THE STABILITY OF IONS IN BUNCHED-BEAM MACHINES*)

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1. INTRODUCTION

The beam bunches ionize the molecules of the residual gas encountered on their path. Under certain conditions the ions may accumulate, giving rise to an increased gas pressure and to direct space-charge effects which produce an incoherent tune shift. In this paper, the conditions leading to the accumulation of ions are established for various cases of bunched beams, together with the maximum ion density which can be reached. An application to the SPS $p\bar{p}$ Collider is also given. (orig./HSI)

2. DEFINITIONS AND SYMBOLS[1])

d_m = neutral molecule density (m^{-3})

d_i = ion density (m^{-3})

d_p = particle density (m^{-3})

η = d_i/d_p = neutralization factor

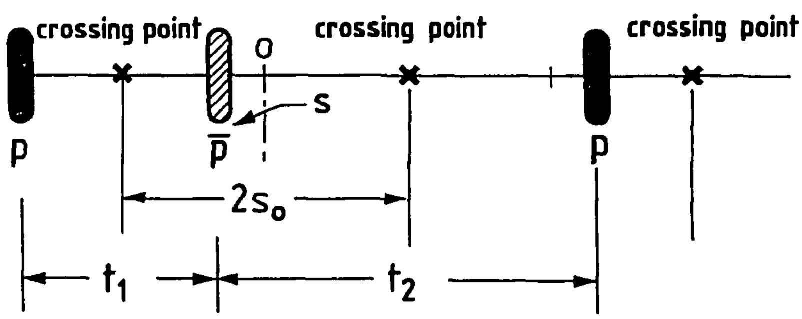
N = total number of particles per beam

n = number of bunches per beam

ℓ_B = length of bunch (m); B = bunching factor = $2\pi R/n\ell_B$

$\sigma_{x,y}$ = r.m.s. values of transverse particle distribution

T = revolution time = $n(t_1 + t_2)$



u = azimuth relative to crossing point = $(t_1 - t_2)/(t_1 + t_2) = S/S_0$

a = kick parameter (s^{-1})

r_0 = classical particle radius (m)

R = average machine radius (m).

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*) *Editorial Note:* In the original programme this lecture was entitled "Vacuum and Ion Trapping". In view of the limited time, it was felt better to omit the general aspects of "Vacuum", i.e. nuclear scattering and Coulomb scattering, since these are not in any way unique to antiproton machines. The reader may also appreciate that trapping in coasting beams is omitted.

In the local coordinate system, where x is horizontal, y vertical, and z axial:

$$d_p = \frac{dN}{dxdydz} = \frac{N}{n}\frac{1}{\ell_B}\frac{1}{2\pi\sigma_x\sigma_y} \quad \text{(at centre of bunch)} \tag{1}$$

$$d_i = \eta d_p = \frac{N_i}{Bn\ell_B}\frac{1}{2\pi\sigma_x\sigma_y} \; . \tag{2}$$

The time needed by one particle to produce one ion is called the ionization time τ_i:

$$\tau_i = \frac{1}{d_m\sigma_i c} \; .$$

Typically $\tau_i < 10$ s for partial pressures in the range 10^{-9} to 10^{-10} Torr. Therefore, if only the first ionization is considered, the final ion density in the case of accumulation is independent of the actual pressure. But ions accumulated in the beam region are exposed to a second ionization with a cross-section comparable to that of the first ionization. In such a case of equilibrium the ion density should not exceed the neutral molecule density, provided ions with still higher charge are unstable at given azimuth (which is not always the case):

$$d_i = \frac{\sigma_i(+)}{\sigma_i(++)} d_m \rightarrow d_i \leq d_m \; . \tag{3}$$

The neutralization is complete when

$$N_i = N \rightarrow \eta_{max} \; .$$

For continuous beams:

$$(d_i)_{max} = d_p \; , \quad \eta_{max} = 1 \; . \tag{4}$$

For bunched beams:

$$B(d_i)_{max} = d_p \; , \quad \eta_{max} = \frac{1}{B} \; . \tag{5}$$

3. CONDITIONS TO START ION ACCUMULATION

Firstly, we consider the case of a collider with bunches of positive and negative particles. At a given azimuth an ion sees successively the focusing (or defocusing) forces induced by the passage of the bunches, and it drifts freely between the bunches. If a vertical dipole field is applied, the horizontal transverse and longitudinal motion will be coupled, while the vertical motion is independent of the magnetic field. The vertical motion is the simplest and therefore is considered first. Some other simplifying assumptions are made, namely:

i) linear electric field inside the bunch,

ii) transverse bunch dimensions constant around the machine,

iii) transverse dimensions of the ion cloud also constant and equal to those of the bunch,

iv) same number of particles in each bunch.

(Later on, non-linear electric fields, varying beam and ion-cloud dimensions, and bunch-to-bunch intensity fluctuations, will be considered.)

