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Laboratoire de l'Accélérateur Linéaire

BEAM-BEAM DRIVEN COUPLING

A Mechanism for Vertical Blow-up of Flat Beams

P. BAMBADE

Seminar given at L.B.L. (Berkeley) and SLAC (Stanford), December 1983

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SUMMARY :

Among the many models and mechanisms proposed to explain the observed beam-beam effects in storage rings, one -the two dimensional single resonance model- appears to be particularly well suited to investigate and understand the familiar growth of vertical dimensions commonly seen in flat-beam operated e^+e^- storage rings. In this paper, after a quick derivation of the method and a comparison of the results obtained with existing experimental data, the assumptions are discussed in order to explain why this elementary model does well in this particular feature of the beam-beam interaction.

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INTRODUCTION

After almost twenty years of efforts, the beam-beam interaction remains a major limitation in the performances of most colliding beam accelerators. The main features are the following :

1) Due to their electromagnetic interaction, the counter-rotating beams blow-up with a corresponding saturation in luminosity.

2) Beyond a threshold in current, the beam lifetime drops, more-less drastically till the beam is lost, indicating that a large fraction of particles reach the physical or the dynamical aperture of the ring.

In both cases, the overall integrated luminosity is severely affected, and indeed, most machines have been disapointing from this point of view. Many models[1) 2) 3)] have been developped and have contributed to some understanding but none has really established itself by its ability to match or predict the experimental observations. Furthermore, the different mechanisms proposed are usually only relevant to some aspect of the beam-beam problem at the time, whereas in a real machine, the various features of the behaviour are intermingled, rendering the experimental investigation essential, interactively with the modelling effort. One example of this lack of completeness in our picture is the old controversy between coherence and incoherence as a dominating feature of the problem. In the case of flat-beam operated e^+e^- storage rings, it is here shown that one effect -the apparence of a strongly blown-up vertical dimension, in particular in the tails- can be explained with quite some success in the frame of incoherence, by the simplest model possible : the two dimensional single-resonance model.

THE SINGLE RESONANCE APPROACH

In each intersection region, the bunches composing the two counter-rotating beams interact electromagnetically. Assuming that one of the beams is much stronger than the other, remaining thus unperturbed in the interaction or that incoherence dominates the problem, the motion of individual particles can be considered (this will be developped later). In this model, the motion of a single particle in one of the beams, colliding with a counter-rotating rigid beam, is investigated.

Radiation damping and diffusion from quantum fluctuations are not included in the analysis as the resonant growth we study is a much faster process (this will also be developped later). The motion can thus be described with the following hamiltonian :

$$H(X,\dot{X},Y,\dot{Y},\theta) = H_0 + H'(X,Y,\theta) \tag{1}$$

where H represents the unperturbed (single beam) motion throughout the ring and H' the perturbation due to the beam-beam interaction, given by :

$$H'(X,Y,\theta) = V_0 \sum_{k=-\infty}^{+\infty} e^{-ikS\theta} \int \frac{1-e^{-\left(\frac{X^2}{1/f+T} + \frac{Y^2}{f+T}\right)}}{\sqrt{(1/f+T)(f+T)}} dT \tag{2}$$

in which $X = x/\sqrt{2\sigma_x\sigma_y}$, $Y = y/\sqrt{2\sigma_x\sigma_y}$ are dimensionless transverse coordinates, S the superperiodicity and $f = \frac{\sigma_y}{\sigma_x}$ the aspect ratio of the beams ($f = 1$ for round beams, $f \ll 1$ for flat beams).

Expression (2) is the product of the electromagnetic potential deriving from a gaussian charge distribution, as calculated in[15,16], and of a series of δ-functions accounting for the succession of localized kicks, in a thin lens approximation.

The strength of the perturbation is contained in V_0. It is expressed in eq.(3) in terms of the superperiodicity S and the main physical parameter of the beam-beam interaction : ξ i.e. the linear tune shift per interaction (assuming equal tune shifts in the two planes : $\xi_x = \xi_y = \xi$).

$$V_0 = S\,\xi^* = \frac{S\,r_e}{2\pi\gamma}\;\frac{N\,\beta_y}{\sigma_y(\sigma_x + \sigma_y)} \tag{3}$$

in which N is the number of particles per bunch, r_e the classical electron radius and γ the relativistic factor.

