4. US National conference on pressure vessel and piping technology
San Antonio; TX (USA) 17-21 Jun 1984
CEA-CONF--7326

APPRAISAL OF ALLOWABLE LOADS BY SIMPLIFIED RULES

D. MOULIN, R.L. ROCHE, C.E.A. Saclay

Abstract

This paper presents a simplified method of analysis of buckling of thin structures like those of L.M.F.B.R's. The edification of the method is very similar to methods used for buckling of beams and columns having initial geometric imperfections and buckling in the plastic range.

Particular attention is paid to the stress hardening of material involved (austenitic steel) and to possible unstable post buckling of thin structures.

The analysis method is based on elastic calculation and diagrams that take into account various initial geometric defects.

1 - INTRODUCTION

Mechanical structures like those of L.M.F.B.R. that do not bear particularly high pressure can be built with rather thin plates and shells. As a consequence, this may result in such slenderness that buckling is very likely to occur if large compressive stresses are present.

Buckling may involve modifications of structure shape that are far enough from the initial one to compromise a satisfactory behavior.

Therefore, careful analysis of buckling behavior of LMFBR structures is necessary. These analysisesare very difficult and costly, and it is desirable to perform them, only in the final design analysis. At the beginning it is very useful to be able to make simplified analysis in order to make a good choice of shape and thickness of shells.

The aim of this paper is to give indications on such a simplified method for preliminary analysis of buckling of shells. This method is very similar to methods used for columns having initial geometric imperfactions and buckling in the plastic range (1).

2 - DESCRIPTION OF THE PROBLEM

Structures made of thin shells cannot, in general, be constructed exactly in accordance with nominal dimensionnal specifications. This results in modifications , sometimes important, in the stress distribution. In effect, the nominal geometrical drawings of the structure are such that loads are sustained without occurence of bending stress. But if the shape is not strickly perfect, because of unavoidable fabrication tolerances, bending stresses are produced. These bending stresses can be far higher than the membrane ones that were strictly necessary for loading resistance if the shape had been perfect.

This amplification of stress due to fabrication tolerance can be still increased by the new shape modifications produced by the loading of the structure. The interactive effect of initial defects and their amplification by loading, may result in the complete destruction of the structure by instability. In such situations it is worth noticing that plastic behavior play an important role.

3 - PREVIOUS STUDIES. DIAGRAM PRESENTATION

The stability of membrane structures had been very firmly studied since the beginning of 19° century . A more recent book has been used for a long time as a reference one for buckling problems : it is the one of TIMOSHENKO (2).

The main part of the work is devoted to elastic stability, this means to classical bifurcation analysis. Nevertheless practical problems revealed that plastic phenomenon cannot be neglected. Since 1889, ENGESSER (3) proposed a simple method to take it into account. This method consisted in the use of the stress-strain curve for non linear elasticity analysis. The subsequent work of VON KARMAN (4) and of SHANLEY (5) permitted to increase the efficiency of the method and demonstrated very clearly that it gives acceptable conservative estimation of the real critical load.

However, all these works dealt with perfect or as-perfect structures. It was demonstrated that geometrical defects play an important role. To sum up, all these works together with those of BAUSCHINGER (6) (1889), CONSIDERE (7) (1889) and TETMAJER (8) (1903) show that, if it is taken into account plasticity, and initial geometrical defects amplified by imposed loading, the allowable loads can be estimated by a diagram that has the presentation given by figure 1, where

- σ_E is the stress corresponding to elastic bifurcation calculation
- Su is the ultimate tensile stress
- Sy is the elastic limit stress
- σ_{may} is the allowable stress

This presentation which is very close to a two criteria approach like the one proposed by TOWNLEY (9) in fracture mechanics field, is the one adopted in the present paper.

The main difficulty in the construction of such a diagram is to take into account the stress hardening of materials, like austenitic steel, that are mainly concerned in practical problems. In addition, it must be noticed that instability during post-buckling does not always show the same indifference characteristic like in classical cases as underlined by KOITER (10).

4 - BASIS OF THE METHOD USED TO CONSTRUCT REDUCTION LOAD DIAGRAM FOR BUCKLING

More detailed indications on the sources used to set up this method can be found in (11). It is presented here the equations used to draw the reduction load diagram for buckling. The effects taken into account in the formulation concern : defect amplification during loading, stress-hardening of the materiel, reduction of critical load due to instability, amplification of bending stress and distribution of plastic stress through the thickness.

4.1 - Evolution of the defect under loading

This problem is well known. It is stated here that the fundamental mode is predominant. As a consequence of this, the geometrical defect, noted f is increased in accordance with an hyperbolic law which is given, if buckling is a stable process, by the following equation :

$$f = \frac{fo}{1 - \frac{\sigma}{\sigma_c}} \qquad (1)$$

where -fo is the initial defect (when the load is not applied)

- σ corresponds to the applied load
- σ_c correspond to bifurcation load (critical stress)

In such a situation the curves giving the evolution of geometrical defect as a function of the ratio $\frac{\sigma}{\sigma_c}$ have schematic shapes given by figure 2.

4.2 - Critical stress

By reference to the evolution of defect presented in the preceeding section, it is clear that the critical load can only be reached with a non finite defect. In other words, its value defines the equation of the assymptotic line of the defect-load curve.

The value of the critical load is here fixed from the elastic bifurcation analysis (that gives the stress noted σE) reduced by two coefficients noted Ap for plasticity and Ai for instability. This can be written in the following equation :

$$\sigma C = \sigma E \times Ap \times Ai \qquad (2)$$

4.2.1 - Elastic bifurcation stress σE

The value of the elastic bifurcation stress noted σE is calculated with the nominal geometrical configuration of the structure (the defect value is zero) and it corresponds to the fundamental mode. This value can be found in classical handbook like (12) for various shapes and types of loading.

4.2.2 - Reduction coefficient for plasticity. Ap.

The first proposal for elastic plastic bifurcation was made by ENGESSER (3). He proposed to use the tangent modelus Et in lieu of YOUNG's modulus to appraise the critical load of columns. This way is very conservative for structures considered here, therefore it was chosen the recommandations of GERARD (13) for spherical vessels under external pressure and thin cylinders under axial compressive loads (these two cases are among the worse ones, as far as post buckling behaviour is concerned). This method is well documented and experimentally verified (14) (15) in some cases. Then the Ap value is :

$$Ap = \left(\frac{1-v^2}{1-\mu^2} \; Es \; Et \right)^{1/2} \frac{1}{E} \qquad (3)$$

where $\mu = 0,5 - (0,5-v) \dfrac{Es}{E}$

- v is the elastic Poisson ratio
- E the Yound's modulus
- Es and Et are respectively the secant and the tangent modulus obtained on the stress-strain curve of the material for a "reference stress" noted σ^* (see figure 3). The meaning of this quantity is explained here after and is chosen in order to give an acceptable definition of plasticity

distribution through the thickness. As an illustration of this effect figure 4 presents the curve giving, for an an austenitic steel at room temperature, the value of the coefficient Ap as a function of the ratio $\frac{\sigma^*}{Su}$.

