THE PHASE IN THE AHARONOV-BOHM EXPERIMENT

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Abstract

The phase of the wave function scattered by an Aharonov-Bohm solenoid containing the quantized flux $(h/e).n$ where n is an integer is shown to be $(-1)^n$ times that of the free one .A two-solenoid experiment is proposed to observe the difference between the cases when n is even or odd.

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A quarter of a century ago Aharonov and Bohm [1] proposed to scatter an electron beam on a long,thin solenoid containing a magnetic flux ϕ.They argued that,although the electrons move in a region where the electromagnetic field strength vanishes,there will be an interaction between the particles and the vectorpotential.According to their prediction - confirmed by the observation 2,3 -

(i) The interference pattern observed on a screen behind the solenoid depends on the phase factor

(1) $$\chi = \exp (ie/\hbar) \oint \vec{A}.d\vec{x} = \exp(ie\phi/\hbar).$$

In particular,if the enclosed flux is quantized,

(2) $$\phi = (h/e).n \text{ ,where } n = \pm 1, \pm 2, \ldots \;,$$

then the interference pattern is the same as with no flux at all.From this one concludes generally

(ii) If (2) holds,the solenoid is unobservable.

In this paper we argue that (i) is correct,but (ii) is false. More precisely,one can decide whether the integer n in (2) is even or odd.

Our proof of this statement is based on the expression of the quantum-mechanical propagator given by Feynman [4]:

(3) $$K(b,a) = \int \exp[iS(\gamma)/\hbar]\, \mathcal{D}\gamma$$

We drop the z-variable and work in the plane with the position of the solenoid removed (fig.1).The configuration space is then not simply connected so the theory of [5] applies.

Observe that the classical action is decomposed as

(4) $$S(\gamma) = S_o(\gamma) + \int_\gamma e\vec{A}.d\vec{x}$$

where S_o is the action associated to the free dynamics.

Let us choose a and b as on Fig.1, and denote ϱ an arbitrary fiducial path from a to b. We assume without loss of generality that ϱ does not wind around the solenoid. If γ is now any path from a to b then $\delta = \varrho^{-1} \circ \gamma$ is a loop through a so it has a homotopy class $[\delta]$ labelled by an integer k, the number of times it winds around the solenoid.

Clearly, the action (4) becomes

$$(5) \qquad S(\gamma) = S_o(\gamma) + e \int_{\delta} \vec{A}.d\vec{x} - e \int_{\varrho} \vec{A}.d\vec{x}$$

The second term here depends only on the homotopy class $[\delta]$. Its value is in fact $e\, n \cdot \Phi$, if $[\delta] = n$. The third term is independent of γ. Consequently, the integral (3) becomes

$$(6) \qquad K = c. \left(\sum_{n=-\infty}^{\infty} \chi^{n} . K^{o}_{n} \right)$$

where K^o_n is the "partial propagator" of a free particle defined by integration over those paths labelled by n, and the phase factor c reads

$$(7) \qquad c = \exp(-\frac{ie}{\hbar} \int_{\varrho} \vec{A}.d\vec{x}).$$

In scattering experiments of the type proposed by Aharonov and Bohm only the absolute value of the propagator plays a role. So if Φ and Φ' differ by an integer multiple of h/e, $|K| = |K'|$, and we get the same interference pattern in both cases. This confirms (i).

So far, so good. Observe however that the propagators are not identical: the phase factor c may take different values. If a and b are far enough - and this is the usual situation in the experiment - the third term in (5) is approximately half the flux, so

$$(8) \qquad c \simeq \exp(-ie\Phi/2\hbar)$$

In particular, if the flux is quantized as in (2), the propagator is simply

$$(9) \qquad K = (-1)^{n} . K^{o}$$

since $\chi = 1$ now and the sum in the bracket in (6) gives just the free propagator K^o.

We conclude that the sign depends on the parity of n. Does this fact imply any physical consequence? We claim that it does.

Remeber first an analogous situation: for spin n/2 particles the quantum operator of a 2π-rotation is $(-1)^n$. This has been known for a long time but nobody paid any attention to this fact until Aharonov (again!) and Susskind, and Bernstein [6] pointed out that the minus sign for n odd may become important in interference experiments.

