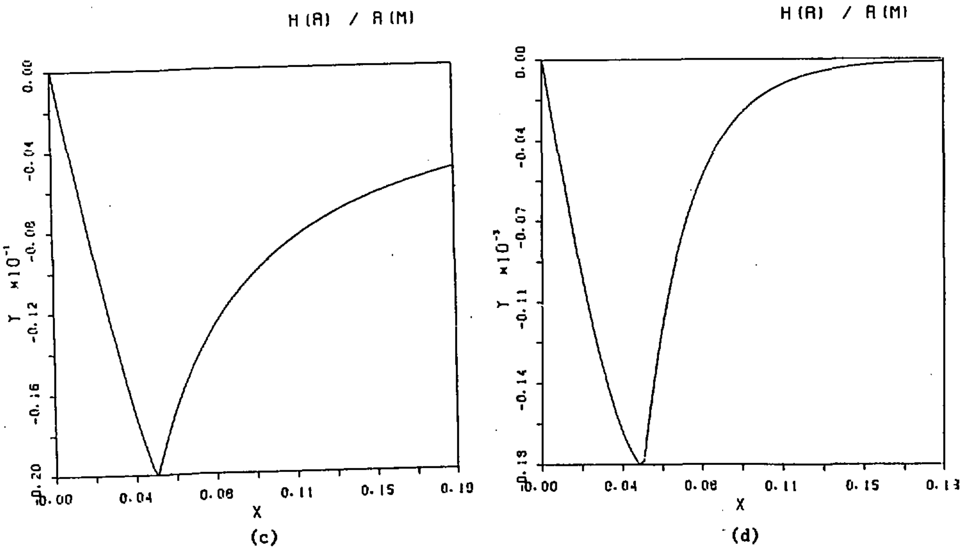
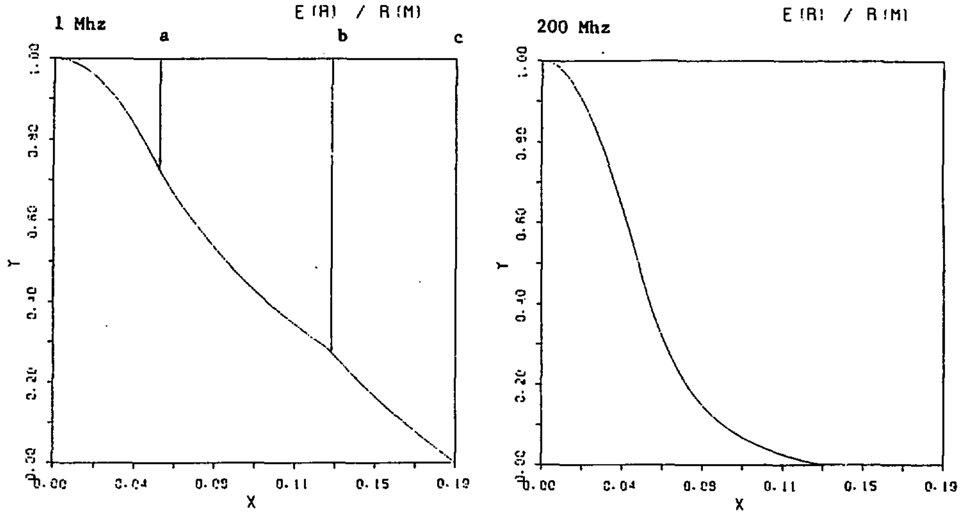