4.2.3 - Reduction coefficient for instability. Ai.

The amplification of the defect value, because of instability in buckling process, is introduced in a manner very similar to plasticity.

If buckling is unstable the evolution of defect with load cannot be represented by the illustration proposed in the classical elastic bifurcation analysis (see figure 2). In this case, when a load, noted σ_i, is reached (σ_i is less than σ_E), it is no longer necessary to increase the load to increase the defect. The defect-load curves are represented by the curves presented in figure 5.

Due to the simple formulation adopted, the reduction of critical load due to instability is to be taken into account by an equation of the form :

$$f = \frac{fo}{1 - \frac{\sigma}{\sigma_E . Ai}} \qquad (4)$$

where Ai is the coefficient of reduction. This means that the critical stress (obtained for an infinite value of the defect) is not the elastic bifurcation one : σ_E but Ai σ_E. The assymptotic line is vertical. Therefore the corresponding adopted defect-load curve is given by the figure 6.It can be compared with a curve corresponding to the stable path and the "real" unstable path that is to be taken into account in a conservative manner. The problem is to choose the Ai value, so that, for a given load value, the defect value used in the calculation, is higher than the real one, in practical design problems.

To reach this goal it was taken into account the reduction of elastic bifurcation stress due to a representative unstable process in buckling. A litterature survey told us that this Ai value can be generally given as a function of initial defect value by an equation of the form :

$$(1 - Ai)^2 = K \frac{fo}{e} Ai \qquad (5)$$

where - Ai is the ratio $\frac{\text{unstable elastic bifurcation stress}}{\text{stable elastic bifurcation stress}}$

- fo is the value of the initial defect
- e is the value of thickness
- K is a coefficient depending on the mechanical configuration concerned (shape of the shell, type of loading, interaction of different buckling modes).

The value of the K coefficient retained, corresponds to the elastic bifurcation analysis of a cylinder under axial compression made by KOITER (16)

$$K = \frac{3}{2} \sqrt{3(1-\nu^2)} \quad \text{which is close to } 2,5$$

As an example of the comparisons made, it is easy to show that in the case of sphere under external pressure calculated by HUTCHINSON (17) for a square mode combination) the coefficient value is $\frac{27\sqrt{3}}{32}$ which is close to 1,5.

The curves drawn in figure 7 give another illustration of the comparison.

Therefore the coefficient of reduction due to instability is defined by the equation :

$$Ai = \frac{1}{2} \left(2 + 2,5 \frac{fo}{e} - \left(\left(2+2,5 \frac{fo}{e}\right)^2 - 4\right)^{1/2} \right)$$

which corresponds to the resolution of the equation (5) with a K value equal to 2,5.

It must be here recalled that two kinds of diagrams are to be proposed. The first one intends to take into account the reduction due to instability by the coefficient Ai. The second one can be used only in the case where it may be justified that buckling is not unstable (in this case Ai = 1).

4.3 - Amplification of bending stress

It is well known that for the structures, the shape of which are close to perfect ones, the bending moment M is equal to membrane stress N multiplied by the geometrical defect as shown in the figure 8.

According to shell theory and with an hypothesis of complete plastic distribution through the thickness, the bending stress σb created by the geometrical defect is given by the following equation :

$$\sigma_b = \sigma_m \cdot 4\frac{f}{e} \qquad (6)$$

4.4 - Reference stress

As quoted before, it is necessary, now, to introduce a stress quantity that must permit to give an acceptable evaluation of the plasticity in the thickness concerned. This quantity is used to define the coefficient of reduction for plasticity Ap (see figure 4). According to the principles of simplification stated before, it is taken a global formulation that usually defines the membrane stress-bending stress relation corresponding to a limit loading.

$$\sigma^* = |\sigma m| + |\sigma b| \qquad (7)$$

5 - DETERMINATION OF DIAGRAMS

Putting together all the equations (1) (2) (6) and (7), established to take into account all the particular effects identified, and eliminating the quantities σ_b and f, the fundamental following equation is obtained

$$\sigma^* = \sigma m \left(1 + \frac{4\frac{fo}{e}}{1 - \frac{\sigma m}{ApAi\, \sigma_E}} \right) \qquad (8)$$

This equation describes all the equilibrium states of the structure, i.e. all the stable states. In other words, if it is known :

- the perfect initial geometry of the structure (therefore the value of σE)
- the initial defect value fo/e
- the material relevant behavior giving the Ap value as a function of σ^* value

it is possible to deduce the maximal allowable stress σm for the structure. Let us call it σm max. Then a diagram giving σm max as a function of σE can be drawn (see figure 1). A diagram can be drawn from each material stress-strain curve and is also a function of the initial defect value. The diagram, drawn according the fundamental equation (8), describes the domain of stability.

In this domain the maximal value of σm is obtained for a given σE value and a given $\frac{fo}{e}$ value by finding the σ^{*} corresponding value. In other words σm max is obtained as the maximal value of σm obtained by the fundamental equation (8) when σ^{*} is varying between 0 and Su.

5.1 - An example of reduction diagram drawn for an austenitic steel

In order to present an example of determination of a buckling reduction diagram that take into account various initial defect values, it was chosen to draw it for an austenitic steel, the material Ap - σ^{*} characteristic of which is given in figure 4.

As an illustration the figure 9 gives the results for values of $\sigma E \geqslant 100$ MPa, obtained, when the instability reduction is applied and when not, for $\frac{fo}{e}$ equal to zero, 1 and 2.

It is worth noticing that reduction due to instability increases when defect increases.

5.2 - Comparison with other methods

It is always necessary to make some simplifications and hypotheses in order to have a method of analysis easy to use. However the resulting diagram must have to be drawn with a certain amount of conservatism because various practical problems must be covered. It is therefore advisable to evaluate the margin obtained by this method.

As an illustration of this, it is simply presented the comparisons made with results obtained by an inelastic descriptive buckling analysis (18) for two internal components of L.M.F.B.R. called V1 and V2. The component V2 was also submitted to experimental investigation and its experimental maximal load was determinated. In the comparison between elastic proposed method and inelastic one it is important to notice that the same stress-strain curve was used for material behavior characterization.