The unperturbed problem represented by H_0 is integrable. Its sol-

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* note : for derivation, eq. (12) can be used at vanishingly small amplitudes ; the exact expression of the tune shift ξ is described for ex. in [21]

ution is the familiar betatron motion[4] (dimensionless) :

$$\begin{cases} X = \sqrt{(1+1/f)\alpha_x} \quad \cos(Q_x\theta+\phi_x) \\ Y = \sqrt{(1+f)\alpha_y} \quad \cos(Q_y\theta+\phi_y) \end{cases} \tag{4}$$

in which $Q_{x,y} = \frac{1}{2\pi}\int_0^L \frac{ds}{\beta_{x,y}(s)} \simeq \frac{R}{\beta_{x,y}}$ are the betatron tunes and $\{\alpha_x, \alpha_y, \phi_x, \phi_y\}$ are dimensionless constants of the motion. Allowing these constants to vary with time when writing the equations of hamilton with the full hamiltonian of (1) yields :

$$\begin{cases} \frac{d\alpha_x}{d\theta} = -\frac{\partial H'}{\partial \phi_x}\left[X(\alpha_x,\phi_x,\theta),\ Y(\alpha_y,\phi_y,\theta),\theta\right] \\ \frac{d\phi_x}{d\theta} = \frac{\partial H'}{\partial \alpha_x}\left[X(\alpha_x,\phi_x,\theta),\ Y(\alpha_y,\phi_y,\theta),\theta\right] \end{cases} \tag{5}$$

and similar equations for the y-motion. $\alpha_{x,y}$ now have the physical significance of "emittance-variables". They are normalized to the sum of the natural beam emittances in the two planes :

$$\alpha_{x,y} = n_{x,y}^2 \ \frac{\varepsilon_{x,y}}{2(\varepsilon_x+\varepsilon_y)} \quad \text{with } \varepsilon_{x,y} = \frac{\sigma_{x,y}^2}{\beta_{x,y}} \tag{6}$$

Furthermore, the use of the canonical variables $\{\alpha_{x,y} ; \phi_{x,y}\}$ has eliminated the integrable part from the equations : (5) is now a set of four equations of Hamilton with :

$$H'(A_x,A_y,\phi_x,\phi_y,\theta) = S\xi \sum_{k=-\infty}^{+\infty} e^{-ikS_x\theta}$$

$$\int_0^\infty dT \ \frac{1-e^{-\left[\alpha_x \frac{\sqrt{f+1}}{\sqrt{f+T}}\cos^2(Q_x\theta+\phi_x) + \alpha_y \frac{f+1}{f+T}\cos^2(Q_y\theta+\phi_y)\right]}}{\sqrt{(1/f+T)(f+T)}} \tag{7}$$

As shown in[5], it is preferable to avoid expanding (7) in a polynomial (<u>alternating</u>) series and truncating it since the convergence is very slow. However, it is easy, using the following expansion into modified Bessel functions :

$$e^{-x\cos y} = I_o(x) + 2\sum_{n=1}^{\infty} (-1)^n \cos(ny)\; I_n(x) \tag{8}$$

to develop (7) in a series of <u>resonant terms</u> :

$$\begin{aligned} H' = S\xi \sum_{k=-\infty}^{+\infty} e^{-ikS\theta} \Big[& H_{oo}(\alpha_x,\alpha_y) + 2\sum_n H_{no}(\alpha_x,\alpha_y)\; \cos\, 2n(Q_x\theta+\phi_x) \\ & + 2\sum_m H_{om}(\alpha_x,\alpha_y)\; \cos\, 2m(Q_y\theta+\phi_y) \\ & + 4\sum_{nm} H_{nm}(\alpha_x,\alpha_y)\; \cos\, 2n(Q_x\theta+\phi_x)\cos\, 2m(Q_y\theta+\phi_y) \Big] \end{aligned} \tag{9}$$

in which :

$$\begin{aligned} H_{oo}(\alpha_x,\alpha_y) &= \int_o^{\infty} \frac{1 - e^{-\left(\frac{\alpha_x}{2}\frac{1/f+1}{1/f+T} + \frac{\alpha_y}{2}\frac{f+1}{f+T}\right)} I_o\!\left(\frac{\alpha_x}{2}\frac{1/f+1}{1/f+T}\right) I_o\!\left(\frac{\alpha_y}{2}\frac{f+1}{f+T}\right)}{\sqrt{(1/f+T)(f+T)}}\, dT \\ H_{nm}(\alpha_x,\alpha_y) &= (-1)^{n+m-1}\int_o^{\infty} \frac{e^{-\left(\frac{\alpha_x}{2}\frac{1/f+1}{1/f+T} + \frac{\alpha_y}{2}\frac{f+1}{f+T}\right)} I_n\!\left(\frac{\alpha_x}{2}\frac{1/f+1}{1/f+T}\right) I_m\!\left(\frac{\alpha_y}{2}\frac{f+1}{f+T}\right)}{\sqrt{(1/f+T)(f+T)}}\, dT \end{aligned} \tag{10}$$