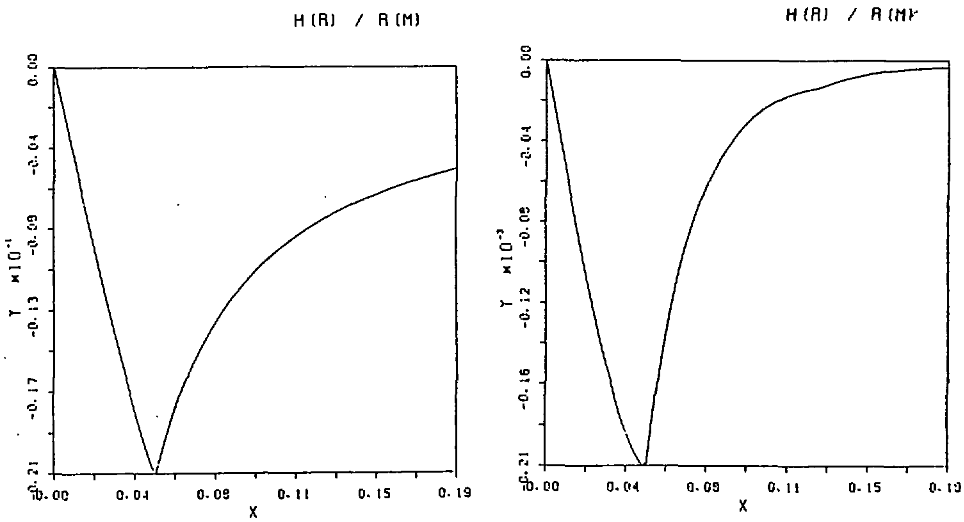
Fig. 4) $E_z(r),H_\Theta(r)$ for metal wall matching ($\mu = \mu_2(f)$;b=13; c=d=19 cm)



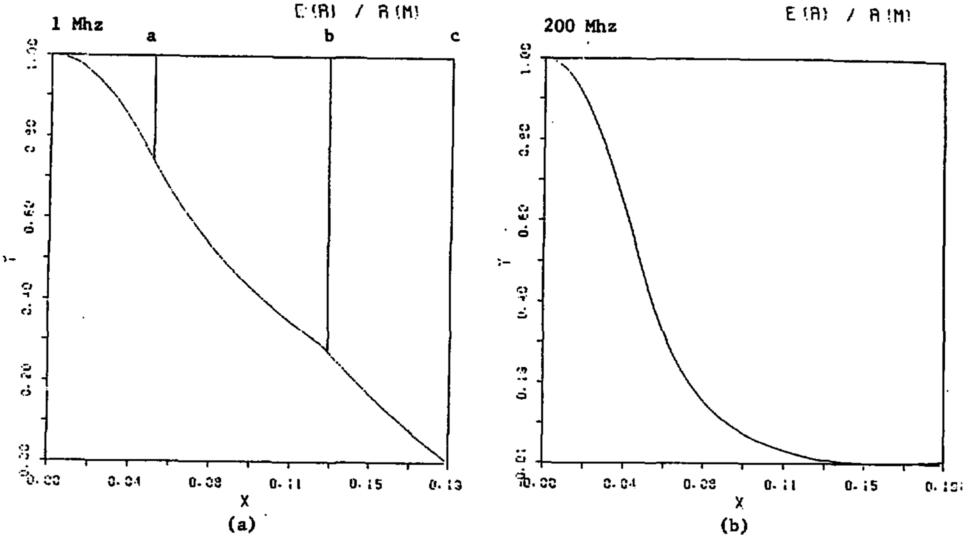


(a) (b)