Let us introduce:

v_i = velocity of ions (small, << c)

$\Delta p_i = m_i \, \Delta v_i = \int_0^{\Delta t} F \, dt = F \, \Delta t$ (thin lens)

$\vec{F} = e\vec{E}_y$ [B_x gives a negligible contribution because $B_x = E_y(v/c^2) = E_y(\beta/c) \approx (1/c)E_y$]

$\Delta t = \frac{\ell_B}{\beta c}$

$$\Delta v_i = \frac{eE_y \Delta t}{m_i} = \frac{eE_y \ell_B}{\beta c m_i} = ay \tag{6}$$

a = kick parameter = inverse focal length

$$\begin{pmatrix} y \\ \dot{y} \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \begin{pmatrix} y \\ \dot{y} \end{pmatrix}_0 \tag{7}$$

$$E_y(\text{inside bunch}) = \frac{e\rho}{2\varepsilon_0} y \quad \text{with uniform charge density} \tag{8}$$

$$\rho_0 = \frac{N}{n} \frac{1}{\ell_B} \frac{1}{2\pi\sigma_x\sigma_y} \quad \text{(at centre of bunch)} \tag{9}$$

$$\frac{dE_y}{dy} = \rho_0 \frac{1}{\varepsilon_0} \frac{e}{1 + (\sigma_y/\sigma_x)} \tag{10}$$

$$E_y = \frac{N}{n} \frac{1}{\ell_B} \frac{e}{2\pi\varepsilon_0\sigma_y(\sigma_x + \sigma_y)} y \tag{11}$$

$$a = \frac{N}{n} \frac{2r_0 c}{\beta\sigma_y(\sigma_x + \sigma_y)A} , \tag{12}$$

where m_i = mass of ion = Am_{proton}.

The complete period of forces on the ion is

$$M = \underbrace{\begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix}}_{\text{Drift}} \begin{pmatrix} 1 & 0 \\ -\bar{a} & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix}}_{\text{Drift}} \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \tag{13}$$

$\bar{a}$ being equivalent to a for the antiproton and of the same sign, and

$$\text{trace } M = 2 \cos \mu_0 . \tag{14}$$

The motion is stable (trapping) if:

$$-1 < \cos \mu_0 < 1$$

$$\cos \mu_0 = 1 + \delta - \lambda(1 - u^2) \ , \tag{15}$$

where

$$\delta = \frac{1}{2}(a - \bar{a})(t_1 + t_2) = \frac{1}{2}(a - \bar{a})\frac{T}{n}$$

$$\lambda = \frac{1}{8}\, a\bar{a}(t_1 + t_2)^2 = \frac{1}{8}\, a\bar{a}\left(\frac{T}{n}\right)^2$$

$$u = \frac{t_1 - t_2}{t_1 + t_2} = \frac{n}{T}\,(t_1 - t_2) \ .$$

One can define dimensionless quantities called critical masses in the following way:

$$A_c = \frac{1}{4}\, a\, \frac{T}{n}\, A \ , \qquad \bar{A}_c = \frac{1}{4}\, \bar{a}\, \frac{T}{n}\, A \tag{16}$$

$$\delta = 2\,\frac{A_c - \bar{A}_c}{A} \ , \qquad \lambda = 2\,\frac{A_c \bar{A}_c}{A^2} \ . \tag{17}$$

A_c and $\bar{A}_c$ are not constant around the machine (because of a and $\bar{a}$ which depend on σ_x and σ_y).

However, in the following the approximation is made that a and $\bar{a}$ are independent of the azimuth.

Let us consider some simple cases:

i) $\bar{A}_c = 0$ (one beam of positive particles)

$\delta > 0$, $\lambda = 0$, ions are always unstable;

ii) $A_c = 0$ (one beam of negative particles)

$\delta < 0$, $\lambda = 0$, ions with $A > A_c$ are stable;

iii) $A_c = \bar{A}_c$ (colliders with equal bunches of positive and negative charges)

a) at $u = 0$ (midway between crossing points), $\delta = 0$, ions with $A > A_c$ are stable.

b) at $u = 1$ (at crossing points), $\delta = 0$, ions at limit of stability.

c) $0 < u < 1$, $\delta = 0$, ions with $A > A_c(1 - u^2)^{\frac{1}{2}}$ are stable.

The critical mass can be expressed by

$$A_c = \frac{N}{n}\,\frac{r_0}{n}\,\frac{\pi R}{\beta\sigma_y^2[1 + (\sigma_x/\sigma_y)]} \ . \tag{18}$$

There is more danger of ion accumulation when A_c is small, close to the masses existing in the residual gas. In general, as is seen from Eqs. (12) and (16), A_c tends to be small when:

i) the revolution time T is small,

ii) the number of bunches n is high,

iii) the number of particles per bunch is small,

iv) γ is small (low-energy machines),

v) the beam is large.

