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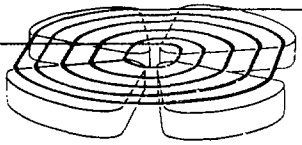

FISSION-FRAGMENT KINETIC-ENERGY DISTRIBUTIONS FROM A TWO-DIMENSIONAL FOKKER-PLANCK EQUATION

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GANIL P. 84.09

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Abstract

We calculate the fission-fragment kinetic-energy distribution for the compound nucleus ^{213}At by simultaneously taking into account spreading in a stretching degree of freedom and fluctuations in a fission degree of freedom. This is done in terms of an approximate solution of a two-dimensional Fokker-Planck equation obtained by propagating Gaussian bundles in momentum space. When compared as functions of nuclear temperature with experimental data, our calculated mean kinetic energies are in approximate agreement and our calculated variances are slightly too small.

Introduction. When a nucleus undergoes fission, the fragments arrive at infinity with a distribution in kinetic energy that arises from two distinct physical effects. The dominant effect is a spreading near scission in a stretching degree of freedom transverse to the fission degree of freedom, which contributes a lower Coulomb repulsion between stretched fragments than between compact fragments. The second effect is fluctuations in the kinetic energy in the fission degree of freedom itself, arising from the dissipation of collective energy into single-particle excitation energy. The former effect has been treated both in terms of the static potential energies of two touching spheroids [1] and by solving the Hamilton equations of motion in several collective coordinates for the nonviscous descent of a fissioning nucleus from the vicinity of its saddle point [2]. The second effect has been treated in terms of a Fokker-Planck equation for one collective fission degree of freedom by solving the time-dependent equation numerically [3,4] and also by use of Kramer's stationary solution for an inverted oscillator [5,6].

It is our purpose here to treat simultaneously both of these contributions to the fission-fragment kinetic-energy distribution, which we do in terms of a two-dimensional Fokker-Planck equation for the distribution function in phase space of fission and stretching collective coordinates and their conjugate momenta. For an initial distribution corresponding to a quasi-stationary flow through the vicinity of the fission saddle point [7], the solution is obtained by propagating forward in time a distribution function that is approximated at each point in coordinate space by Gaussian bundles in momentum space [8]. In this way the coupling between the two collective degrees of freedom is taken into account automatically.

Collective model. We describe the dependence upon time t of the distribution function $f(q_1, q_2, p_1, p_2, t)$ in phase space of collective coordinates q_1 and q_2 and conjugate momenta p_1 and p_2 by the two-dimensional Fokker-Planck equation

$$\frac{\partial f}{\partial t} + (M^{-1})_{ij} p_j \frac{\partial f}{\partial q_i} - \left[\frac{\partial V}{\partial q_i} + \frac{1}{2} \frac{\partial (M^{-1})_{jk}}{\partial q_i} p_j p_k\right] \frac{\partial f}{\partial p_i} = \gamma_{ij} (M^{-1})_{jk} \frac{\partial}{\partial p_i}(p_k f) + T\gamma_{ij} \frac{\partial^2 f}{\partial p_i \partial p_j} \qquad (1)$$

where repeated indices are summed over from 1 to 2. The nuclear temperature T, which is measured here in energy units, is assumed to be constant, but the potential energy of deformation V, the inertia tensor M, and the dissipation tensor γ are all functions of the collective coordinates q_1 and q_2.

Our two collective coordinates are defined for reflection-symmetric and axially-symmetric nuclear shapes in terms of the c-h parametrization of Brack et al. [9]. In order to display our results in terms of coordinates that are more directly related to quantities of physical interest, we transform from c and h to two new collective coordinates defined as the distance ρ between the centers of mass of the two halves of the fissioning nucleus and the quadrupole moment ξ of one half.

As a function of these two coordinates we calculate the potential energy of deformation $V(\rho,\xi)$ as the sum of the surface and Coulomb energies of the liquid-drop model [10] and the proximity potential that corrects for the nonzero range of the nuclear force [11]. The resulting potential-energy surface is shown in fig. 1 for the compound nucleus ^{213}At. For estimating the inertia and the dissipation tensor we approximate our configuration at each ρ and ξ by two equivalent spheroids. The resulting diagonal inertia tensor $M(\rho,\xi)$ is calculated for incompressible irrotational flow, for which a closed analytic expression is available [12].

