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FEASIBILITY STUDY OF STOCHASTIC COOLING OF BUNCHES IN THE SPS

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The average luminosity of the SPS collider could be improved if the slow blow-up of transverse emittances due to beam-beam and intrabeam scattering effects were to be reduced by a transverse cooling system. We ~~shall~~ examine the parameters of such a system and propose a technological approach which seems better suited to the case of a few bunches circulating in a large machine.

1. COOLING PARAMETERS

The cooling rate of transverse emittance is, in the best of all cases, given by:

$$\frac{1}{\tau} = \frac{W}{N} \tag{1}$$

where W is the bandwidth of the cooling system and N the number of particles in a continuous coasting beam[1]. For a bunched beam, and neglecting for the moment the longitudinal dynamics, we replace N by N_{eff} [2]:

$$N_{eff} = N_b/(\Delta t \cdot f_o)$$

N_b	:	number of particles in the bunch
Δt	:	bunch duration
f_o	:	revolution frequency.

N_{eff} is the total number of particles in an equivalent coasting beam having the same density.

If we take $N_b = 10^{11}$, $\Delta t = 5$ ns [with a new 100 MHz RF system[3]] and $W = 8$ GHz, we obtain $\tau = 16$ h, which is of the same order of magnitude as the observed blow-up rates[4]. It means that, to be useful, a cooling system for the SPS must work very close to the optimum defined by equation (1).

In the more precise expression[1]:

$$\frac{1}{\tau} = \frac{W}{N_{eff}} \{2 g \xi - g^2 (\mathcal{M} + U)\} \tag{2}$$

the quantities $\mathcal{M} \geq 1$ (mixing factor) and $\xi \leq 1$ (unequal gains for all particles) should be unity, whereas U (electronic to Schottky noise ratio) should be zero, in order to achieve optimum cooling rate given by (1).

A good mixing factor ($\mathcal{M} = 1$) also ensures that feedback via the beam is unimportant.

In the particular case of very low electronic noise (U = 0), it is conceivable to double the cooling rate by using two identical systems spaced by a quarter betatron wavelength.

2. BANDWIDTH REQUIREMENTS

The lower frequency of the system is determined by the condition of good mixing (complete overlap of betatron bands).

For a bunched beam, and for high enough frequencies, the half width of the betatron lines is approximately

$$\Delta f = n\, a_m\, f_s$$

where n is the revolution harmonic number, a_m the maximum synchrotron phase oscillation amplitude in azimuth around the ring, and f_s the maximum synchrotron frequency in the bunch. This is equivalent to the coasting beam case where

$$\Delta f = \eta\, n\, f_0 \frac{\Delta p}{p}$$

for sufficiently high frequencies. ($\Delta p/p$ is the half momentum spread).

The condition $\mathcal{M} = 1$ (good mixing) is equivalent to the condition for the betatron bands to overlap:

$$\Delta f \simeq (^1/_4)\, f_o .$$

In the SPS case, $\Delta p/p$ is limited by the chromatic aberrations of the machine with its low beta insertions. Taking the present value $\Delta p/p \simeq \pm 7 \times 10^{-4}$, we find a lower limit for n, and hence for the lowest frequency of the system ($f_{min} \simeq 8$ GHz).

The mixing between pick-up and kicker, which reduces the factor ξ in equation (2) limits the upper frequency. Unfortunately, the pick-up to kicker distance cannot be made very small, due to the extra delays in the signal transmission along the vertical shafts. It seems unrealistic to consider a pick-up to kicker distance smaller than two sextants of the machine, and consequently the cooling stops for particles having the maximum energy deviation ($\xi = 0.5$) at $f_{max} = 16$ GHz, if the same gain as for optimum cooling of particles at the centre of the distribution is applied. There is however the possibility of choosing a different gain, optimized for the whole distribution and including the phase shifts introduced by beam feedback, which will result in finite cooling times for all particles.

3. USE OF AN HARMONIC CAVITY

In the case of a bunched beam, the betatron Schottky bands are not uniformly populated. The synchrotron satellites, spaced by f_s (synchrotron frequency) having a line width $\mu \cdot \Delta f_s$ (μ = satellite number, Δf_s synchrotron frequency spread) overlap only for very high values of μ. To avoid the corresponding reduction of cooling rates for bunches of a moderate length ($\Delta t \times f_{RF} \approx 0.5$), it is necessary to increase Δf_s by an harmonic RF system. Also, with extremely small Δf_s values for conventional single RF bunches, the beam feedback would reduce the closed loop Schottky signal to negligible values (to zero actually, if $\Delta f_s = 0$)[11].