4. THE ION ACCUMULATION LIMIT

Once started, accumulation continues until the defocusing force due to the ion space charge is strong enough for an equilibrium to be reached. At this point, it should be stressed that the successive bunch passages modify the size of the ion cloud and consequently the magnitude of the space-charge force. However, in this analysis it is assumed that the ion cloud is constant in size and that the force is linear. Obviously now the drift-times of Section 3 are replaced by thick defocusing lenses:

$$M_{ion} = \begin{pmatrix} \cosh \xi & \frac{1}{\sqrt{K}} \sinh \xi \\ \sqrt{K} \sinh \xi & \cosh \xi \end{pmatrix}$$

where in the familiar magnetic case

$$\xi = \ell\sqrt{K} \ , \quad K \propto \frac{\text{gradient}}{p_0} \ .$$

In the case of ions:

$$K = g^2 = \frac{e}{Am_p} \frac{\partial E_y}{\partial y} \ , \quad t \text{ equivalent to } \ell$$

$$\xi \equiv gt \ , \quad g(s^{-1})$$

$$M_{ion} = \begin{pmatrix} \cosh gt & \frac{1}{g} \sinh gt \\ g \sinh gt & \cosh gt \end{pmatrix} \tag{19}$$

For one period:

$$M = M_{i_1} \begin{pmatrix} 1 & 0 \\ -\bar{a} & 1 \end{pmatrix} M_{i_2} \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$$

The stability condition ($-1 < \cos \mu < 1$) is:

$$\cos \mu = \cosh g(t_1 + t_2) + \frac{1}{2}(a - \bar{a}) \frac{1}{g} \sinh g(t_1 + t_2) - \frac{1}{2}(a \cdot \bar{a}) \frac{1}{g^2} \sinh gt_1 \sinh gt_2 \ . \tag{20}$$

With the following approximations (which are valid for $gt \lesssim 2$):

$$\sinh gt \simeq gt, \quad \cosh gt \simeq 1 + \frac{1}{2}(gt)^2$$

$$\cos \mu = \cos \mu_0 + \frac{1}{2}\left(g \frac{T}{n}\right)^2 \tag{21}$$

where $\cos\mu_0$ is the ½ trace of the matrix when there is no accumulation. The equilibrium is reached when $\cos\mu = 1$:

$$\frac{1}{2}\left(g\,\frac{T}{n}\right)^2 = \lambda(1 - u^2) - \delta\ . \qquad (22)$$

But g is related to the ion density d_i as follows:

$$\frac{\partial E_y}{\partial y} = \frac{d_i}{\varepsilon_0}\,\frac{e}{1 + (\sigma_y/\sigma_x)} \qquad (23)$$

$$g^2 = \frac{4\pi r_0}{A}\,c^2\,\frac{d_i}{1 + (\sigma_y/\sigma_x)}\ . \qquad (24)$$

Hence

$$\frac{d_i}{1 + (\sigma_y/\sigma_x)} = \frac{1}{2\pi r_0}\,\frac{n^2}{(2\pi R)^2}\,A\left[\lambda(1-u^2) - \delta\right]$$

where d_i is the final ion density reached. It is appropriate to consider some particular cases.

4.1 Accumulation limit in single-beam machines with negative charges

In this case

$$A_c = 0\ ,\quad \bar{A}_c \neq 0$$

$$\cos\mu = 1 - 2\,\frac{\bar{A}_c}{A} + 2k\,\frac{d_i}{A}\ , \qquad (25)$$

where

$$k = \pi r_0\left(\frac{2\pi R}{n}\right)^2\,\frac{1}{1 + (\sigma_y/\sigma_x)}\ (m^3)\ .$$

Equation (25) is the stability condition for an ion of mass A in a machine where ions have already accumulated to a density d_i.

At the beginning, when $d_i = 0$, only $A_1 > \bar{A}_c$ can accumulate. When $d_i > 0$,

$$A_2 > \bar{A}_c - kd_i \quad (A_2 > A_c)\ .$$

The limit of d_i (independent of ion mass) is

$$\bar{A}_c - kd_i = 0\ . \qquad (26)$$

The process starts if, at a given azimuth

$$A_{max} > \bar{A}_c\ .$$

4.2 Accumulation limit in colliders

In this case:

$$A_c = \bar{A}_c$$

$$\cos \mu = 1 - 2\left(\frac{A_c^2(1-u)^2}{A^2} - k\frac{d_i}{A}\right) . \tag{27}$$

At a given azimuth u_0, at the beginning $d_i = 0$:

$$A_1 > A_0 , \quad A = A_c(1-u_0^2)^{\frac{1}{2}} . \tag{28}$$

If in the residual gas only ions of mass A_1 are present:

$$kd_i = \frac{A_0^2}{A_1} . \tag{29}$$

If, in addition to ions of mass A_1, also ions of mass $A_2 > A_1$ exist, they are progressively eliminated because

$$kd_i = \frac{A_0^2}{A_1} \text{ stable} ,$$

while $\cos \mu > 1$ (unstable) for ions of mass A_2.