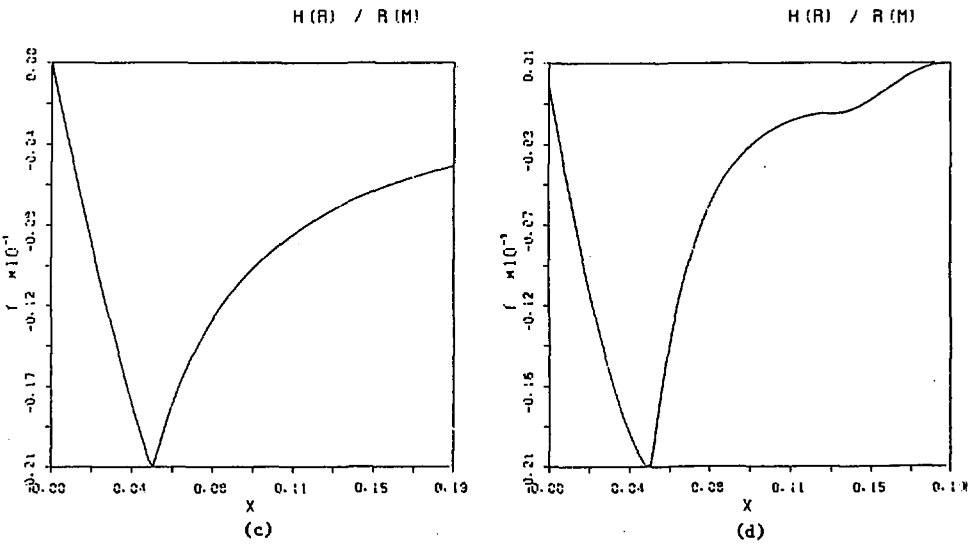
Fig. 5) $E_z(r),H_\Theta(r)$ for ferrite matching ($\mu = \mu_1(f)$;b=13; c=d=19 cm)

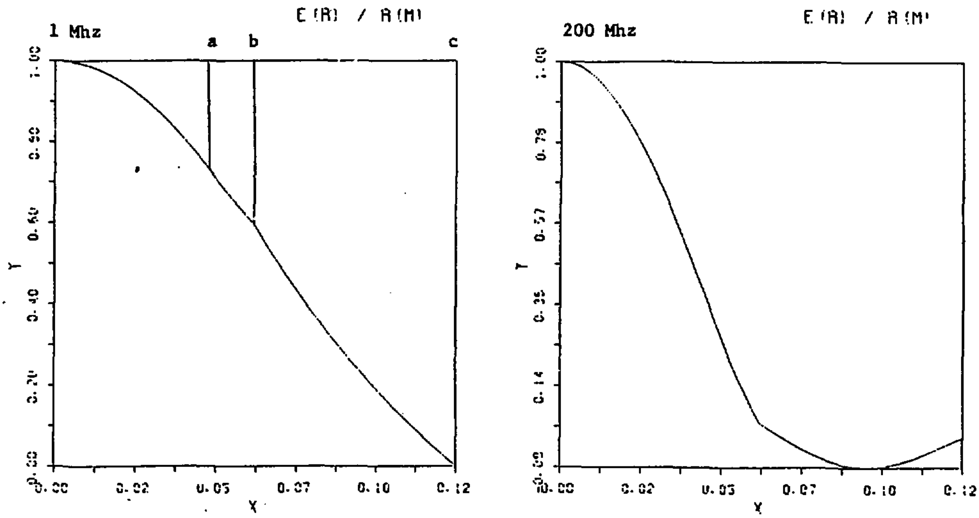


(c) (d)



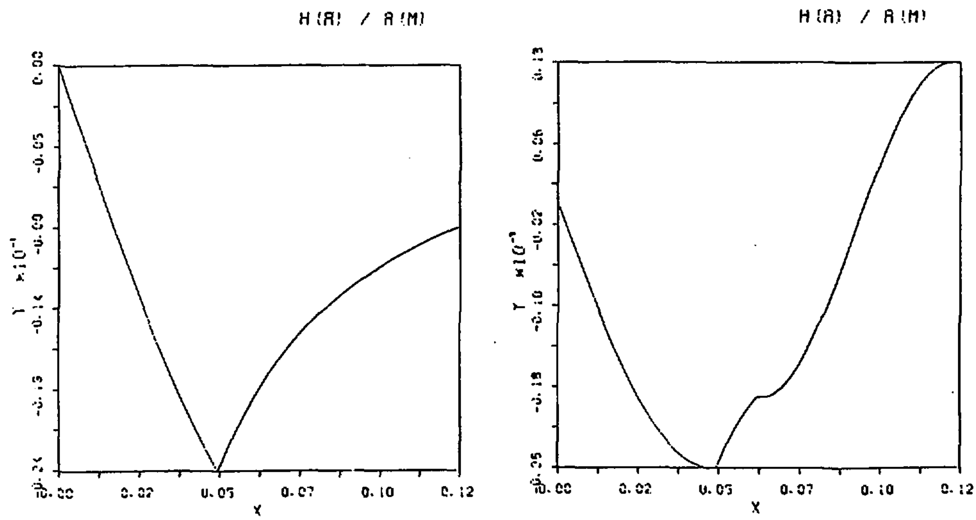
<u>Fig. 6</u>) $E_z(r), H_\Theta(r)$ for ferrite matching ($\mu = \mu_2(f)$; b=13; c=d=19 cm)