The single-resonance approximation consists in keeping only <u>one</u> of the oscillating terms in (9), assuming that the corresponding phase is nearly constant, which in the case of a resonance of order $2|n| + 2|m|$ occurs when :

$$2n\, Q_x + 2m\, Q_y - kS = 2\Delta q \ll 1 \tag{11}$$

and assuming that all other resonant terms of "low order" oscillate fast enough to even out over a few turns. A slow beating of the variables then dominates the behaviour and can be calculated as the motion becomes <u>integrable</u>. Obviously, no résonance is really isolated as a high order resonance line can always be found very close to any point in túne diagram. However, an integrable approximation of a non-integrable problem can sometimes be quantitatively very good (see last paragraph).

An indication of the relative strengths of the resonances is contained in the order of magnitude of each exciting term H_{nm} normalized to the leading term H_{oo}, i.e. H_{nm}/H_{oo}. For typical amplitudes, this ratio gets smaller and smaller as the order of the corresponding resonance is increased, due to the fast decrease with order of the modified Bessel functions in (10). For example $H_{11}/H_{oo} \sim 10^{-3}$. The weak resonance excitation produces a distorsion of the phase space trajectories in such a way that the motion of resonant particles gets trapped in islands rather than turning on the usual circles, as shown in figure 1 for a 6^{th} order one-dimensional resonance.

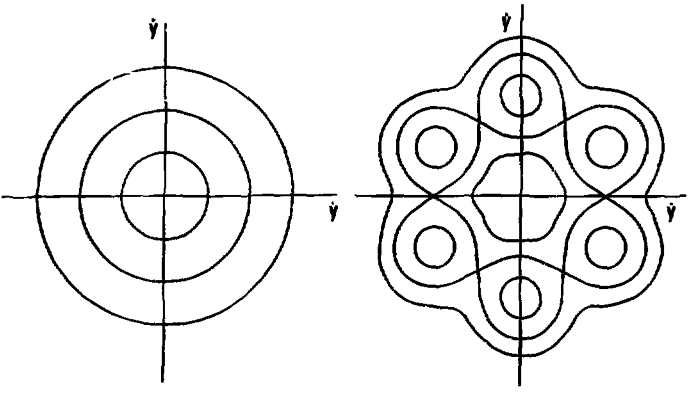

Fig. 1 : Distorsion of phase space in the vicinity of a 6^{th} order one-dimensional resonance

The leading term H_{oo}, present even when all resonances are far away, is responsible for the amplitude-dependent detuning produced in the beam-beam interaction, as can be seen from :

$$\frac{d\phi_{x,y}}{d\theta} = \frac{\partial H'}{\partial \alpha_{x,y}} \simeq S\,\xi\,\frac{\partial H_{oo}}{\partial \alpha_{x,y}} = S\Delta Q_{x,y}(\alpha_x,\alpha_y) \tag{12}$$

This results in a spread of the tunes that transforms the working point into an area of size $S\xi$ in tune diagram, as shown in figure 2a,b) for round and flat beams[2]. In order to obtain a large luminosity, the biggest tune spread possible has to be fitted into a region free of low order resonance lines (Fig. 2.c). The effect, when the tune spread overlaps a resonance line, is that islands will appear in phase space for particles close to that line, at amplitudes such that their effective tunes ($Q_{x,y} + S\Delta Q_{x,y}$) are exactly on the line. The smallness of the islands for relevant amplitudes has led one to consider these beam-beam excited resonances unable to explain, alone (i.e. without any additional enhancing mechanism[2]), the rather strong effects observed. We shall see in the next paragraph that things are different in the case of very flat beams.