If ions of mass $A_3 < A_1$ also exist, then:

i) At the beginning of the process the ions of mass A_3 are unstable.

ii) At the end, they can be stable if

$$\cos \mu = 1 - 2\left(\frac{A_0^2}{A_3^2} - \frac{A_0^2}{A_3 A_1}\right) > -1 , \tag{30}$$

which happens when

$$\frac{A_0^2}{A_3^2} - \frac{A_0^2}{A_3 A_1} < 1$$

$$A_3 > \frac{A_0}{2}\left[\left(1 + 4\frac{A_1^2}{A_0^2}\right)^{\frac{1}{2}} - 1\right] . \tag{31}$$

For example, when $A_1 = A_0$,

$$A_3 > 0.62 A_0 . \tag{32}$$

4.3 The ion ladder

Can the process described in Section 4.2 continue, namely lighter and lighter ions chasing the ones immediately above in the mass scale?

For example, consider the case of an azimuth u, for which

$$A_0 = A_c(1-u^2)^{\frac{1}{2}} = A_1 = 44 \text{ (mass of } CO_2^+) . \tag{33}$$

The CO_2^+ i[illegible] accumulate up to a density

$$(d_i)_1 = \frac{1}{k}\,\frac{A_0^2}{A_1}\,. \tag{34}$$

At this density, the next lighter ions CO^+ with $A_2 = 28$ can start to accumulate since

$$\frac{A_0^2}{A_2^2} - \frac{A_0^2}{A_1 A_2} = 0.90 < 1\,. \tag{35}$$

The density reached will be:

$$(d_i)_2 = \frac{1}{k}\,\frac{A_0^2}{A_2}\,. \tag{36}$$

The next lighter ions H_2O^+ with $A_3 = 18$ cannot start to accumulate since

$$\frac{A_0^2}{A_3^2} - \frac{A_0^2}{A_2 A_3} = 2.13 > 1\,. \tag{37}$$

In this case the "ion ladder" stops at $A_2 = 28$.

If a mass $A_3 = 22$ existed, the "ion ladder" could continue. The propagation of the phenomenon depends very much on the residual gases existing in the vacuum chamber and on the initial value of A_0.

5. EFFECTS OF THE ACCUMULATED IONS ON THE CIRCULATING BEAMS

Still under the assumptions of Sections 3 and 4, we examine some of the consequences of the presence of ions on the circulating beams.

5.1 Tune shifts

The tune shift induced by a local quadrupole of strength K(s) is given by

$$\Delta Q = \frac{1}{4\pi}\int \beta(s)K(s)\,ds\,, \tag{38}$$

where

$$K(s) = \frac{e}{\gamma m_p c^2}\,\frac{\partial E_i}{\partial x,y}$$

with

E_i = electric field due to ions,

m_p = rest mass of the particle.

But

$$\frac{\partial E_i}{\partial x,y} = \frac{d_i}{\varepsilon_0}\,\frac{e}{1 + (\sigma_{x,y}/\sigma_{x,y})} \tag{39}$$

Introducing the classical particle radius

$$r_0 = \frac{e^2}{4\pi\varepsilon_0 m_p c^2}\,,$$

one obtains

$$(\Delta Q_y)_i = \frac{r_0}{\gamma} \int \frac{d_i}{1 + (\sigma_y/\sigma_x)} \beta_y(s)\, ds$$

$$(\Delta Q_x)_i = \frac{r_0}{\gamma} \int \frac{d_i}{1 + (\sigma_x/\sigma_y)} \beta_x(s)\, ds \quad (40)$$

It is interesting to compare Eqs. (40) with the incoherent tune shift due to the space charge of the particles themselves:

$$(\Delta Q_{x,y})_{sc} = \frac{r_0}{\gamma} \int \frac{d_p}{1 + (\sigma_{x,y}/\sigma_{y,x})} \frac{\beta(s)}{\gamma^2}\, ds \quad (41)$$

Since $d_i = \eta\, d_p$, one has finally:

$$(\Delta Q_{x,y})_i = \eta \gamma^2 (\Delta Q_{x,y})_{sc} \quad (42)$$

5.2 Increased gas pressure

When only one type of ion is present in the residual gas:

$$P_{tot} = P_i + P_m$$

$$d_i = 9.65 \times 10^{24} \frac{P_i}{T} \; (m^{-3}\ {}^\circ K\ Torr)$$

$$d_m = 9.65 \times 10^{24} \frac{P_m}{T} \; (m^{-3}\ {}^\circ K\ Torr)$$

$$P_{tot} = \frac{1}{9.65} \times 10^{-24}\, T\, d_m \left(1 + \eta \frac{d_p}{d_m}\right).$$

But η varies around the machine and so does P_{tot}. One can define an average $\langle P_{tot} \rangle$:

$$\langle P_{tot} \rangle = \frac{1}{2\pi R} \int_0^{2\pi R} P_{tot}(s)\, ds .$$

The calculation must be repeated for each gas component.