(a) (b)

Fig.7) $E_z(r),H_\theta(r)$ for ferrite matching ($\mu = \mu_2(f)$;b= 6; c=d=12 cm)



(c) (d)

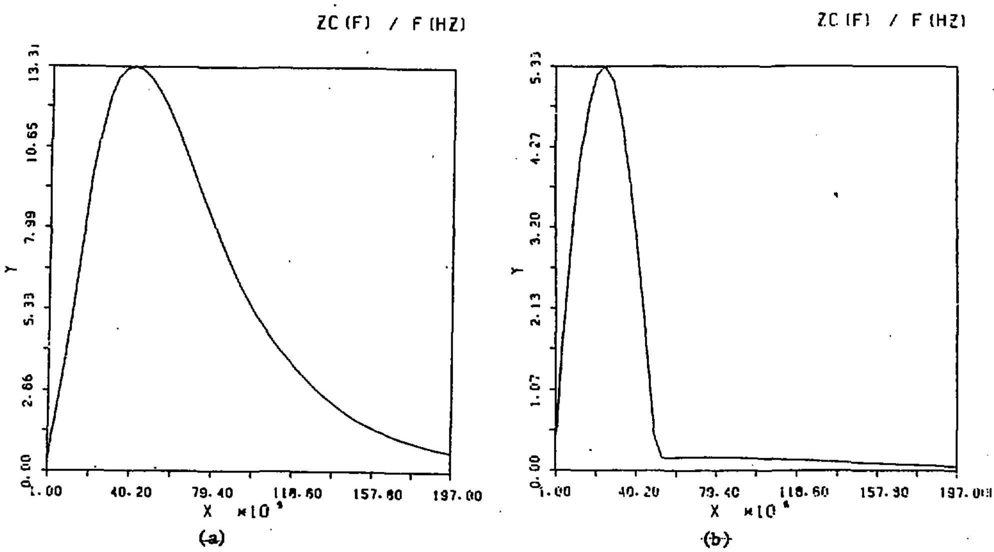
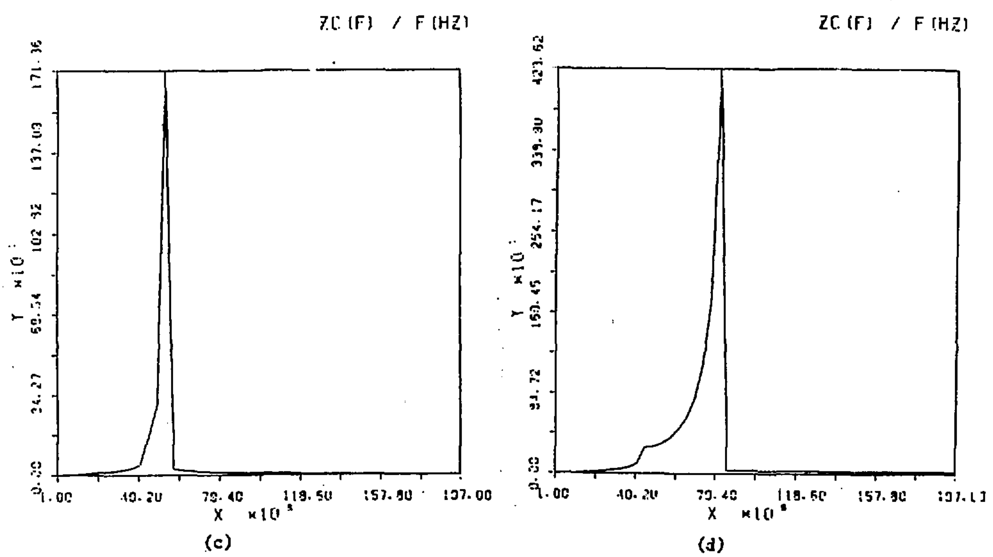