The results of the comparison are presented in the following table :

	M_I	M_S	$\frac{\sigma_E}{Su}$	$M_{I\ exp}$	$M_{S\ exp}$
V1	4,8	2,1	0,08		
V2	2,5	1,7	0,38	2,6	1,8

where M_I : $\frac{\text{Maximal load obtained by the proposed method (instable)}}{\text{Maximal load obtained by a descriptive inelastic method}}$

M_S : $\frac{\text{Maximal load obtained by the proposed method (stable)}}{\text{Maximal load obtained by a descriptive inelastic method}}$

$\frac{\sigma_E}{Su}$: $\frac{\text{elastic bifurcation membrane stress}}{\text{ultimate tensile stress}}$

$M_{I\ exp}$ and $M_{S\ exp}$: same definitions like in the calculation comparison

The maximal load is here obtained by experimental investigation.

These results clearly show that in both cases, where elastic bifurcation stress is rather low (σ_E/Su $\leq$ 0,4), the proposed method, based on diagram, is conservative wether buckling is judged stable or unstable but with larger safety margin in the last case.

Similar conclusions are obtained, if it is applied a somewhat different method to construct diagrams that takes into account the influence of defect in buckling (19). This method is very close to the one proposed here, except that plasticity effect is introduced in the inelastic, bending moment-curvature curve integrated from the stress-strain curve of the material, and that evolution of defect, in instable buckling case, is more precisely represented by the following equation :

$$\sigma_m \frac{f}{e} = \sigma_E \left(\frac{f - fo}{e}\right) + \mu'_1 \left(\frac{f - fo}{e}\right)^2$$

where the nomenclature is the same as before,
The value of $\mu'1$ is chosen according to similar criteria as those presented to choose the coefficient K used to determine the coefficient Ai (see 4.2.3). It intends to take into account, the same sensibility to imperfections in unstable buckling, as in a representative calculated configuration (cylindrical shell under axial compression).

As an illustration of the comparison between the proposed method and the one proposed in (19), allowable load, for defect value $\frac{fo}{e}$ = 1, is

5.3 - Recommendations

The study clearly reveals that the proposed method, based on elastic calculations and on correction of imperfection by a reduction diagram, produces conservative results. It is therefore possible to propose it, for a buckling rule to be used in preliminary design stage. The defect value must be evaluated from tolerance fabrications written on the drawings project.

The french construction code R.C.C.M.R. (Recueil des règles de conception et de construction des matériels mécaniques des ilots nucléaires RNR) (20) proposes a design rule based on diagrams obtained by the method presented here. This rule intends to prevent instability by the limitation of primary stress. This code also proposes a rule based on the diagrams that takes into account the interactive effect of cyclic secondary stresses with load-buckling. A background to this rule is presented in (21).

7 - CONCLUSIONS

- Thin shell structures of high slenderness must be designed according to buckling risk that may lead to instability.
- Geometrical imperfections, resulting from fabrication tolerances, play an important role in buckling process of thin structures.
- A method based on simplifications of real behavior, that permits a simple analytical formulation is presented.
- The method is usable in preliminary design.
- The reduction of buckling resistance (knock-down factor) due to geometrical imperfection is taken into account by diagrams.
- Two kinds of diagrams are proposed wether buckling is anticipated to be stable or unstable.
- A diagram can be drawn from the stress-strain material curve.
- Comparisons made, demonstrated that acceptable conservative results are predicted by the method and permit the formulation of a design buckling rule to limit primary stresses.

R E F E R E N C E S

(1) W.E. AYRTON, J. PERRY "ON STRUTS", Engineer 62 (1886), 464-465, 513-515

(2) S. TIMOSHENKO "Théorie de la stabilité élastique" Paris et Liège, Librairie Polytechnique Ch. Beranger (1943)

(3) F. ENGESSER
"Schweiz Bauzeitung" vol. 26, page 24 (1895)
et "Z. ver. deut. Ing.", vol. 42, page 927 (1898)

(4) Th. V. KARMAN
"Forschungsarbeiten" n° 81 (1910) Berlin

(5) F.R. SHANLEY, J. Aero. Sci, 14, 261 (1947)

(6) I. BAUSHINGER
"Mitt. mech. tech. Lab. tech. Hoschschule"
Münich n° 15 (1889)

(7) A. CONSIDERE
"Congrès International des procédés de construction"
Paris, vol. 3, page 371 (1889)

(8) L. TETMAJER
"Die Gesetze der Knickung und der zusammengesetzten Druckfestigkeit der technisch wichtigsten baustoffe"
3ème édition, Leipzig et Vienne (1903)

(9) C.H. TOWNLEY, A.R. DOWLING
The effect of defects on structural failure. A two "criteria approach" Int. Journ. of Pressure Vessel and Piping Vol.3 n°2
April 1975

(10) W.T. KOITER
"On the stability of elastic equilibrium"
These Delft, H.J. Paris Amsterdam 1945. American translation in NASA TT-F-10 833 (1967)

(11) D. MOULIN, R. ROCHE
"Estimation de l'effet des tolérances géométriques sur la tenue des coques minces"
Rapport EMT/SMTS/80/40 (22.10.1980)

(12) HANDBOOK OF STRUCGURAL STABILITY, edited by COLUMN RESEARCH COMMITTEE OF JAPAN, CORONA PUBLISHING COMPANY LTD.TOKYO (1971)

(13) GERARD G. HANDBOOK OF STRUCTURAL STABILITY - NACA-TN 3783 Aug. 1957

(14) GERARD G. "On the role of initial imperfection in plastic buckling of cylinders under axial compression"
J. Aerospace Sci, 29, p. 744.745 (1962)

(15) A.K. DHALLA, D.S. GRIFFIN "Verification of simplified design method for plastic buckling of cylinders"
paper L 1513, 6th SMIRT Conf. Paris (1981)

(16) W.T. KOITER "The effect of axisymmetric imperfection on the buckling of cylindrical shells under axial compression"
Koninkl. Ned. Akad. Weterschap Proc. B.66, 265-279 (1963)

(17) J.W. HUTCHINSON "Imperfection sensitivity of externally pressurised spherical shells" J. App. Mech. 34 (1967) 49-55

(18) A.COMBESCURE "Elastoplastic buckling of shells : proposition of a methodology" Paper n°11 in this conference

(19) J. DEVOS, A. HOFFMANN "Stabilité des coques minces - Diagrammes de réduction de charge incorporant l'instabilité post-critique"
Rapport DEMT/SMTS/LAMS 83-32 (2.08.1983)

(20) AFCEN "Projet de Règles de conception et de construction des matériels mécaniques des ilots nucléaires RNR,RCC-MR"
Edition provisoire décembre 1983

(21) D. BROUARD, P. COUSSERAN, J. LEBEY, D. MOULIN, R. ROCHE,
"Progressive buckling caused by cyclic straining"
Paper L 15/4, 6th SMIRT Conf. Paris 1981