From $\langle P_{tot} \rangle$ it is possible to deduce the nuclear and multiple scattering equivalent pressures following the standard procedure described in Guignard[2] (pages 31 to 34).

For nuclear scattering, the beam intensity I decreases as follows:

$$\frac{1}{I} \frac{dI}{dt} = c\, n_{NS} \sigma_{n,N} , \quad (43)$$

where

n_{NS} = equivalent density of nitrogen atoms for nuclear scattering (m^{-3}).

$\sigma_{n,N}$ = total cross-section for the beam particles on nitrogen (m^2).

$$n_{NS} = 1.93 \times 10^{25} \frac{P_{NS}(\text{Torr})}{T(°K)} . \qquad (44)$$

P_{NS} is the equivalent nitrogen pressure giving the same effects for nuclear scattering:

$$P_{NS} = \frac{1}{2\sigma_{n,N}} \sum_i \left(P_i \sum_j \sigma_{n,j}\right) , \qquad (45)$$

where

- i is numbering the type of gas in the mixture,
- j is numbering the type of atoms in a molecule of the i^{th} gas. If there are m identical atoms B in a molecule, the summation on j should include all of them and can be replaced by $m\sigma_{n,B}$.
- P_i is the pressure of the i^{th} gas (Torr),
- $\sigma_{n,j}$ is the total nuclear cross-section for the beam particles on the j^{th} atom (m^2).

For multiple scattering, the increase of the transverse beam dimensions is given by

$$\frac{1}{\sigma_{x,y}} \frac{d\sigma_{x,y}}{dt} = 1.09 \times 10^{-23} \frac{\bar{\beta}_{x,y}}{p^2 \pi E_{x,y}} n_{MS} , \qquad (46)$$

where

- $\bar{\beta}_{x,y}$ is the average amplitude function around the machine (m),
- p is the beam momentum (GeV/c),
- $E_{x,y}$ is the beam emittance defined as $4\sigma^2_{x,y}/\beta_{x,y}$ (rad·m),
- n_{MS} is the equivalent density of nitrogen atoms for multiple scattering (m^{-3})

The atom density n_{MS} is related to the nitrogen equivalent multiple scattering pressure P_{MS} (Torr) by

$$n_{MS} = 1.93 \times 10^{25} \times 2 \frac{P_{MS}(\text{Torr})}{T(°K)} , \qquad (47)$$

P_{MS} is the equivalent nitrogen pressure giving the same multiple scattering effect as the actual residual gas:

$$P_{MS} = \frac{1}{2G_N} \sum_i \left(P_i \sum_j G_j\right)$$

where

- i is numbering the type of gas in the mixture,
- j is numbering the atoms in a molecule of the i^{th} gas. If there are m identical atoms B in a molecule, the summation on j should include all of them and can be replaced by mG_B,
- P_i is the pressure of the i^{th} gas (Torr),
- G_j is the absolute gas factor for the j^{th} atom (see Table 1 of Ref. 2).

6. APPLICATION TO THE SPS $p\bar{p}$ COLLIDER

The analysis developed so far with the specified simplifying assumptions can be applied to the SPS $p\bar{p}$ Collider. The kick parameter a and the critical mass A_c can be expressed by

$$a = r_0 c \frac{N}{n} \frac{1}{\sigma^2} \frac{1}{A} = 45.9\left(\frac{N}{n} \frac{1}{10^{11}}\right) \frac{10^6}{\sigma^2\,(\mathrm{mm})} \frac{1}{A} \tag{48}$$

$$A_c = \frac{1}{4}\, a \frac{T}{n}\, A \quad = 11.5\left(\frac{N}{n}\frac{1}{10^{11}}\right)\left(\frac{T}{n}\right)\frac{1}{\sigma^2\,(\mathrm{mm})}\ , \tag{49}$$

where T is the revolution time in μs.

In the case of six bunches (n = 6) and p = 270 GeV/c, which will be the nominal configuration once the antiproton intensity is increased, and with

$$\frac{N}{n} = 10^{11} \quad \text{and} \quad \beta\gamma E_x = \beta\gamma E_y = 29\ (\mu\mathrm{m\cdot rad})\ ,$$

one has

$$A_c = \bar{A}_c = 44 \quad (\text{equal to A of } CO_2^+)\ .$$

The azimuthal range of stability is:

$$\left(1 - \frac{A^2}{A_c^2}\right)^{\frac{1}{2}} \leq u \leq 1\ . \tag{50}$$

CO_2^+ (A = 44)	start(s) to accumulate for	$0 \leq u \leq 1$
CO^+ and N_2^+ (A = 28)	" " " "	$0.77 \leq u \leq 1$
H_2O^+ (A = 18)	" " " "	$0.91 \leq u \leq 1$
O_2^+ (A = 16)	" " " "	$0.93 \leq u \leq 1$
H_2^+ (A = 2)	" " " "	$0.99 \leq u \leq 1$.