With a 100 MHz RF system in the SPS, the present 200 MHz cavities can be used as a second harmonic RF system, which will provide a large frequency spread $\Delta f_s \approx f_s$ (Fig. 1). In this situation the longitudinal dynamics is very close to the coasting beam case and Equation (1) with $N = N_{eff}$ can be applied. Schottky power distribution over a betatron band is more or less uniform in this case.

The problem of longitudinal diffusion due to RF noise is certainly enhanced by the presence of an harmonic system, because a large Δf_s means a small overall phase loop gain. Experiments have been carried out in the SPS using the 800 MHz harmonic cavity to evaluate this effect[5]. The equilibrium lifetime turns out to be a relatively weak function of the synchrotron frequency distribution: measurements have shown a decrease of the lifetime by factors between 3 and 5 when using an harmonic cavity to provide maximum Δf_s. Therefore this effect should not be a problem as the RF noise limited lifetime is in the order of several hundred hours.

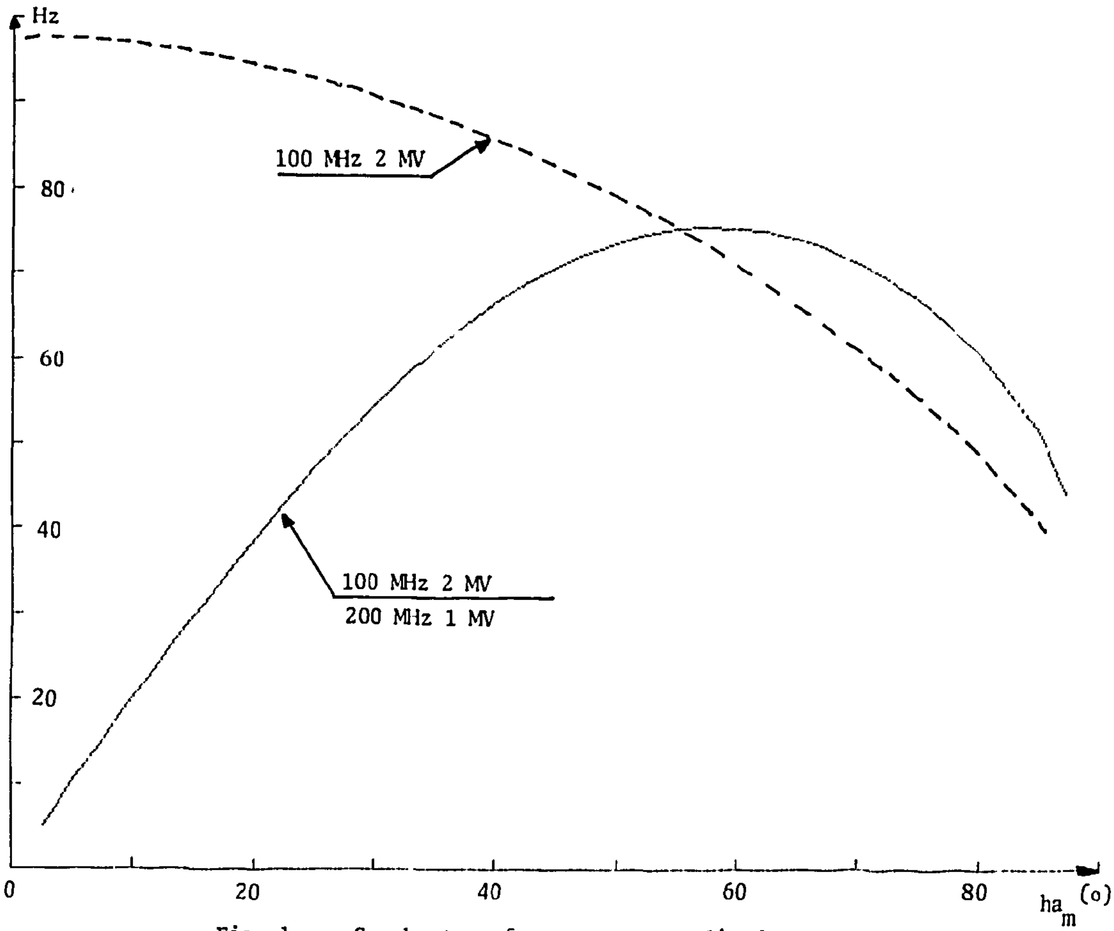


Fig. 1 - Synchrotron frequency vs amplitude

4. SIGNAL PROCESSING

The transmission of a signal having an 8 GHz bandwidth over 2 kilometres looks a very difficult problem (dispersion of the transmission, stability of delay, etc.). But we notice that this large bandwidth is only needed for a very short time Δt (bunch duration), and we can considerably slow down the transmission rate of the information by spreading the data to be transmitted in time.