Fig. 8) Coupling impedances according to conditions of Fig. 4,5,6,7



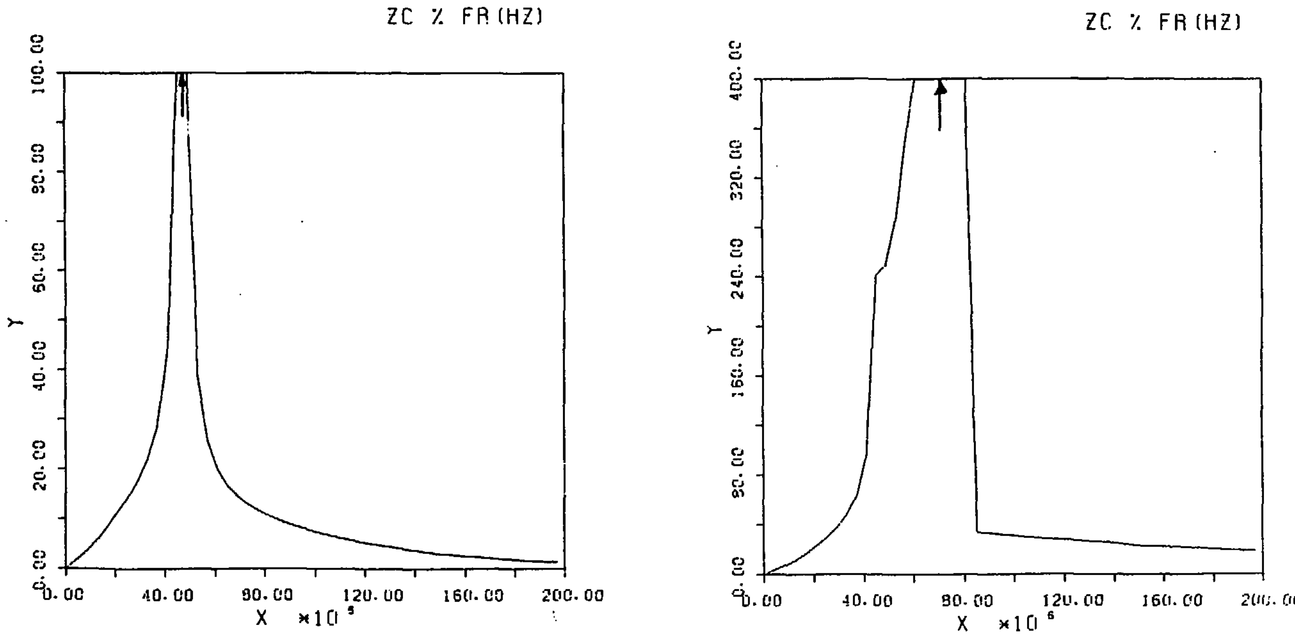


Fig. 9) Coupling impedances of Fig. 8c,d (stretched Z_c - scale)