For a given type of ion of mass A, the maximum density $(d_i)_{max}$ is reached at u_0:

$$(d_i)_{max} = \frac{1}{k}\ \frac{A_c^2(1-K_0^2)}{A} = \frac{A}{k}\ . \tag{51}$$

In an azimuthal range where several types of ions can accumulate, the lightest ones chase all other types. One has

$$(d_i)_{max} = \frac{1}{k}\ \frac{A_c^2(1-u_0^2)}{A_{min}}\ . \tag{52}$$

For a round beam ($\sigma_x = \sigma_y$), the parameter k defined in Section 4.1 is

$$k = \pi r_0 \left(\frac{2\pi R}{n}\right)^2 \frac{1}{2} = 3.19 \times 10^{-12}\ (\mathrm{m}^3)\ . \tag{53}$$

$$\left(d_i\right)_{max} = \frac{A}{k} = 0.31 \times 10^{12}\ A\ (\mathrm{m}^{-3})\ . \tag{54}$$

The neutralization factor $\eta (= d_i/d_p)$ is then given by

$$\eta_{max} = \frac{A}{k}\frac{n}{N}\,\ell_B(2\pi\sigma_x\sigma_y) = 19.48\ \ell_B\sigma_x\sigma_y A\ . \qquad (55)$$

The density of the neutral molecules d_m is obtained from

$$d_m = 9.65\times 10^{24}\ \frac{P_m(\mathrm{Torr})}{T(^\circ K)} = 3.2\times 10^{22}\ P_m(\mathrm{Torr}) \qquad (56)$$

Figure 1 gives for each azimuthal range the ion density reached at this stage as a continuous line of parabolic form with the corresponding type of ions.

But the phenomenon of double ionization (very probable for a collider where beams circulate for hours) limits the ion density d_i to d_m. In Fig. 1 d_m is indicated for $P_m = 10^{-10}$ (Torr). It is seen that for such a partial pressure the neutralization factor $\eta \leq 1.4\times 10^{-4}$. Since the partial pressures of all gas components, except hydrogen, are below $\sim 2\times 10^{-11}$, the neutralization factor η should be $\leq 2.8\times 10^{-5}$.

Other phenomena, such as the ion ladder, which might occur in the n-range between 0.8 and 1.0, do not change this basic result.

Moreover the instability of trapped ions in the horizontal plane in some ranges due to the dipole field (see Section 7.4) further reduces the neutralization factor to 1 or 2×10^{-5}.