The time available is determined by the distance between bunches and also the difference in time of flight of the beam and the signal.

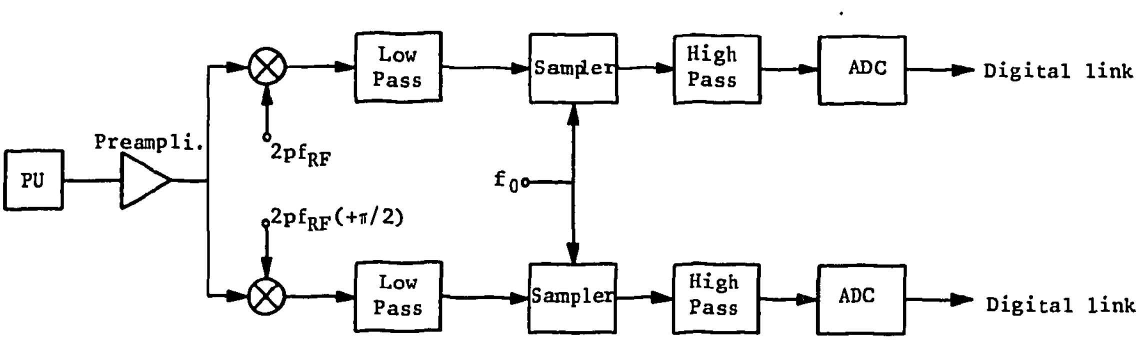
There are M independent numbers characterizing the signal per bunch and per turn, where[9,10]:

$$M = 2\,W\,\Delta t$$

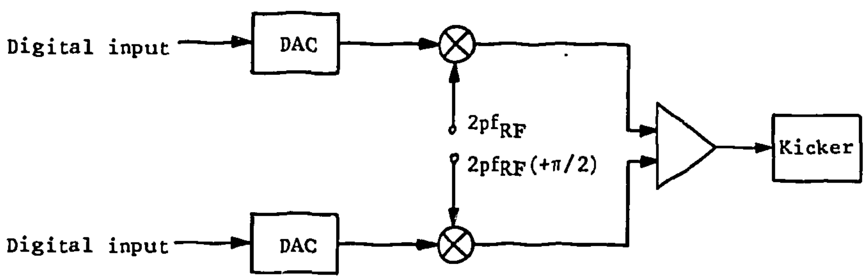
$M = 80$ for $W = 8$ GHz and $\Delta t = 5$ ns.

These M numbers can be extracted by Fourier analysis of the beam signals at multiples 2p of the RF frequency [for $\Delta t = 1/(2\,f_{RF})$]. A possible layout is schematized in Fig. 2. The pick-up signal is mixed with the RF frequency harmonic by two quadrature mixers followed by low pass filters. It is necessary to have M/2 such channels to extract the information characterized by only M numbers which need to be transmitted. They appear as the heights of the pulses delivered by the low-pass filters.

It may be convenient in such a system to use M/2 narrow-band pick-ups with the advantage of a high sensitivity and a good directivity (independent cooling systems for proton and antiproton beams).



M/2 channels at the pick-up side



M/2 channels at the kicker side

Fig. 2 - Layout of the system.

Sampling of the pulses at the revolution frequency automatically provides rejection of the stationary part of the pick-up signal (i.e. the high frequency components of the bunch waveform). Only the fluctuating Schottky components are retained for analog to digital conversion.

Data transmission can be done in serial digital form to slow down the transmission rate and benefit from noise immunity inherent to digital technology.

At the receiving end the Schottky signals are converted into analog pulses and mixed up in frequency to restore the original signal. The kickers also may be narrow band and directional. The overall delay of the transmission is only determined by the proper phasing of the local oscillators (absolute clocks) synchronized on multiples of the RF frequency. The dispersion and delay stability of the transmission link are relatively unimportant.