Indeed, if we split a beam of spin 1/2 particles into two coherent partial beams, rotate the spin of one one them using a magnetic field, then, after recombination, there will be a destructive/resp. constructive interference depending on the parity of the times the spin has been rotated. This has been observed subsequently in beautifully performed neutron-interference experiments [7].

Our strategy to observe the $(-1)^n$ factor in (9) is now clear: (Fig.2.a)

(1) split an electron beam to two coherent partial beams by a biprism a;

(2) scatter both partial beams on thin, identically-shaped cylinders c_1 and c_2 one of which - say c_1 - contains an AB solenoid.

If (9) is correct, there will be a destructive interference for n odd and a constructive interference for n even. So the parity of n is observable.

Interestingly, the experiment we propose could be performed. In their recent AB experiment Möllenstedt and his collaborators [3] were able to separate coherently the partial beams as widely as 200 μm and construct glass tubes with diameter 60 μm, which contain AB coils.

Alternatively, one could use a superconducting circuit [8] the current of Cooper pairs can be regarded as a coherent beam and the interference can be observed by mean of Josephson junctions (Fig.2.b). Notice that flux quantization - contrarily to the case encountered in observing 2π rotations [9] - does not rule out our effect, since any integer value of n is now allowed.

The same conclusion as above is obtained from scattering theory [10]. Indeed, the momentum-space representation of the S-matrix acts on the subspace of constant angular momentum m (an integer) by mere phaseshift:

$$(10) \qquad S_m = \begin{cases} \exp\left[ie\Phi/2\hbar\right] & \text{for } m > e\Phi/h \\ \exp\left[-ie\Phi/2\hbar\right] & \text{for } m < e\Phi/h \end{cases}$$

In paricular, if the flux is quantized as in (2), we get

$$(11) \qquad S = (-1)^n$$

in agreement with the previous results.

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Figure captions

Fig.1. Path integral in the Aharonov-Bohm experiment

Fig.2.a A two-solenoid experiment to observe the phase in the AB scattering. If the flux is quantized in even multiples of the basic unit one should observe a constructive interference. If the flux is an odd multiple, the interference should be destructive.

Fig.2.b. Realization of the previous experiment with superconducting circuits. The Cooper pairs can be regarded as a coherent beam and the Josephson junctions work as interferometers [8].

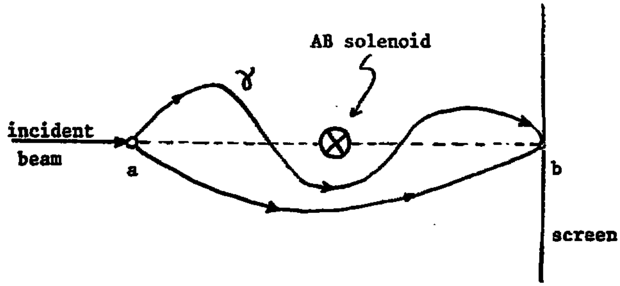

fig.1.

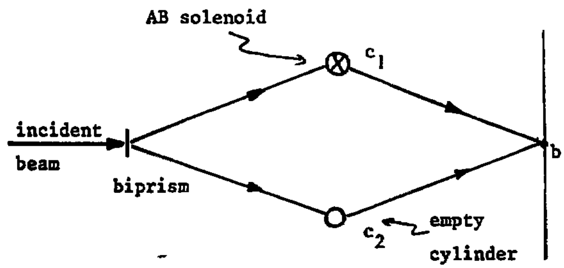

fig.2.a.

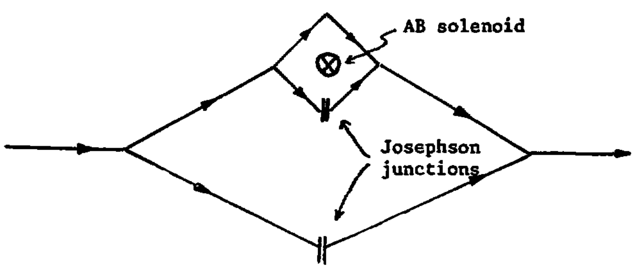

fig.2.b.