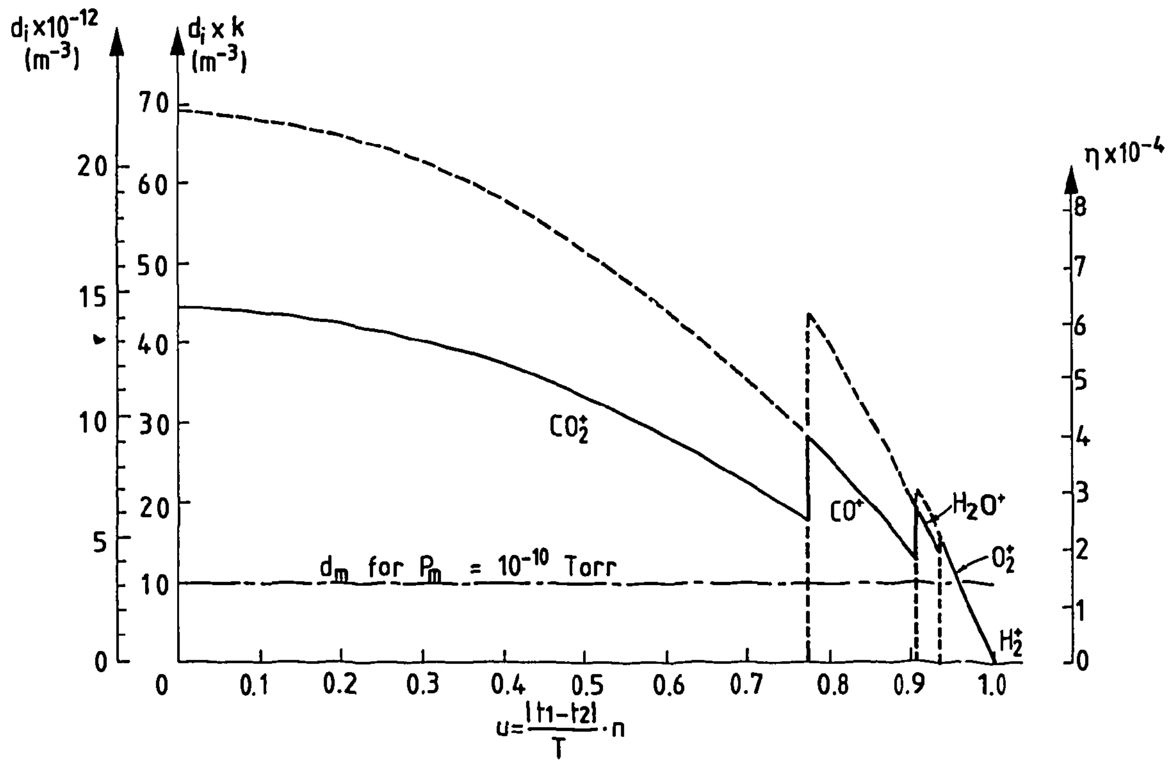

Fig. 1 Ion stability conditions for the SPS $p\bar{p}$ Collider.

7. ACCUMULATION LIMIT WITH MORE REALISTIC ASSUMPTIONS[3)]

The assumptions of Section 3 (linear electric field inside the bunch, constant transverse beam dimensions around the machine, and equal bunches) are certainly not occurring in practice.

How do the results obtained in the previous section change with more realistic assumptions? Various studies were carried out during the Workshop on $p\bar{p}$ in the SPS of March 1980[3)].

7.1 Variation of beam dimensions with the azimuth

Kissler[3)] completed the treatment, taking into account the azimuthal variation of the beam dimensions around the machine due to the betatron functions. The results are not very different from those already obtained. If anything the average neutralization is somewhat diminished.

7.2 Ion motion in the non-linear electric field of a Gaussian bunch

Gröbner[3)] carried out a computer simulation of the ion motion with the proper field of a Gaussian bunch of cylindrical symmetry:

$$E(r) = \frac{Ne}{2\pi\varepsilon_0 n \ell_B} \; \frac{1 - \exp\,(-r/r_b)^2}{r} \; . \tag{57}$$

Examples of the results obtained are given in Figs. 2 and 3 for the following conditions:

$$\frac{N}{n} = 10^{11} \text{ particles} \; , \quad n = 6 \text{ bunches}$$

$$r_b = 1.41 \times 10^{-3} \text{ (m)} \; , \quad r_v = 2 \times 10^{-2} \text{ (m) vacuum chamber radius} \; .$$

These conditions correspond to $A_c = \bar{A}_c = 44$ of the previous treatment, which implies that ions with a mass larger than A_c are stable and are trapped in the beam.

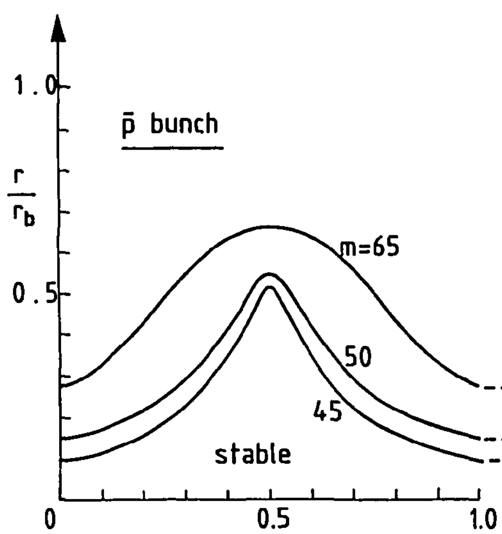

Fig. 2 Ion stability condition with non-linear electric field ($\bar{p}$ bunch, critical mass $\bar{A}_c = 44$, equal bunch spacing).

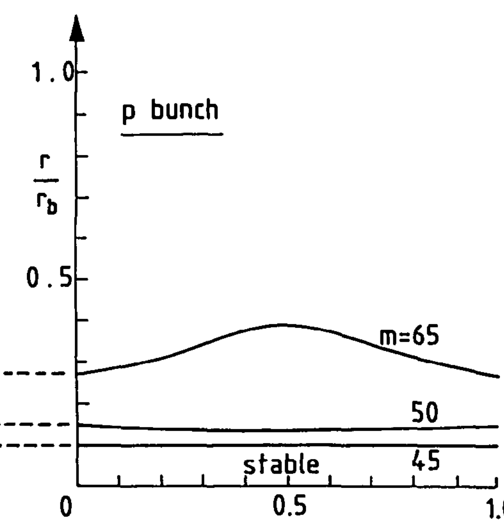

Fig. 3 Ion stability condition with non-linear electric field (p bunch, critical mass $A_c = 44$, equal bunch spacing).

It is seen from Figs. 2 and 3 that even for masses equal to or larger than A_c, only very few of the ions produced remain trapped in the beam. Heavier ions and ions produced at the centre of the bunch can be stable to a larger radius. The proton bunches are much less efficient that the antiproton bunches in producing stable ions. The conclusion is that the non-linear field leads to less ion stability (hence to more favourable conditions) than the linear one.