5. THE PICK-UPS

The two conflicting requirements for the pick-up are the sensitivity and the aperture. The aperture of the SPS is determined during coast by the beam scrapers. For a round beam ($\beta_H = \beta_V = 50$ m) the diameter of the aperture is 14 mm, whereas the r.m.s. oscillation amplitude is ~ 1 mm.

The sensitivity of the pick-up determines the Schottky to thermal noise power ratio (1/U) and the corresponding reduction in cooling rate.

Consider the equivalent coasting beam containing N_{eff} particles. For this case we can easily calculate the U factor for a given pick-up sensitivity. For a bunched beam, the Schottky signal simply looks gated in time for the bunch duration Δt (pick-up bandwidth >> $1/\Delta t$). If, after the first preamplifier, which defines the signal to noise ratio, we gate out the signal for a time Δt before processing, the signal to noise ratio of the bunched beam (N_b particles) will be the same as that of the equivalent coasting beam (N_{eff} particles).

From the following equation, valid for a 3 dB noise figure preamplifiers (see Appendix):

$$\frac{1}{U} = \frac{P_{sch}}{P_{th}} = \frac{N_{eff.}\ e^2\ f_0\ A^2}{2kT\ R}\ Z'^{\,2}_{PU}$$

where A is the r.m.s. betatron oscillation amplitude, Z'_{PU} is the pick-up sensitivity [volts in an R = 50 Ω load/(beam amperes × displacement)], we obtain:

$$\frac{1}{U} = \frac{P_{sch}}{P_{th}} = 10$$

for $N_{eff} = 4 \times 10^{14}$ ($N_b = 10^{11}$), R = 50 Ω and $Z'_{PU} = 3$ Ω/mm.

It seems that such a sensitivity can be obtained with a few loop type couplers[6] scaled down in size to match the aperture (14 mm) and the frequency band (8-16 GHz).

Another possibility is to use the deflecting mode of the well-known disk-loaded waveguide. Much higher sensitivities can be achieved (~ 1000 Ω/mm per m of structure at 8 or 16 GHz), but many pick-ups would be necessary to cover the entire bandwidth.

The pick-ups should of course be movable in order not to intercept the beam during injection and acceleration.

The phase response of the pick-up is not of ultimate importance as each frequency is processed independently. What is more important is the rejection of the coherent longitudinal signal over the entire bandwidth, and this may finally determine the type of pick-up to be used (octave bandwidth loop couplers or narrow-band structures).

For lower intensity beams, e.g. $N_b = 10^{10}$, the factor U becomes unity with the same pick-up, and the optimum cooling rate is halved (τ = 3.2 hours instead of 1.6 hours).

6. ANALOG SIGNAL PROCESSING

Mixing with a multiple of the RF frequency will introduce extra electronic noise in the system. This is because the phase noise of the local oscillator will appear translated at low frequency around each residual longitudinal line (Fig. 3).

The ratio of the residual coherent longitudinal power per line to the transverse Schottky power per band is given by[7]

$$r = 8 \left|\frac{\delta}{A}\right|^2 N_b$$

where δ is the precision with which the pick-up can be centred on the beam. Taking δ = 0.1 mm, A = 1 mm and $N_b = 10^{11}$ we obtain r ≈ 100 dB at low frequencies.