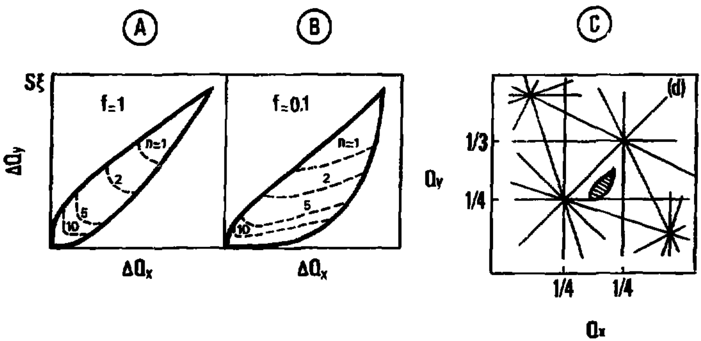

Fig. 2 : Tune-spread for round (a) and flat (b) beam (taken from Ref.[2]). Fitting of the flat beam tune-spread in a region free of "low order" resonance lines in tune space (c). The dotted lines indicate successive beam "enveloppes" i.e. particles such that $x^2/\sigma_{x^2} + y^2/\sigma_{y^2} = n^2$.

NON-LINEAR COUPLING IN A VERY FLAT BEAM

The sensitivity of vertical dimensions in the very flat beams stored in e^+e^- machines has been the subject of much study[10,17,18,19] and worry. The role of non-linear coupling, first stressed by B. Montagne[6] as a relevant mechanism, can be investigated with the above described procedure, yielding, in the case for example of the non-linear resonance of lowest order :

$$2Q_x - 2Q_y - kS = 2\Delta q \ll 1 \tag{13}$$

the following hamiltonian :

$$H'_{RES} = S\xi[H_{oo}(\alpha_x,\alpha_y)+2H_{11}(\alpha_x,\alpha_y)\cos(2n(Q_x\theta+\phi_x)+2m(Q_y\theta+\phi_y)-kS\theta)] \tag{14}$$

Integration is easily performed as two invariants of the motion can be derived. The first :

$$C_1 = \alpha_x + \alpha_y \tag{15}$$

is intuitive since it expresses the exchange of energy between the two coupled planes. It also expresses the stability of this difference resonance. Further eliminating the time-dependence in (14) and using the first invariant of (15) yields a new one-dimensional hamiltonian, only function of the natural coupled variable of the problem :

$$K = \alpha_x - \alpha_y \tag{16}$$

and of a "slow phase" :

$$\psi = \frac{\Delta q}{2}\theta + \frac{\phi_x - \phi_y}{2} \tag{17}$$

$$H_{res}(K,\psi) = S\xi[H_{oo}(K) + 2H_{11}(K)\cos(4\psi)] + \frac{\Delta q}{2}K\ ^{*} \tag{18}$$

This hamiltonian is now a constant of the motion i.e. a second invariant. The sensitivity of the vertical plane can be understood from it. Since the smallness of the excitation, due to the smallness of the driving term H_{11} ($H_{11}/H_{oo} \sim 10^{-3}$ for typical amplitudes), must be looked at in terms of $K = \alpha_x - \alpha_y$, and since $\alpha_y \ll \alpha_x$ ($\alpha_y/\alpha_x \simeq \varepsilon_y/\varepsilon_x = f = 1/16$ in the case of the LEP design[7]), a small resonant beating of K, having to satisfy the invariance of $C_1 = \alpha_x + \alpha_y$, will translate into a much stronger one in α_y. At the contrary, beating in α_x will be weaker.

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* note : All the manipulations of the hamiltonian can of course be carried on, more concisely, with canonical transformations.

For example, an increase due to coupling of 40 % in the vertical amplitude will only mean a 2.5 % decrease horizontally, at one σ in both planes. This would be difficult to observe in the horizontal plane. An illustration is given in figure 3, where the transformation due to coupling of successive "envelopes" of the distribution (particles such that $x^2/\sigma_x^2 + y^2/\sigma_y^2 = n^2$; $n = 1,2,\ldots$) is shown.