7.3 Effects of throbbing of the ion cloud and of bunch-to-bunch fluctuations

Schönauer[3] has calculated the influence of these effects. The first effect (throbbing of the ion cloud) leads to a reduction of the stable phase area for the ions (as already mentioned in Section 7.2 -- see Fig. 4). The second one (bunch-to-bunch intensity fluctuations) is the most prominent and causes a substantial reduction of the neutralization factor.

In fact, the phenomenon is very efficient in creating stop-bands (where ions are unstable), similar to the ones produced by quadrupole errors in normal magnetic FODO structures, but of course much wider because the variance of intensity from bunch to bunch is certainly larger than quadrupole errors. Figure 5 gives the stop-bands appearing in the

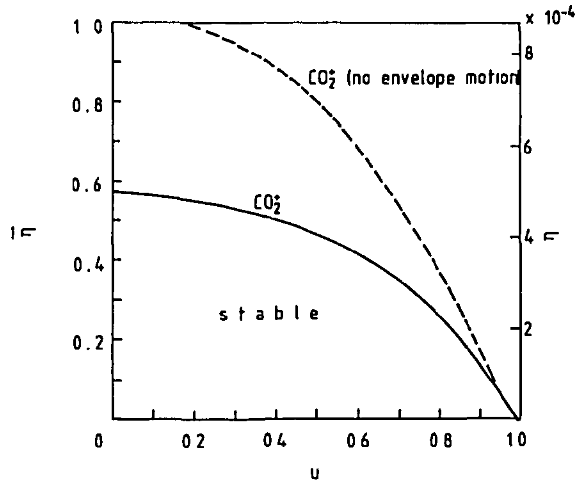

Fig. 4 Effect of throbbing of ion cloud on the stability diagram

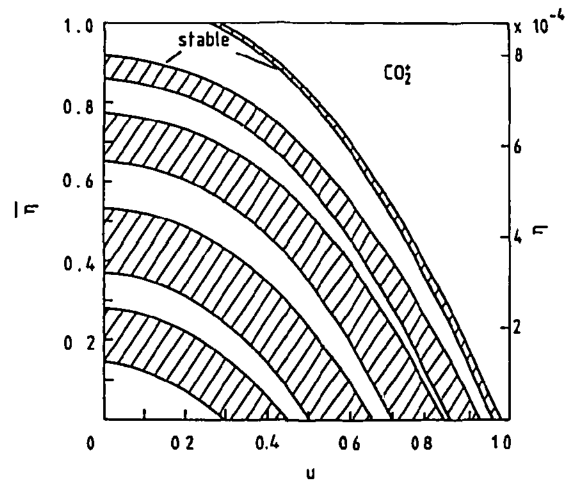

Fig. 5 Stop-bands in ion-stability diagram due to bunch-to-bunch fluctuations

stability diagram (similar to that of Fig. 1) owing to 5% variance in bunch intensity. This feature would reduce substantially the neutralization factor.

7.4 Ion instability in the horizontal plane within dipoles

The combined action of the vertical dipole field and of the electric field of the bunches makes the ions rotate between bunches. The drift-times must therefore be replaced by angular rotations with the cyclotron radian frequency ω

$$\omega = \frac{eB}{Am_p} . \tag{58}$$

It has been shown[1] that for a value of the magnetic field given by

$$B_c = \pi \frac{m_p}{e} \frac{n}{T} A \tag{59}$$

and its multiples, the ion motion is unstable and ions, if trapped vertically, are ejected from the beam region horizontally. The width of the corresponding stop-band expressed in terms of the field range can be calculated in each case. The width varies as the inverse of the order of the stop-band. In general, the stop-band width is a small percentage of the distance between stop-bands.

The global result is that ions are unstable horizontally over a sizeable part of the circumference covered by dipoles, further reducing the neutralization factor.

Potaux[3] calculated the longitudinal (azimuthal) drift of ions due to the field combination mentioned above. The drift velocity is of the order of thermal velocity ($\sim 100\ \mathrm{ms}^{-1}$) and therefore insufficient to affect the neutralization factor.

7.5 Conclusions of Section 7

It is seen than most of the effects described in Section 7 tend to lower the neutralization factor obtained from the "linear" theory. When considering the result of Section 6 for the SPS $p\bar{p}$ Collider, namely $\eta = 1$ or 2×10^{-5}, the effects of Section 7 are likely to bring down the η value well below 10^{-5}. Such a value would not give rise to any detrimental effect to the beam behaviour.

8. CONCLUSIONS

The accumulation of ions, if any, in colliders for either electrons and positrons or protons and antiprotons is unlikely to lead to detrimental effects to the beam behaviour.

In contrast, the accumulation certainly occurs for single-beam machines with negative particles such as electrons.

The analysis made in this paper provides the basis for the evaluation of the neutralization factor and its consequences on the beam.

* * *

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