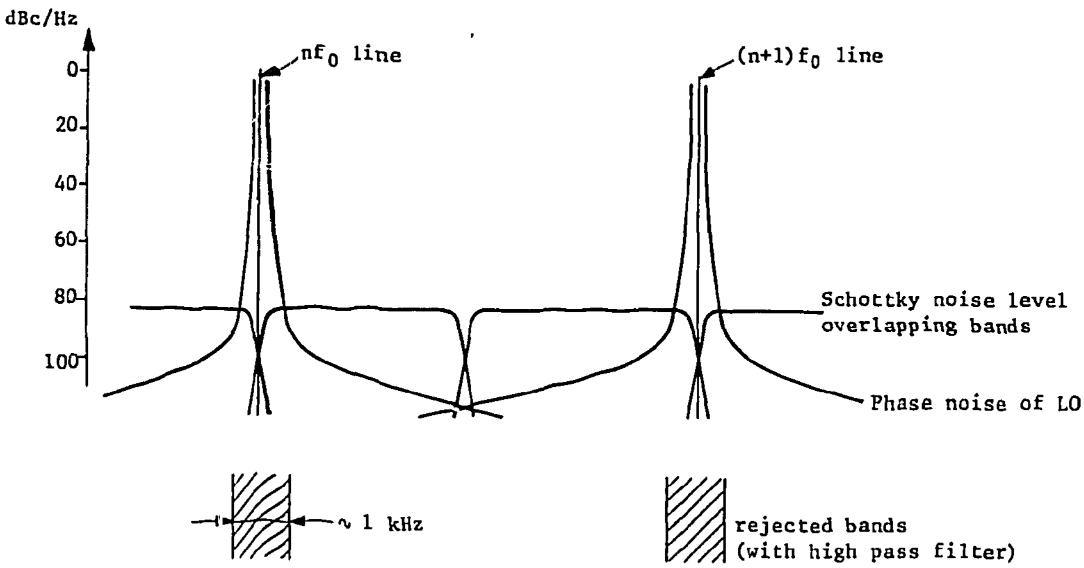


Fig. 3 - Influence of LO phase noise.

Fortunately, the longitudinal components of the beam decrease rapidly with frequency. Taking a parabolic bunch shape (pessimistic case) the coherent line is reduced by more than 60 dB at 8 GHz. Therefore the Schottky noise power density, referred to the residual longitudinal line is -40 dB per half revolution frequency (overlapping bands) or in more usual units -83 dBc/Hz.

At low frequencies (near an nf_o line) the local oscillator noise will dominate (Fig. 3). However, for a typical microwave oscillator in the 10 GHz range, where phase noise can be as low as -90 dBc/Hz at 1 kHz [8)], all frequencies above 1 kHz are almost noise free. The low frequency components will be rejected by the sampler followed by a high pass filter having a cut-off frequency of ~ 1 kHz. The reduction of useful Schottky power (2 x 1 kHz/43.5 kHz) is negligible even for completely overlapping bands.

This analysis is valid if we consider only one revolution line per channel, which implies a filtering time of the order of one revolution period. This is of course not acceptable, as the time delay has to be kept to a minimum, and therefore many f_o lines will be transmitted through the low pass filter (Fig. 2). We take for instance 300 f_o lines (bandwidth 13 MHz) which corresponds to a time delay of about 30 ns. All longitudinal lines will be added coherently, and the same is true for the associated local oscillator noise. For the Schottky power, we notice that, due to the bunched nature of the beam, successive betatron bands are correlated, and consequently they also have to be added coherently. The ratio between Schottky power and phase noise power is therefore not affected by taking many Schottky bands.

In the previous analysis we have assumed that the longitudinal components of the beam are represented by noise-free RF generators. This is of course not true at low frequencies (near and below f_s), but seems a reasonable approximation above 1 kHz (>> f_s) where the beam is extremely stiff.

Another approach to the local oscillator problem is to directly synthesize the $2.p.f_{RF}$ mixing frequencies using the beam itself. A longitudinal pick-up electrode, followed by a step recovery diode and a comb filter can generate very high harmonic frequencies exactly synchronous with the stationary components of the beam signals. The noise introduced by the step recovery diode can be extremely small[a]. One could use a similar arrangement at the kicker end, and automatically obtain the proper delay of the cooling system by using the bunch itself to trigger the kickers.

7. THE TRANSMISSION LINK

A fast analog to digital converter (ADC), with a conversion time less than ~ 10 ns follows the sampler. The number of bits of the ADC should be such that quantization noise is unimportant. If we remember that the r.m.s. Schottky signal is about three times the thermal noise signal (power ratio U ≈ 0.1) it seems that 4 to 5 bits are sufficient, which is no problem with existing hardware (6 to 8 bits, 100 M samples/second).

The number of bits to be transmitted per bunch and per turn is 5 × M if we choose a 5 bit ADC. (400 bits for M = 80). The parallel data at the ADC outputs is converted into serial form for transmission. The time available for this conversion determines the transmission link capacity (Fig. 4).

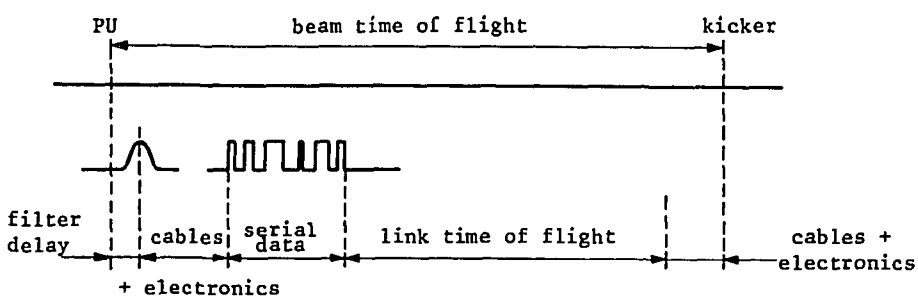


Fig. 4 - Transmission link delays.