$^{0}{}_{0}{}^{0}$

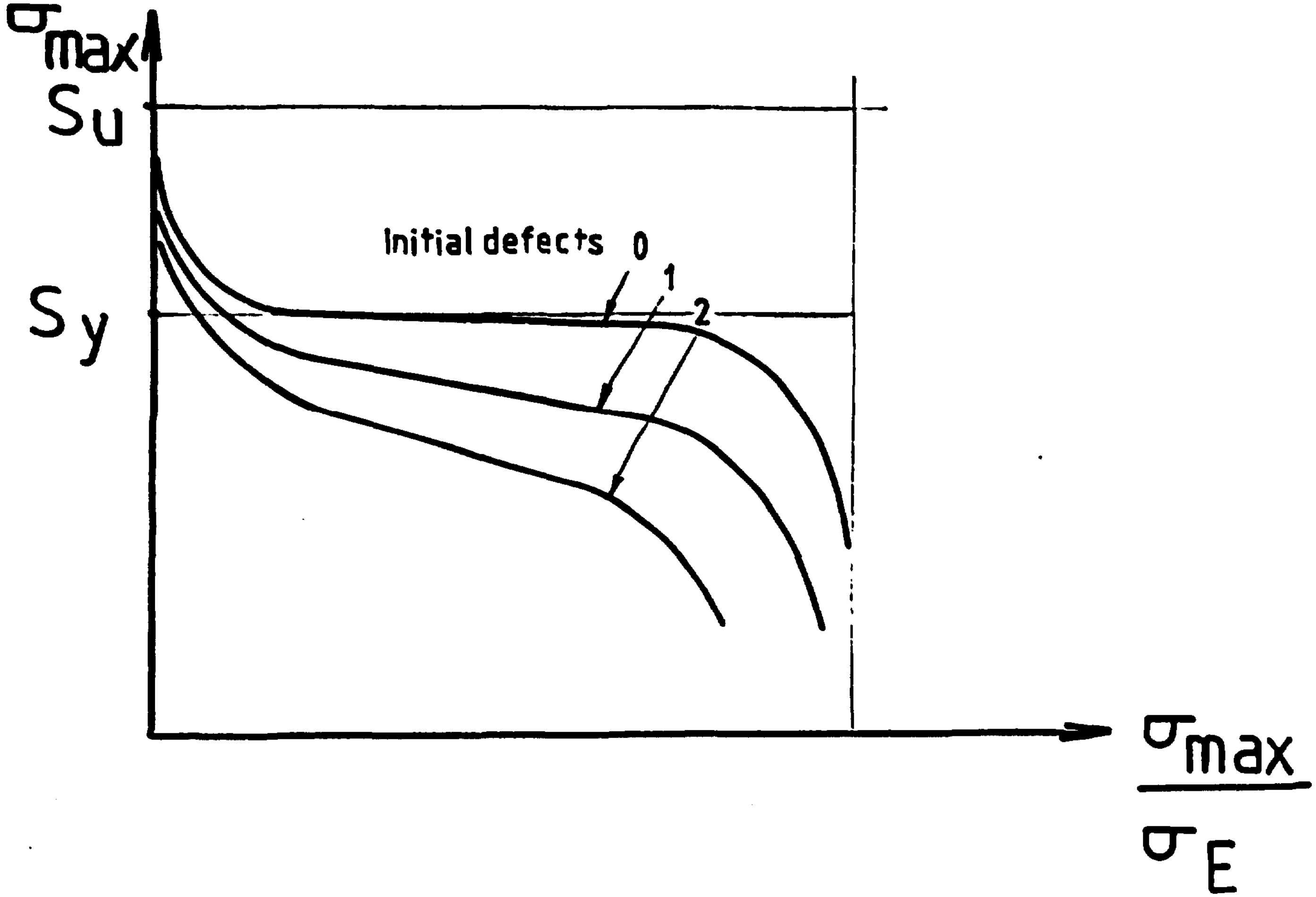

FIGURE 1 Presentation of buckling reduction diagram

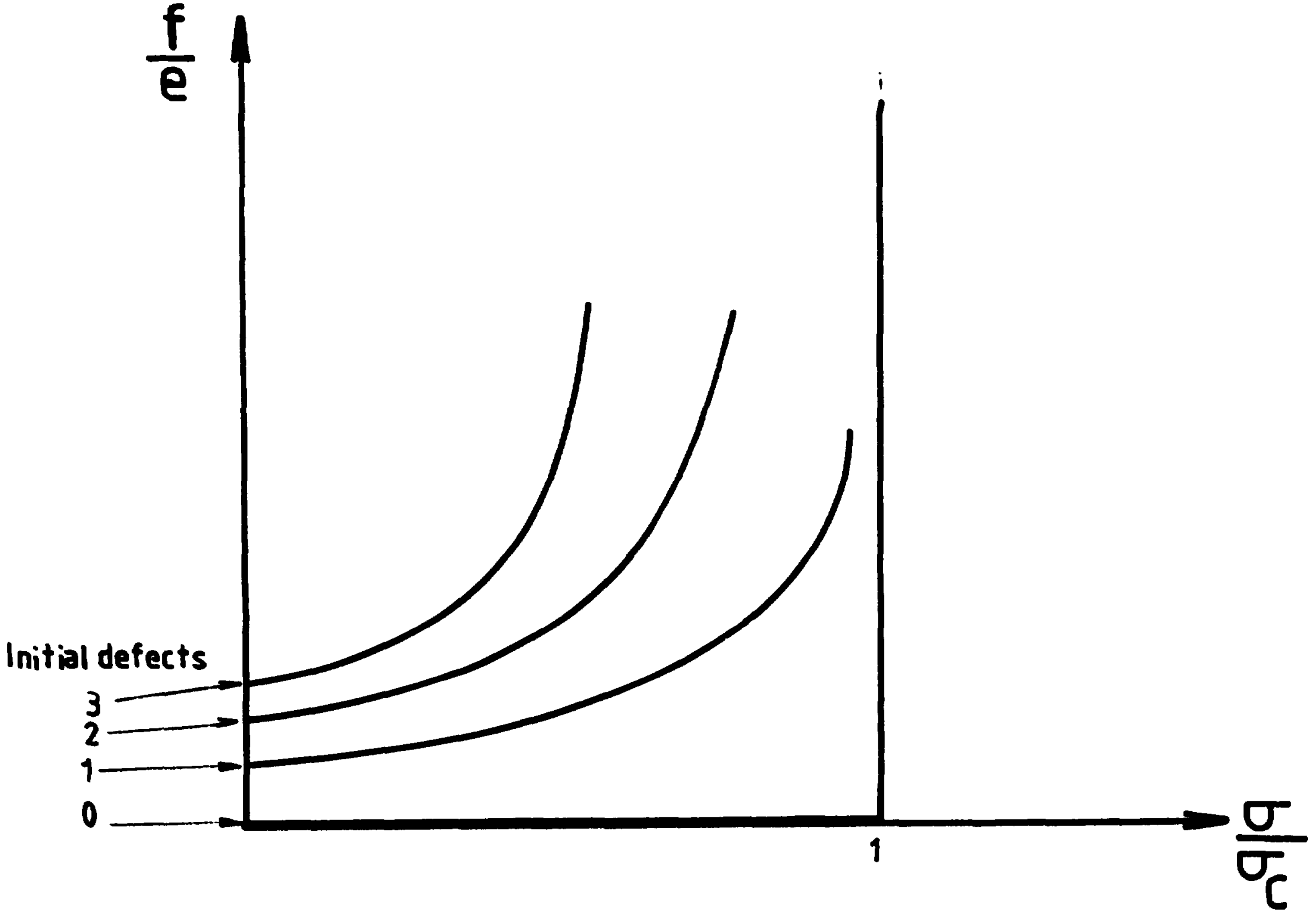

FIGURE 2 Evolution of f/e (stable case)