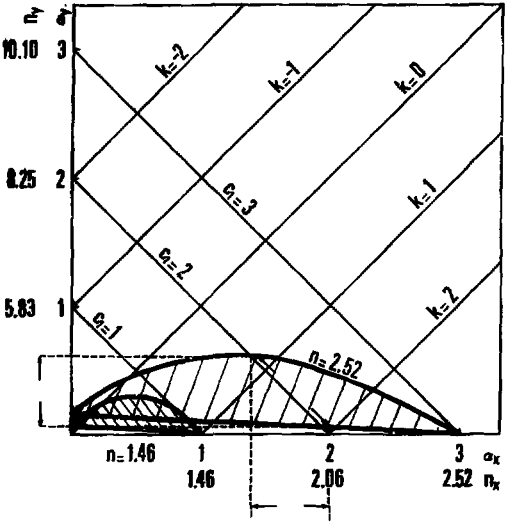


Fig. 3 : Transformation due to non-linear coupling of successive "envelopes" of the beam distribution, i.e. of particles such that $x^2/\sigma_x^2 + y^2/\sigma_y^2 = n^2$; $n = 1,2$. The variables of the diagram are the "emittance variables" (a_x, a_y) defined in eq. (6). The corresponding number of sigmas (n_x, n_y) are also indicated. Particles trapped in the resonance beat on lines of constant C_1, the variable K oscillating between two extremum values. The corresponding beating is much larger vertically and weaker horizontally, in the case of very flat beams (aspect ratio f = 1/16) as can be seen on the scales of (n_x, n_y).

Calculating numerically the integrals involved in (10), the transformations of beam envelopes for $n = 1$, 1.41, 2 and 3 are obtained graphically from invariant (18), in a way illustrated in figure 4.a. The particles move on lines of constant H_{RES} within a sector limited by two curves corresponding to $\cos 4\psi = \pm 1$. Depending on initial conditions, these lines are limited by both curves (case A), the angle ψ describing all values and the corresponding particle rotating uniformly in phase-space, or by only one (case B), the angle ψ being now limited to a restricted set of values corresponding to a particle the motion of which is trapped in an island. The representation in figure 4.a is equivalent to the usual island structure appearing in the phase space of a resonant trajectory (Fig. 4.b). Estimates of the maximum beating in K are obtained from it.

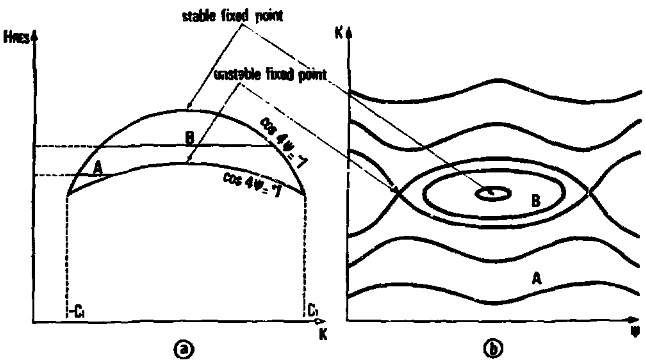


Fig. 4 : Resonant phase-space illustrating usual phase oscillation (A) and resonant trapping (B). The fig. in (a) and (b) are equivalent.

The calculations were performed at three different working points close to the resonance line, as shown in figure 5, for a tune shift $\xi = 0.03$ and a superperiodicity $S = 4$.

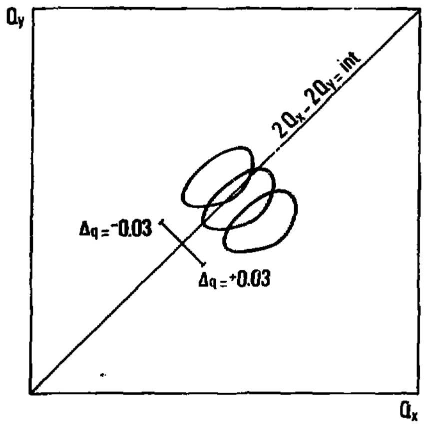
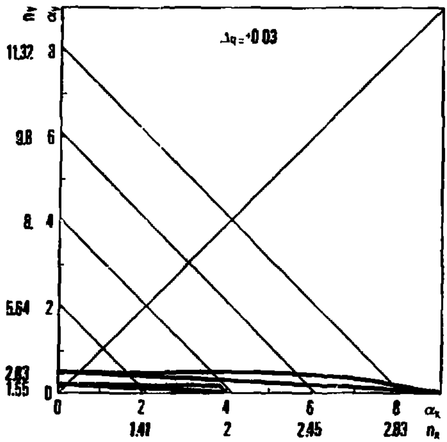


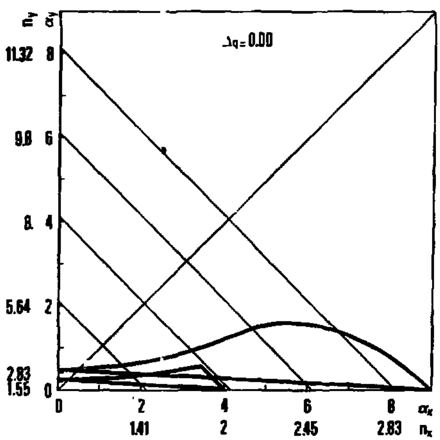
Fig. 5 : Three working points close to the resonance studied, and for which the calculation was performed are here shown with their corresponding tune-spreads.