The difference in time of flight between the beam along the arc and the straight transmission link (at the speed of light) is 1.32 μs if pick-ups and kickers are separated by two sextants of the machine.

The fixed delays in filters and electronics (~ 100 ns) and in the cables from the tunnel to the surface (2 × 100 m, i.e. ≈ 800 ns) leave about 400 ns for parallel to serial conversion. Therefore the transmission link capacity amounts to 400 bits in 400 ns, i.e. 1 Gbit/second.

A microwave radio link is an obvious choice to handle the required capacity, with carrier frequencies in the order of several ten GHz. However, a much more economical solution looks possible using an infrared carrier (λ = 850 nm). Fast modulated (up to 6 GHz) lasers exist on the market as well as fast avalanche photo detectors. The laser is coupled to an optical system which corrects the astigmatism and provides a small divergence beam (~ 1 mrad). Preliminary tests have been carried out at CERN using such a commercial laser (cheap version, bandwidth 800 MHz) and a Cassegrain telescope at the receiving end. The very first results have demonstrated the transmission of 500 MHz bandwidth over 500 m distance. Further tests should determine the influence of the dispersion of atmosphere on the bandwidth of the overall system.

Referring to commercial data for such infrared transmission systems, the availability of the link should be ~ 99% for 2 km distance.

8. KICKERS AND POWER AMPLIFIERS

In the case of bunched beam cooling the required peak power at the kicker, during the bunch passage corresponds to the average power of a cooling system working with the equivalent N_{eff} number of particles, and with optimum gain.

As our system is designed with a very small U, the output power does not depend on the pick-up sensitivity (see Appendix):

$$\hat{P} = \frac{4\beta^2 E^2}{N_{eff}} \, \frac{1}{\beta_K \beta_P} \, \frac{1}{K_\perp^2} \, A^2 \, \frac{W}{Rf_0}$$

$\hat{P}$ is the peak power at the kicker and $K_\perp$ is a dimensionless coefficient which characterizes the kicker sensitivity.

For $N_{eff} = 4 \times 10^{14}$, $E = 270$ GeV, $A = 1$ mm and $\beta_P = \beta_K = 50$ m we obtain:

$$\hat{P} = 1.07 \cdot 10^3 \times \frac{1}{K_\perp^2}$$

If we use conventional loop kickers (length ~ $\lambda/4$ at mid frequency = 6.25 mm, width = 14 mm) the factor $K_\perp$ is of the order of unity. To arrive at a reasonable power (several watts), one would use several tens of these kickers, the total length being of the order of one metre.

The other solution is to use the disk-loaded waveguide whose $K_\perp$ factor is ~ 20 per 10 cm of structure (typical kicker length). There one would use M/2 = 40 such narrow-band kickers (4 m total length), the power per channel being ~ 2 Watts to achieve the same peak deflection as an octave bandwidth system.

Acknowledgements

The experiments on the optical infrared link were proposed and carried out by H. Beger, J. Bosser, J. Mann and D. Stellfeld.

APPENDIX

1. Schottky to thermal noise ratio

For initial betatron phases randomly distributed between 0 and 2π, the transverse dipole moment Schottky signal per betatron band satisfies[7,11]:

$$\langle d_{(n\pm Q)}(t) \rangle = 0 \qquad (A1)$$

$$[d_{(n\pm Q)}(t)]^2_{RMS} = \langle |d_{(n\pm Q)}(t)|^2 \rangle = \frac{N}{2}(ef_0)^2 \langle A^2 \rangle \qquad (A2)$$

where N is the total number of particles in the bunch, e the electric charge, f_o the revolution frequency in the ring, $\langle A^2 \rangle$ the mean square betatron amplitude, averaged over the distribution of particles in the bunch and Q the betatron tune. Note that this would be the RMS dipole moment signal only if the PU (pick-up) filtered out just a single betatron band and it is the same as for a continuous coasting beam containing the same number N of particles. This Schottky signal power is the same for all betatron bands, distributed over their increasing widths with decreasing densities as we go higher in frequency, until the betatron bands begin to overlap, beyond which the power density in frequency remains more or less constant[11].