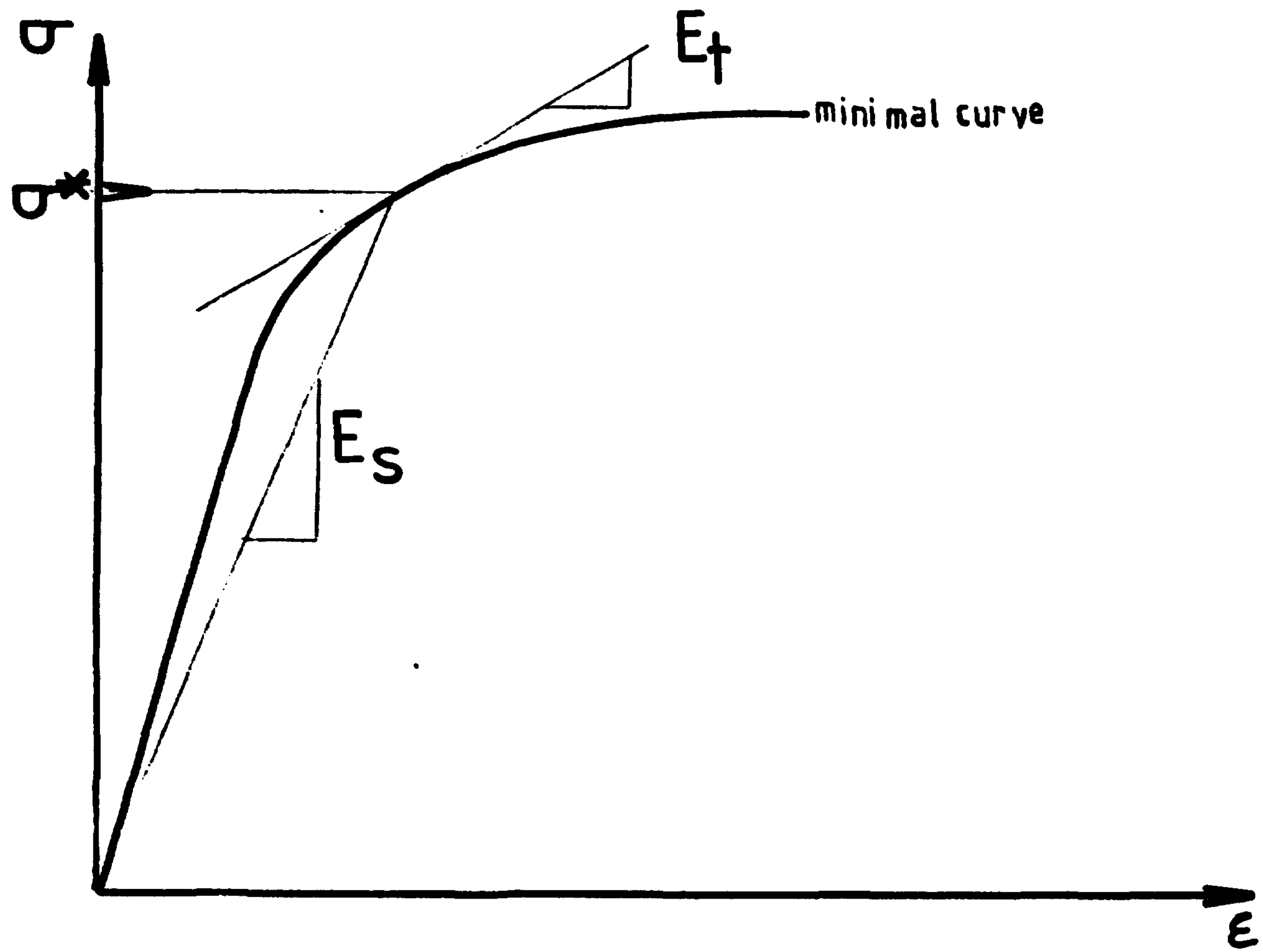

FIGURE 3 Definition of tangent (E_t) and secant (E_s) moduli from tensile curve

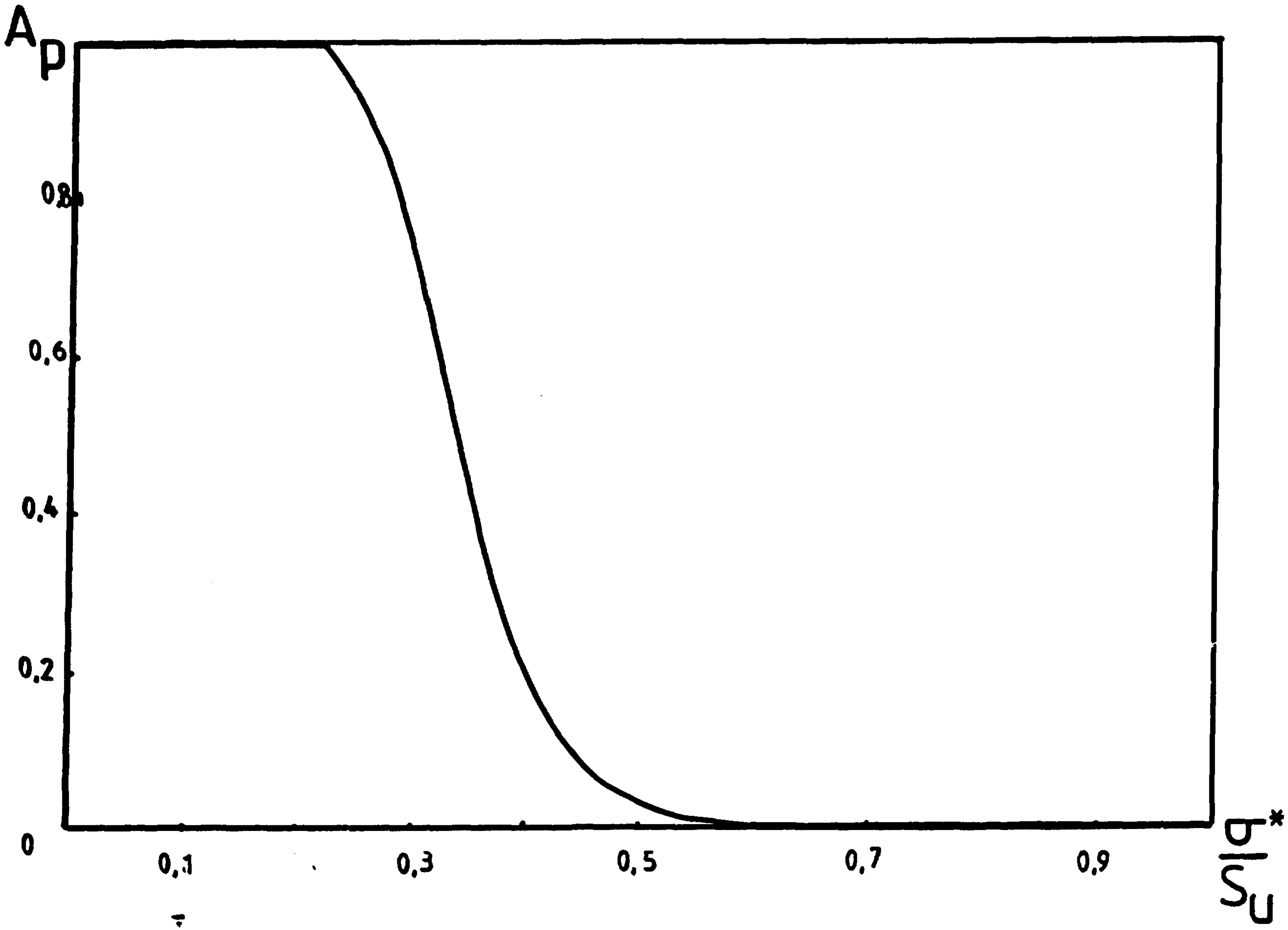