Figures 6 to 8 show how beam envelopes for $n=2$ and 3 ($n^2=x^2/\sigma_x^2+y^2/\sigma_y^2$) are transformed when passing through the resonance. Very clearly, the transfer from horizontal to vertical motion is big in the tails, especially when $\Delta q=-.03$. This is consistent with the preferred side predicted in[6], observed experimentally at PETRA[9] and easily understood (see also[2]) from the almost triangular shape of the tune-spread in the case of a flat beam (Fig. 2.b and 5), allowing the resonance line to be crossed more efficiently from just above than from below.

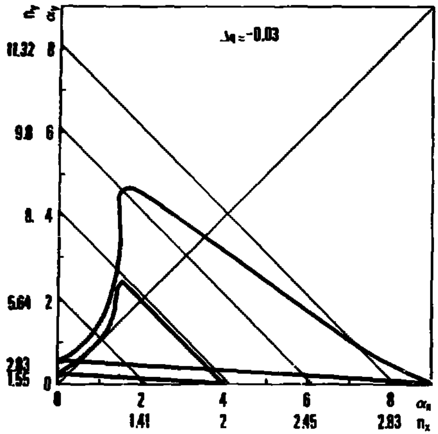
A magnification (Fig. 9) for smaller amplitudes of figure 8 shows that a $n=1$ envelope, corresponding to the core of the beam is not effected very much, whereas the excitation is already strong for $n=1.41$. This suggests that the resonance drives particles mostly on the edges of the core ($n=1.4$ to $n=3$) and in the tails ($n \gtrsim 3$).



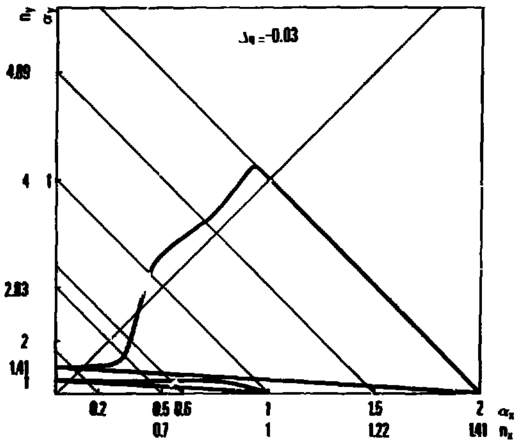
<u>Fig. 6</u> : Transformation of beam envelopes below the resonance (Δq = + 0.03)



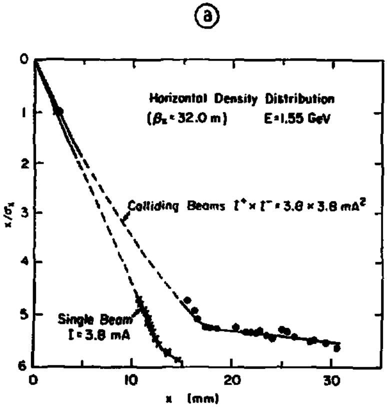
<u>Fig. 7</u> : Transformation of beam envelopes right on the resonance (Δq = 0.00)



<u>Fig. 8</u> : Transformation of beam envelopes above the resonance (Δq = - 0.03)



<u>Fig. 9</u> : Magnification of figure 8 for smaller amplitudes



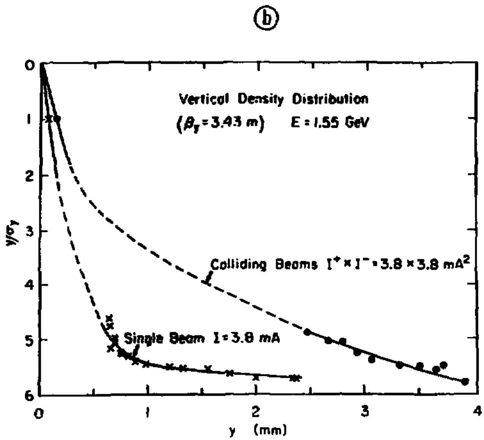


Fig. 10 : Horizontal (a) and vertical (b) particle density distributions as measured in SPEAR[10]. A strong blow-up in the vertical tail is clearly seen.