The dissipation tensor $\gamma(\rho, \xi)$ that describes the conversion of collective energy into single-particle excitation energy is calculated for one-body dissipation[13-15]. The transition from the wall formula that applies to mononuclear shapes to the wall-and-window formula that applies to dinuclear shapes is made in terms of a smooth interpolation involving the ratio of the neck area to the maximum transverse area of a nascent fragment [15].

Propagation in phase space. In order to solve the Fokker-Planck equation (1), the method of propagation with Gaussian bundles was used [8,16]. That means that the distribution function f is, at each point (q_1° , q_2°) in coordinate space, represented by a Gaussian in the momentum space :

$$f(q_1^\circ, q_2^\circ, p_1, p_2) \sim \exp\left(-\frac{1}{2}\, \tilde{X}\, \sigma_\circ^{-1} X\right) \qquad (2)$$

with $\tilde{X} = ((p_1 - p_1^\circ), (p_2 - p_2^\circ))$,

where $\sigma_\circ$ and (p_1° , p_2°) are the local variance tensor and the local mean momenta. This assumption is likely well fulfilled in the large viscosity regime obtained within the one-body dissipation picture.

Propagators in time are associated to the Gaussian bundles and provide us with a solution of the diffusion equation (1). As a matter of fact the assumption (2) substantially reduces the computational effort compared to the general propagator method [4], which could practically not be applied for our configuration space.

The initial distribution is taken at the saddle point where its analytical expression can be derived in the stationary regime [7]. After a few 10^{-21}s the stationary solution is obtained over all the considered potential landscape of the figure 1. Beyond the scission line we have to deal only with conservative forces along ρ and a positive definite stiffness tensor. The distribution function at infinity can therefore be obtained by calculating the time-dependent propagators, with the momentum distributions along the scission line as the initial conditions, in the usual way [4]. In practice it will be sufficient to

integrate the equations of motion until the physical observables, which are to be compared with the experiment, reach their stationary values.

Results. The total energy $E_t(\rho,\xi)$ may be decomposed into :

$$E_c(\rho) = \frac{1}{2} B_{\rho\rho} \dot{\rho}^2 + U_c(\rho) \tag{3}$$

and a remainder $E_r(\rho,\xi)$:

$$E_t(\rho,\xi) = E_c(\rho) + E_r(\rho,\xi) \tag{4}$$

where $U_c(\rho) \sim 1/\rho$ is the monopole-monopole term of the Coulomb interaction between the two halves of the fissioning nucleus. Since $B_{\rho\rho}$ is in our model the reduced mass of the fragments, $E_c(\rho)$ is independent of ξ. The remainder $E_r(\rho,\xi)$ contains at large relative distance the energy stored in the quadrupole vibration and is there independent of ρ, whereas it contains, for small relative distance, all the coupling between the two degrees of freedom ρ and ξ.

The quantities we are interested in are the mean values $\bar{E}_c$ and the variance $\sigma^2_{E_c} = \overline{(E_c - \bar{E}_c)^2}$ of the Coulomb part E_c only, which tend asymptotically to the mean kinetic energies of the fragments and their variances.

The numerical results are compared with the experiments by F. Plasil et al.[17] for the fission of ^{213}At, induced by the ^{4}He + ^{209}Bi-reaction. The angular momentum transferred in this reaction is very small. We verified by an approximate inclusion of the rotation energy that an angular momentum of about $l = 20\hbar$ does not significantly modify the results obtained for $l = 0$. We consequently restricted our calculations to this latter case.