FIGURE 4   Plasticity reduction factor [Gerard formulation] A_p

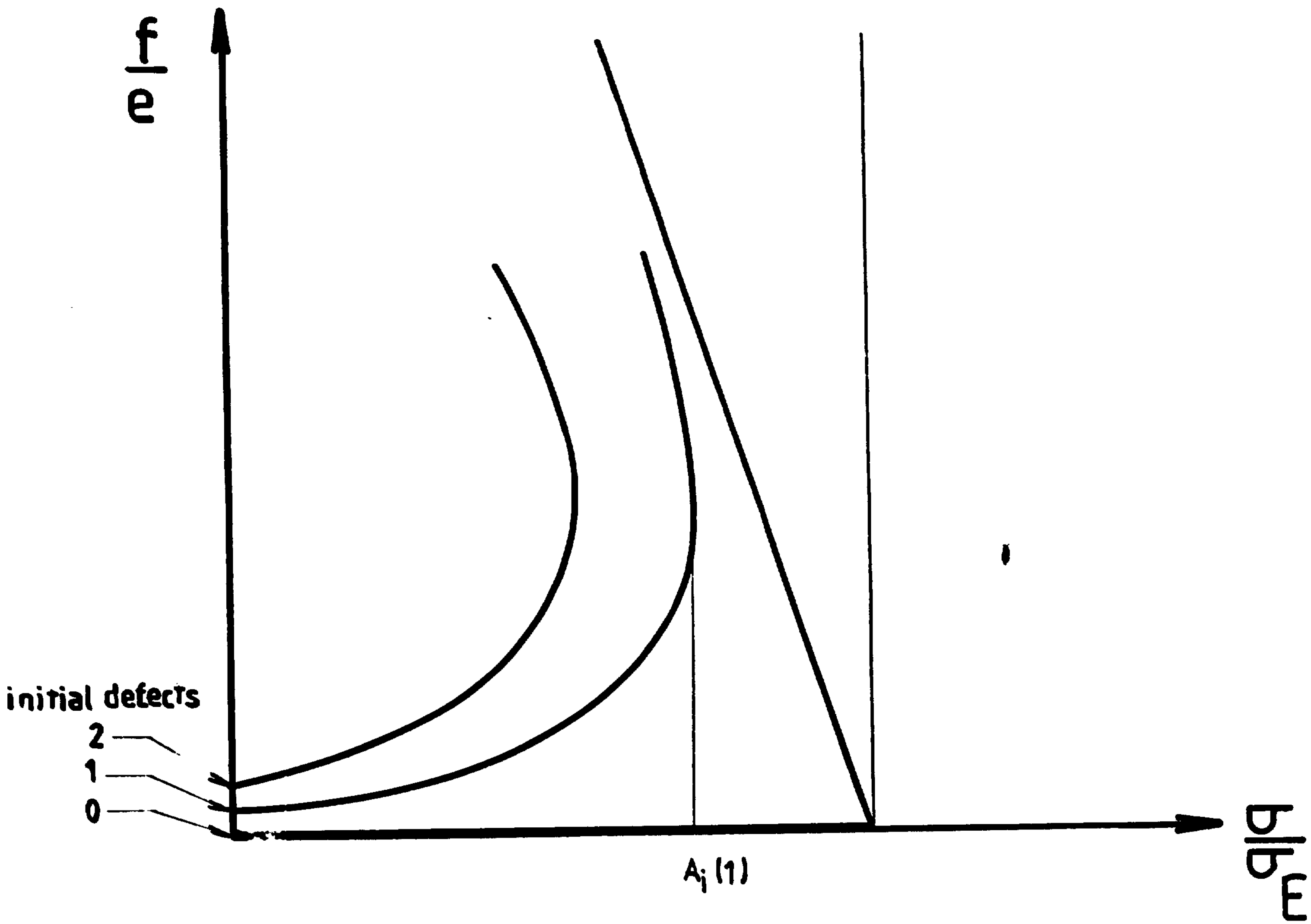

FIGURE 5 Evolution of f/e (unstable case)