Of course, the biggest blow-up being in the tails, the global blow-up of the beam, which can be obtained through a convolution over the whole distribution, is weaker, as was pointed out by S. Kheifets[9]. However, it does not limit the relevance of these results since the blow-up observed is mostly in the vertical tails, as was observed for example at SPEAR[10] (see Fig. 10). A similar observation can be found in[11]. In figure 10, more points, at 2 and 3 σ_y are needed but difficult to obtain through an experiment based on scrapers. A study for different tunes would also be useful.

WHERE DOES THE MODEL BREAK DOWN ?

1) Damping and quantum excitation were not included, which is fine if the characteristic time for the coupled motion is much shorter than the damping time. An estimate of the beating time is obtained using[18] and Hamilton's equation :

$$\frac{dK}{d\theta} = - \frac{\partial H_{RES}}{\partial \psi} = - 8 H_{11} (K) \sin 4\psi \tag{19}$$

$$\Delta\theta \sim \frac{- \Delta X}{8 H_{11} (\bar{K}) \overline{\sin 4\psi}} \tag{20}$$

where the bar means a "typical" value is taken. As shown in figure 11, $\overline{\sin 4\psi} \simeq 1/2$ is rather characteristic as an intermediate between the largest time (infinite) corresponding to the stable fixed point ($\sin 4\psi = 0$) and the shortest corresponding to usual phase oscillations ($\sin 4\psi = 1$). The number of turns amounts to :

$$n_T = \frac{\Delta\theta}{2\pi} \sim .024 \frac{C_1}{S\xi y} \frac{1}{H_{11}\left(\frac{C_1}{2}\right)} \sim 20 \text{ or } 30 \text{ turns}$$

to be compared with damping times of 3000 turns for PETRA and 500 turns for LEP. This is not where the model breaks down.

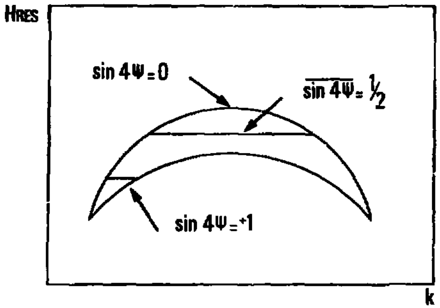


Fig. 11 : Calculation of a characteristics beating time for the resonance studied ; as shown, $\sin 4\psi \simeq 1/2$ is a rather good average between the two extreme values of $\sin 4\psi = 0$ (fixed point) and and $\sin 4\psi = 1$ (usual phase oscillations).

2°) A thin lens approximation was used, permitting the use of δ-functions for the time dependence, instead of an integration of the kicks through the longitudinal extension of the bunch. This is valid if $\beta > \sigma_s$ as shown in[20].

3°) Synchrotron oscillations would be interesting and easy to include. Effects from synchro-betatron satelittes could then be studied, with the limitation that the resonance approximation may break down more drastically as the resonance line density is increased (see 6°) for this point).

4°) Errors in phase advance and/or Beta-functions between the different interaction regions break up the symmetry of the machine (for example for PETRA, $S = 4$ becomes $S = 1$) which is also equivalent to an increase in the resonance line density. This was not included here but could, with however a similar limitation as in 3°).

5°) Weak-strong picture for flat beams :

In the limit of a completely flat beam ($f = \sigma_y/\sigma_x = 0$), an infinite plane with a charge density σ can be considered. It is easily found from symmetry arguments (or from Gauss's th) that the electric field generated is vertical and independent of the distance to the plane : as the plane is infinite, an observer cannot tell, just looking at it, how far it is.

In reality, as the beam is almost flat, the electromagnetic field will only be almost independent of the vertical dimension y, as long as one is not in the core (see Fig. 12).