Figure 2 (resp. figure 3) shows the mean kinetic energy (resp. variance) as a function of the temperature. The mean kinetic energy is predicted almost constant in the considered temperature range. As a matter of fact, it is essentially determined by the prescission kinetic energy, i.e. by the magnitude of the viscosity in the ρ-direction, and by the Coulomb field after the scission region. With the procedure used for calculating the friction coefficients of equivalent spheroids and the liquid drop potential in the c-h parametrization, it turns out that the experimental measurements are reproduced with a good accuracy. The calculated variances show a more pronounced increase with temperature as we could expect from a diffusion process. This dependence on temperature seems to correspond to the trend indicated by the data. The absolute values of our calculation are about four times larger than in the one-dimensional models of ref. [3,4] and are only slightly smaller than the experimental ones. Additional degrees of freedom to elongation and striction, which take into account the entire mass division distribution, could be responsible for this small difference.

On the other hand we have checked that the variances are almost unchanged for different choices of the friction tensor (which is actually fixed by the one-body dissipation picture in the results of the figures 2 and 3), only the slope of the calculated curve increases slightly with decreasing friction strength in the ξ-direction. Finally we would like to indicate that the high values of the viscosity coefficient along the ξ-coordinate hinders the system to reach the bottom of the potential valley near the scission region. Indeed, the stationary values of our observables can only be obtained for interdistances between the fission fragments larger than 50fm.

Outlook. By solving approximately a two-dimensional Fokker-Planck equation we have simultaneously taken into account the effects of spreading in the stretching degree of freedom and fluctuations in the fission degree of freedom on the fission-fragment kinetic-energy distribution. The coupling between these two degrees of freedom is crucial to reproduce experimental values of the variance in kinetic energy.

As future extensions of this work, a three-dimensional Fokker-Planck equation involving also the mass-asymmetry degree of freedom should be solved to yield the variance in the fission-fragment mass distribution [8,16] and the variance in kinetic energy integrated over all mass divisions. Since new experimental data for these quantities are now becoming available at much higher nuclear temperatures [18,19], it should be possible to test the predictions of our approach over a wide range of excitation energy.

J.R. NIX warmly thanks the physicists of GANIL for the kind hospitality extended to him during his stay in Caen. This work was supported by the Commissariat à l'Energie Atomique and the U.S. Department of Energy.

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FIGURE CAPTIONS

Fig. 1 : Contours of the nuclear potential energy of deformation $V(\rho,\xi)$, in units of MeV, for the compound nucleus ^{213}At. The radius R_o of the spherical nucleus is given by $R_o \approx 1.2049\text{fm} \times (213)^{1/3} = 7.196\text{fm}$. The saddle point is indicated by a cross and the scission line determined in terms of zero neck radius is indicated by a dashed line.

Fig. 2 : Dependence of the mean fission-fragment kinetic energy upon nuclear temperature. The curve gives our calculated results and the points give the experimental data of ref. [17], which have been corrected for the effects of neutron emission from the fragments.

Fig. 3 : Dependence of the variance in the fission-fragment kinetic-energy distribution upon nuclear temperature. The curve gives our calculated results and the points give the experimental data of ref. [17], which have been corrected for the effects of neutron emission from the fragments.