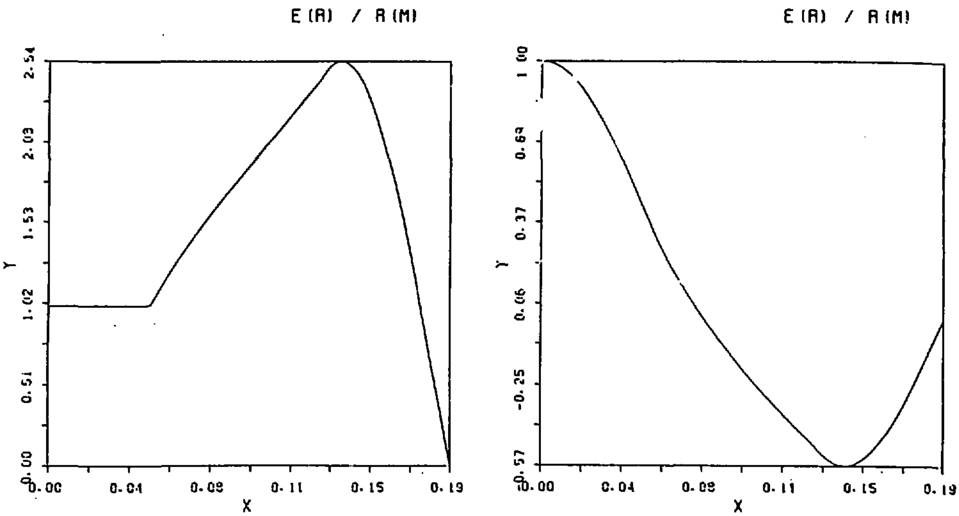
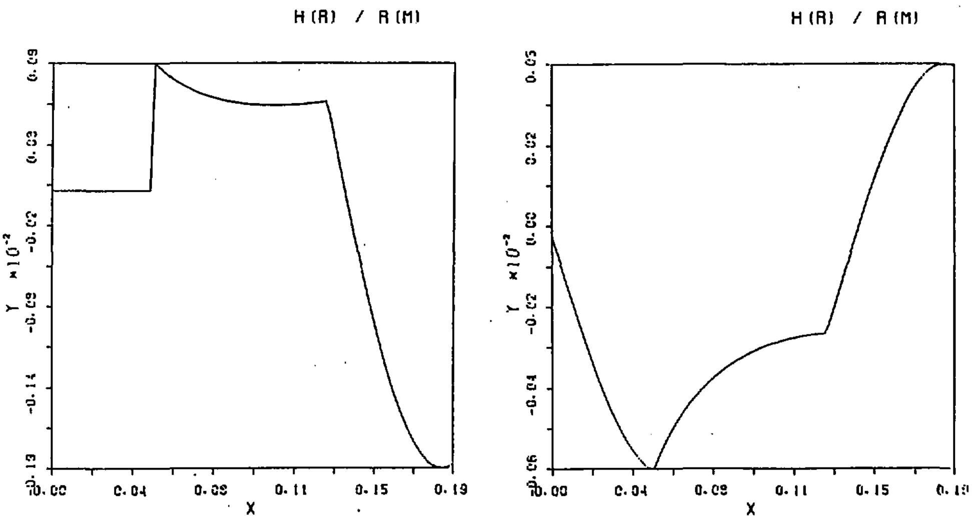
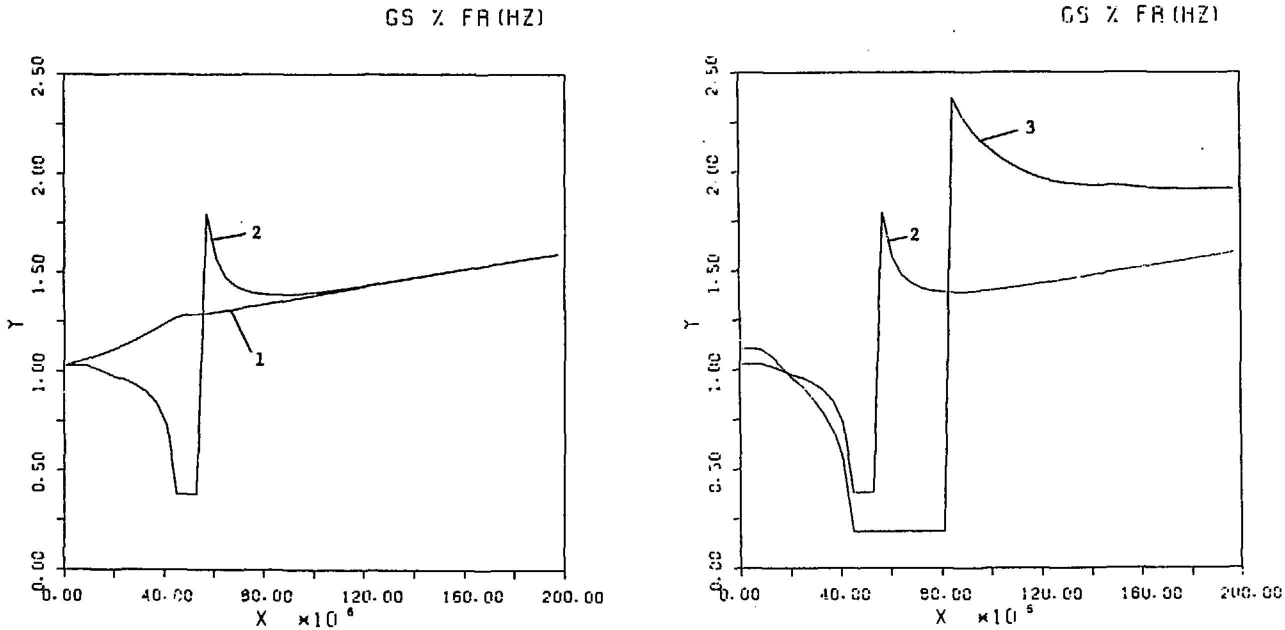