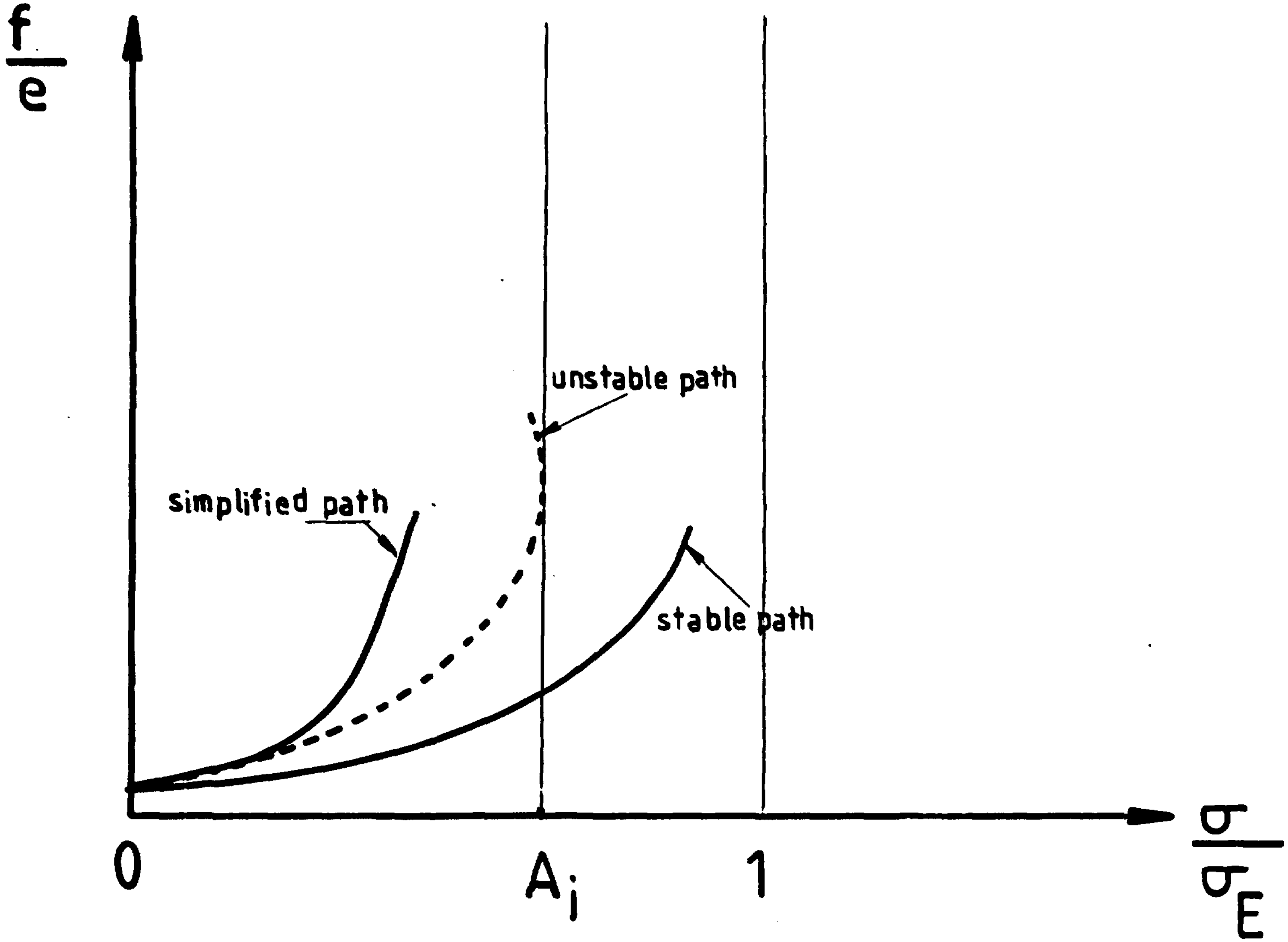

FIGURE 6 Simplified evolution of geometric defect

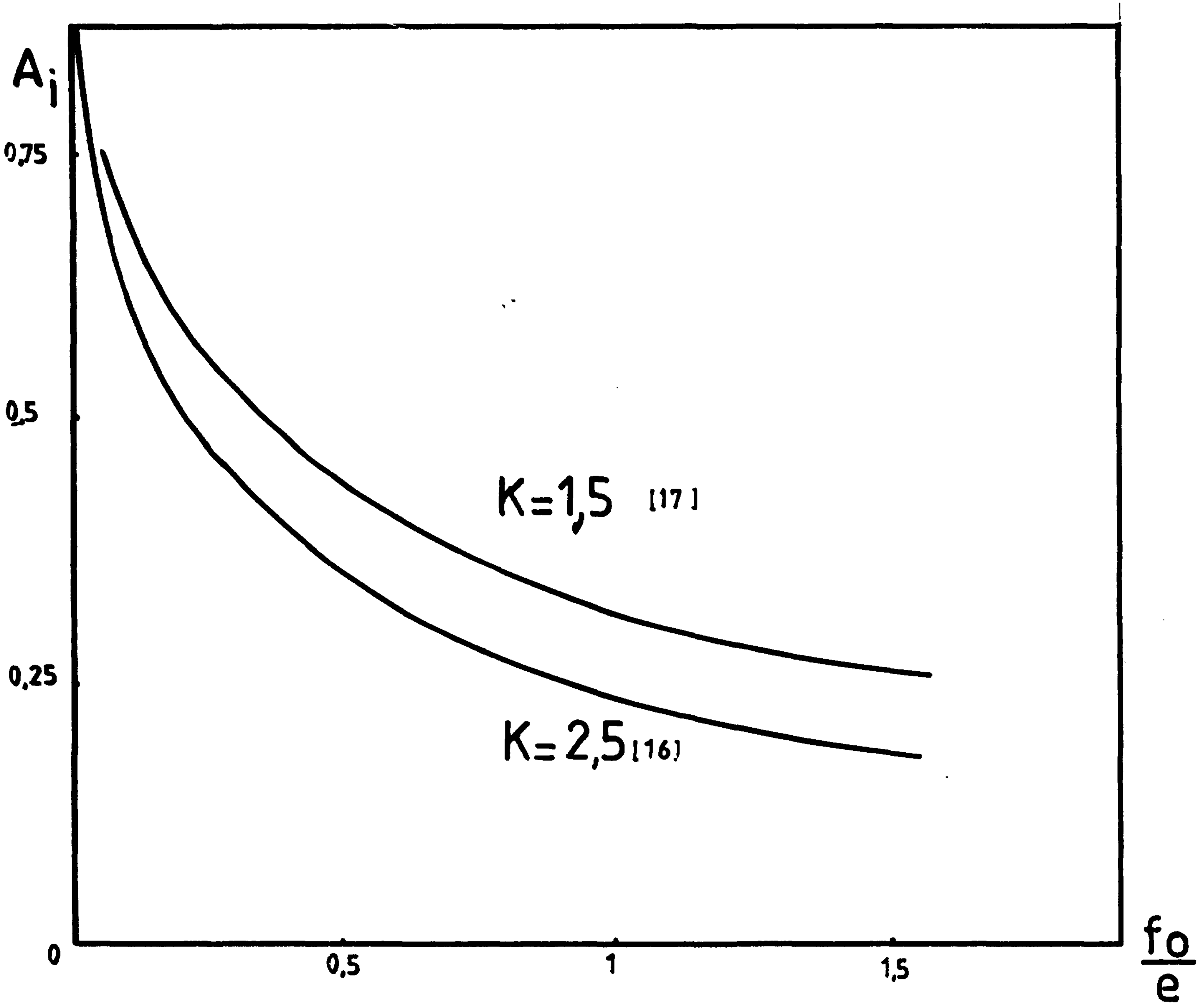

FIGURE 7 Comparison between [16] and [17] instability reduction factor A_i