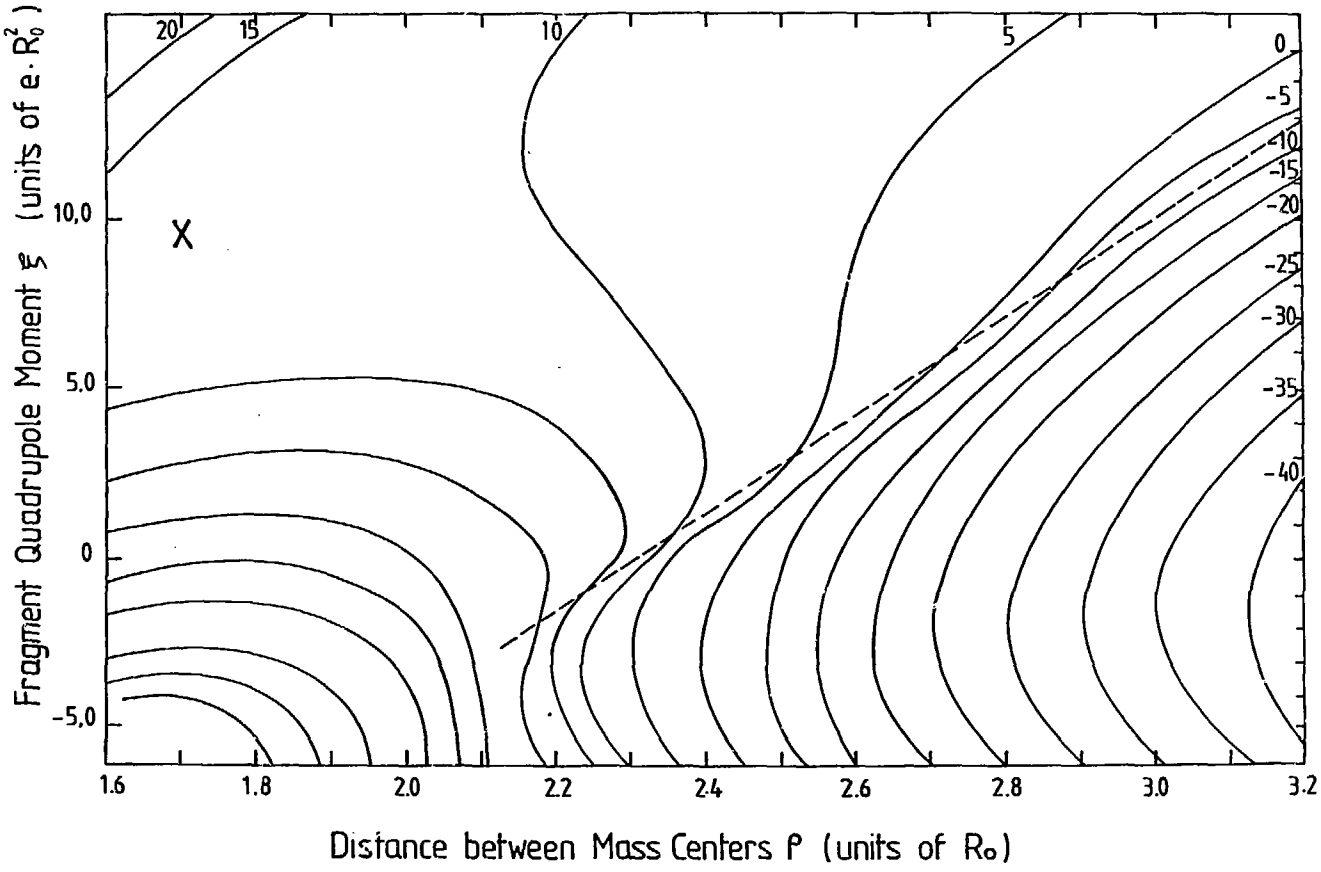
Fragment Quadrupole Moment ℰ (units of e·R₀²)
10,0
5,0
0
-5,0
X
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
Distance between Mass Centers ρ (units of R₀)