Fig. 10) $E_z(r), H_\Theta(r)$ for f=50,60 Mhz (at the peak of Z_c in Fig. 8c)





(1 = Fig. 5; 2 = Fig. 6; 3 = Fig. 7)

Fig. 11) Frequency-dependence of γ_s according to different conditions

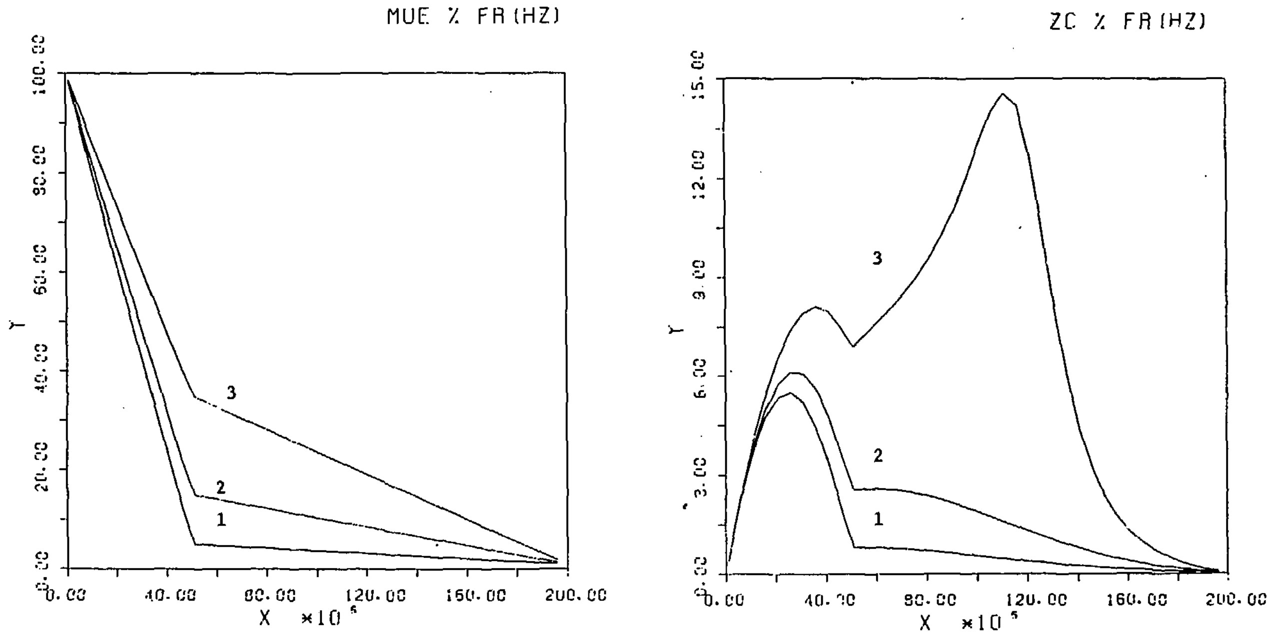
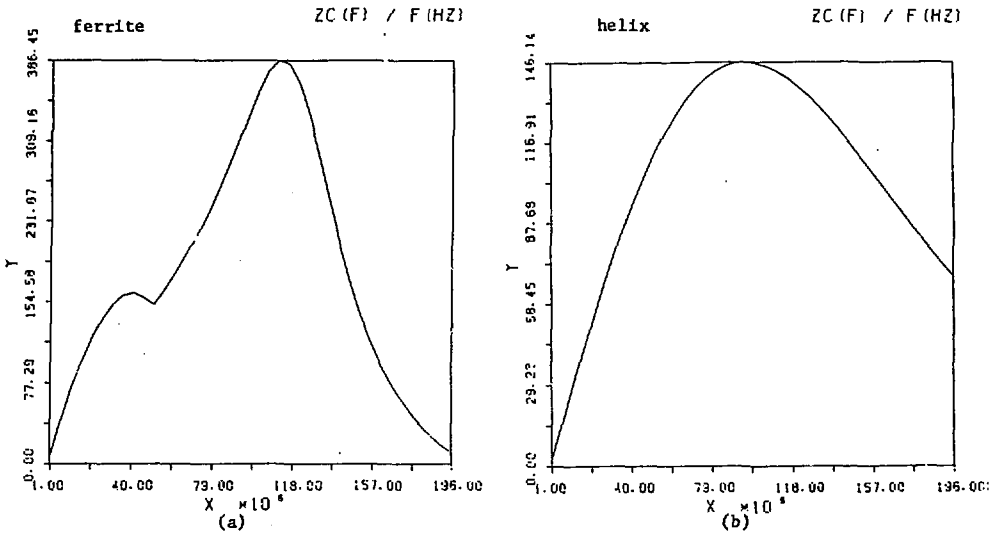
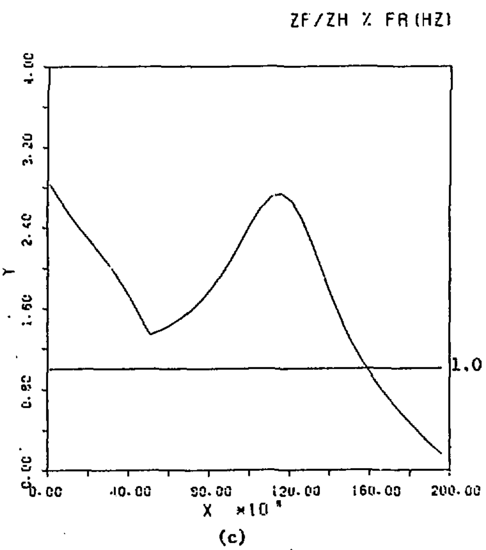


Fig. 12) Coupling impedances for different μ – dependences





a) Z_c for a ferrite pick-up
(b=6, c=12, length=55 cm)

b) Z_c for a helix pick-up
(b=6, d=6.6, length=55 cm, 12 turns)

c) ratio of Z_c (ferrite) (ZF)
and Z_c (helix) (ZH)

<u>Fig. 13</u>) Comparison of Z_c(helix) and Z_c(ferrite)