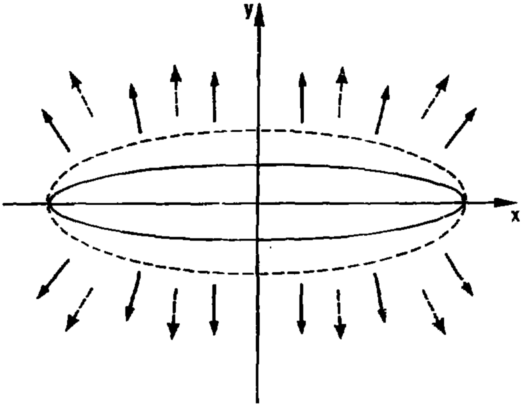


Fig. 12 : Electromagnetic field distribution from a very flat beam. As the fields are almost independent of y in a large region outside of the core, they will not be affected much if the beam blows up vertically by a factor 2.

Now, reversing the argument, one finds that vertical blow up (of a factor 2 for example) of a very flat beam will not affect the field much outside the core (Fig. 12, dotted line). This can also be seen looking at the vertical kick (Fig. 13) for a flat beam.

The leveling off of the kick occurs at a couple of σ_y for round beams, but between 10 and 20 σ_y for a ribbon beam. When the beams get blown-up this leveling off changes location, producing quite a different kick from the round beam, but roughly the same from the flat beam. As an illustration (Fig. 13), a particle oscillating at, say, $3\,\sigma_y$ will be kicked by roughly the same amount from a non blown-up beam as from one blown-up by for example a factor 2.

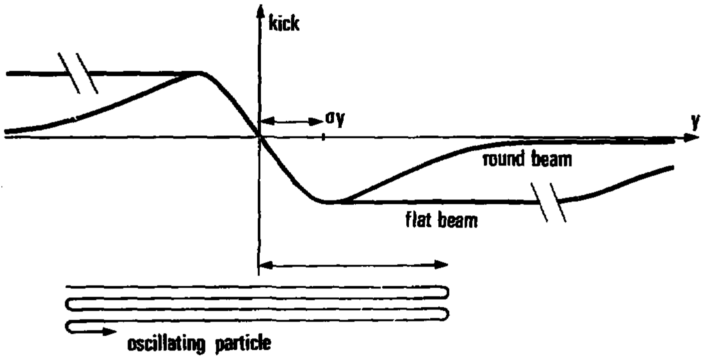


Fig. 13 : As the levelling off of the vertical kick occurs very far away (10 or 20 σ_y) for a ribbon beam, a particle oscillating at a couple of σ_y will get roughly the same kick from a blown-up or a non-blown up distribution.

More analytically, an inspection of Augustin's formula[12] for the flat beam vertical kick also shows that the kick is roughly independent of σ_y :

$$\delta\theta_y \sim \frac{y\,e^{-x^2}}{\sigma_y(\sigma_x+\sigma_y)} \int_0^1 e^{-\frac{y^2 v^2}{2\sigma_y^2}}\,dv$$

$$\delta\theta_y \sim \frac{y\,e^{-x^2}}{\sigma_y(\sigma_x+\sigma_y)} \int_0^{\frac{y}{\sigma_y}} e^{-\frac{u^2}{2}}\,du\,\frac{\sigma_y}{y}$$

$$\delta\theta_y \sim \frac{e^{-x^2}}{\sigma_x} \int_0^{\frac{y}{\sigma_y}} e^{-\frac{u^2}{2}}\,du, \text{ as } \sigma_y \ll \sigma_x$$

$$\delta\theta_y \sim \frac{e^{-x^2}}{\sigma_x} \quad \text{if } y \gtrsim 1.5\,\sigma_y$$

From the above argument, the beam-beam interaction in the case of very flat beams may, for many of its observed aspects, be considered a quasi-weak-strong process even when the two beams are equally strong*. Although this argument is not valid in the core and far out in the tails of the beam, it strengthens the validity of the incoherent picture for at least some beam-beam effects such as lifetime problems, backgrounds and of course vertical blow-up. It also emphasizes the difference between flat and round beams in the frame of the beam-beam interaction.

6°) The single resonance approximation is the most severe limitation. Approximating a non-integrable problem by an integrable one is qualitatively false in the sense that the fundamental "chaotic" property of the system, -i.e. the production, due to the mixing with nearby resonances, of unpredictable chaotic trajectories- is, a priori, suppressed.

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* note : This remark was initiated by H. ZYNGIER.

However, the result can be quantitatively very good (see [13]) up to a threshold in perturbation strength (tune-shift in our case).

"How isolated" the resonance is, and up to what threshold, is now being tested[14] with a simple two-dimensional tracking program, calculating the two invariants of (14) and (15) over a few hundred turns and checking their invariance for different tune-shifts.

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