Figure 1

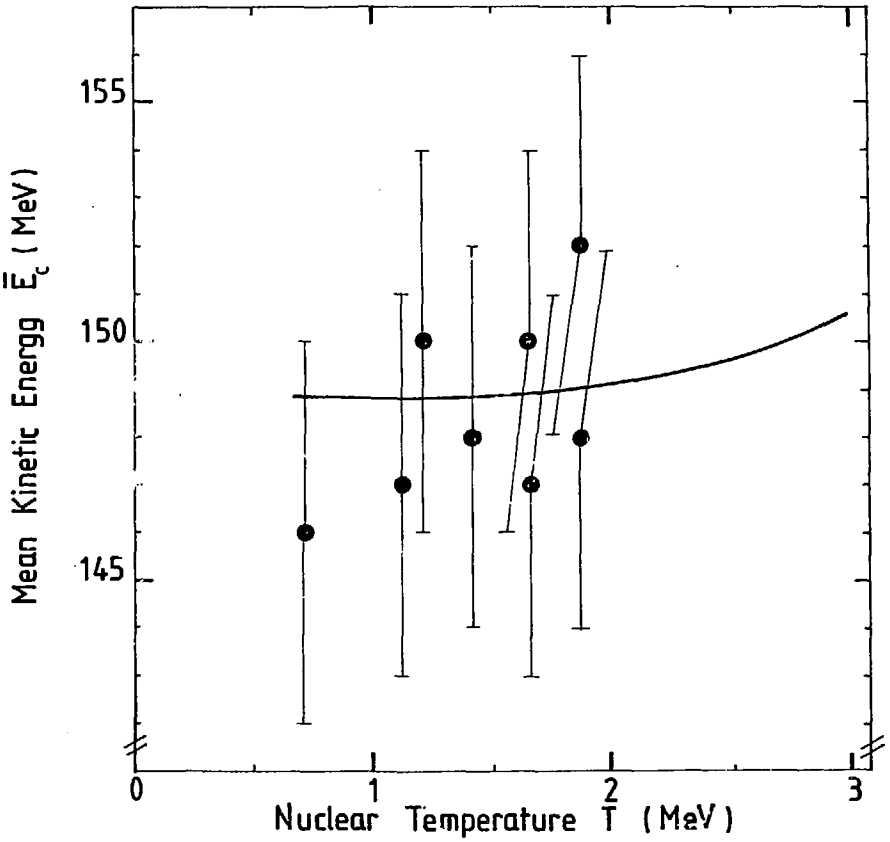
155
150
145
Mean Kinetic Energg $\bar{E}_c$ (MeV)
0
1
2
3
Nuclear Temperature T (MeV)

Figure 2

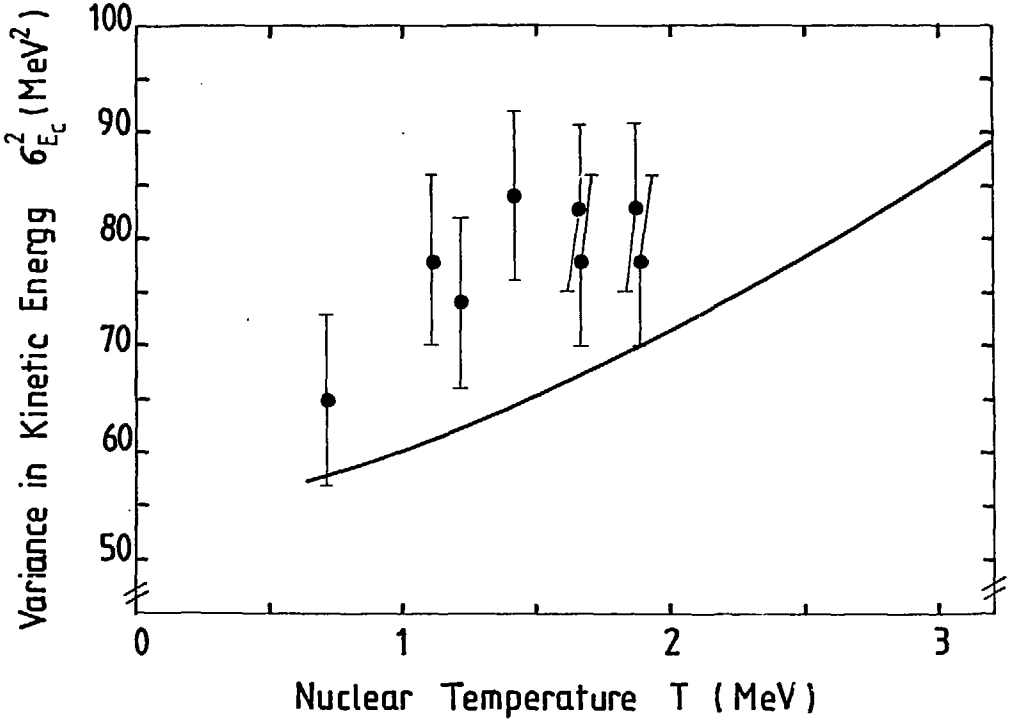
Variance in Kinetic Energg $\sigma^2_{E_c}$ (MeV^2)
100
90
80
70
60
50
0
1
2
3
Nuclear Temperature T (MeV)

Figure 3