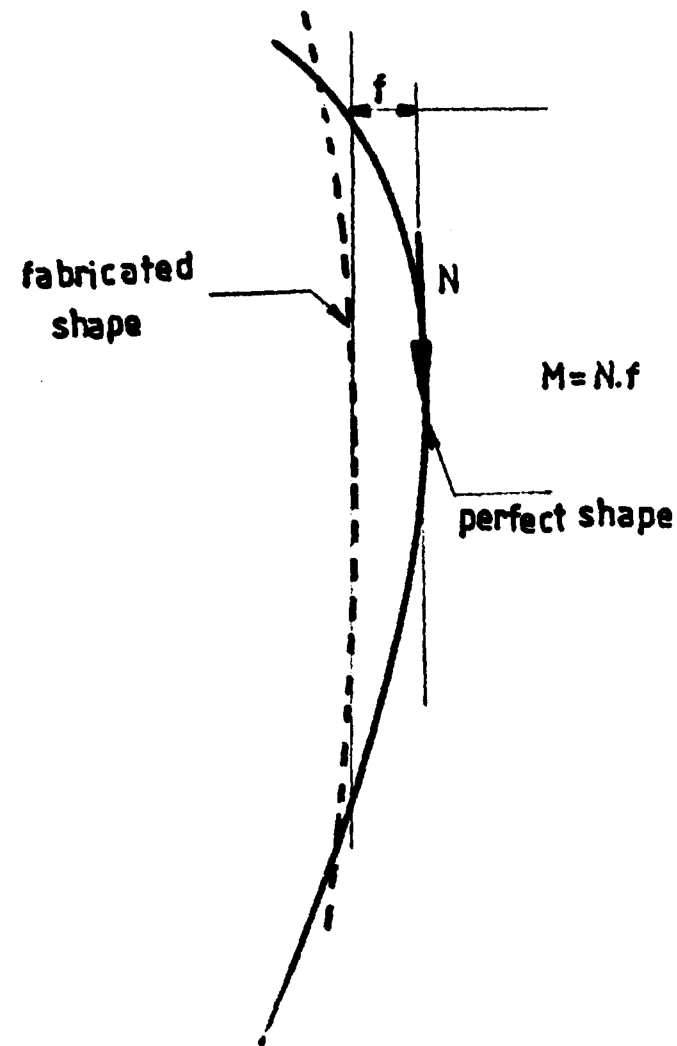

FIGURE 8 Bending moment in deformed menbrane shell

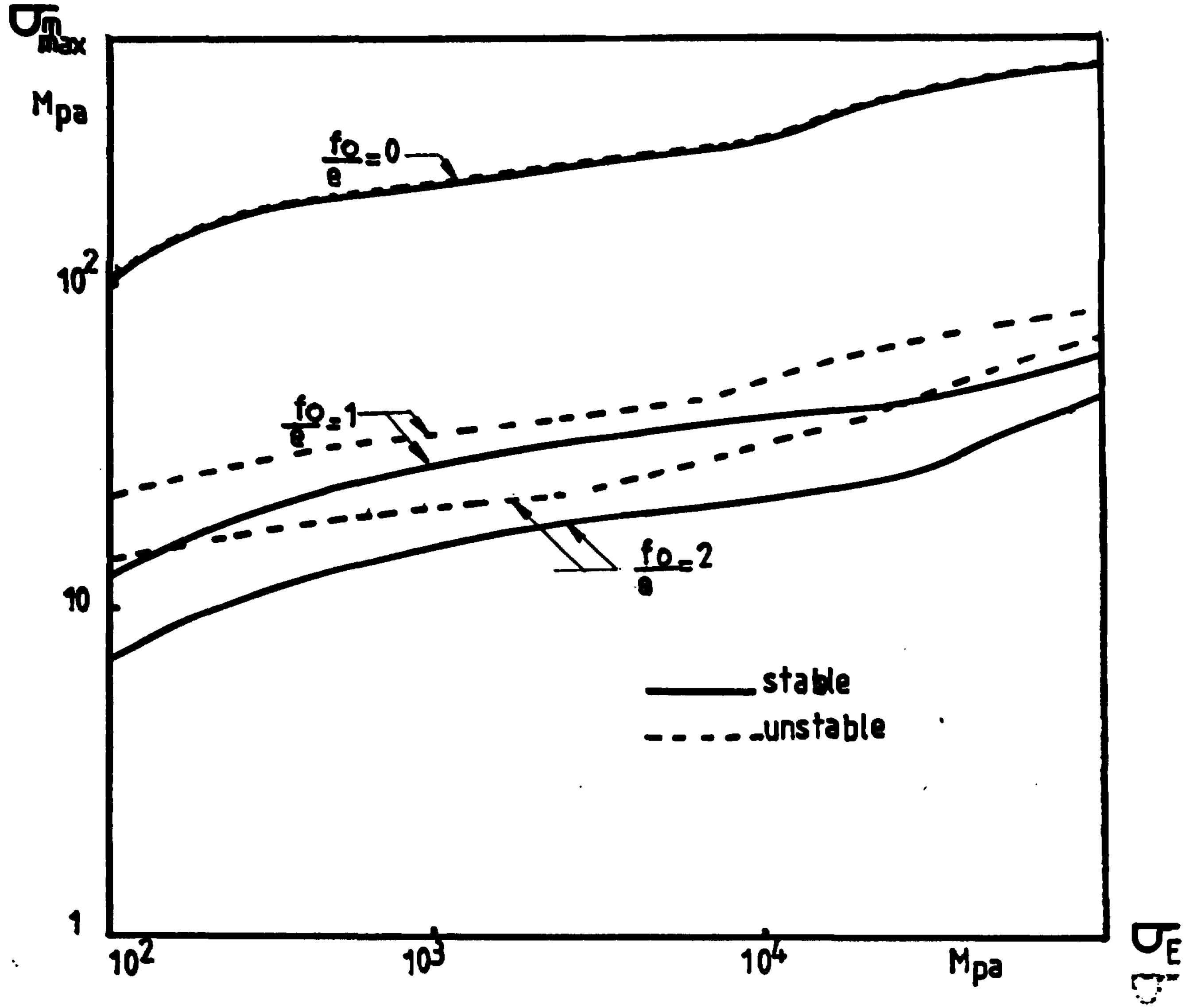

FIGURE 9 Reduction diagrams (austenitic steel at R.T.)