If the PU is characterized by a sensitivity (impedance) of Z'_{PU} in ohms per displacement [volts across a load R/(beam current in amperes × beam displacement)], the Schottky signal power to the preamplifier input dissipated across its characteristic impedance R_{in} (typically 50 Ohms) in watts is:

$$P_{(n\pm Q)} = \frac{V^2_{RMS}}{R_{in}} = \frac{|Z'_{PU}|^2}{R_{in}} \cdot \frac{N}{2}(ef_0)^2 \langle A^2 \rangle \qquad (A3)$$

For a broad band PU with bandwidth W [Fig. 5(a)], there are $2 \cdot W/f_o$ betatron bands and average Schottky power of transverse dipole signal over a full revolution period T_o is just the sum of the individual powers per band and is given by [Fig. 5(b)]:

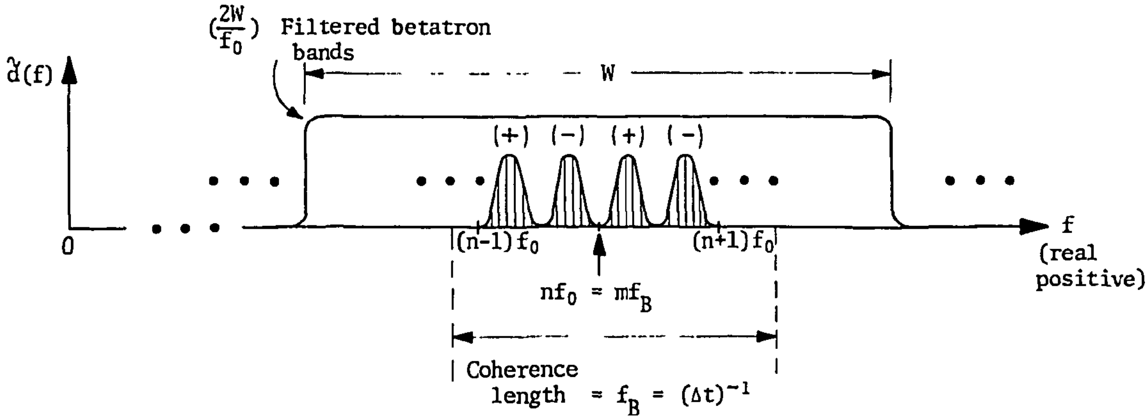
$$[P_W]_{av.} = \frac{|Z'_{PU}|^2}{R_{in}} \cdot \frac{N}{2}(ef_0)^2 \langle A^2 \rangle \cdot \frac{2W}{f_0} \qquad (A4)$$

However, the distribution of this power in time depends on the nature of the PU. We consider two cases.

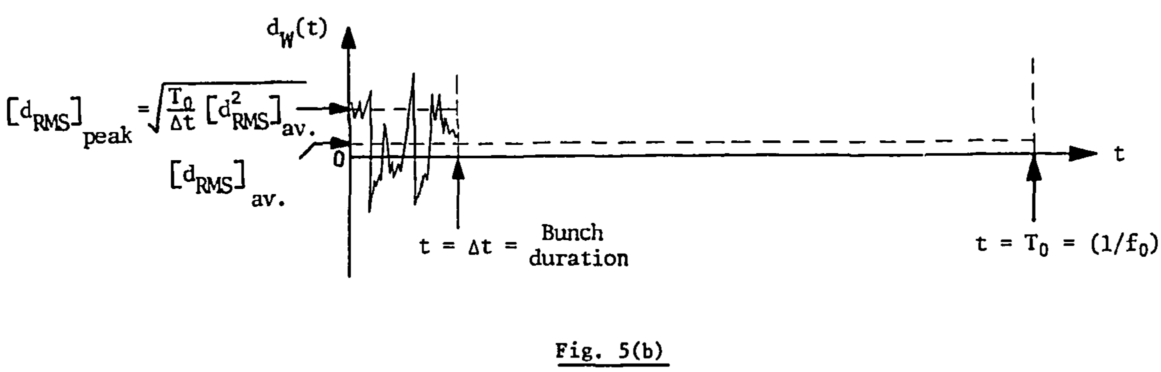
(A) If the PU rise-time is much shorter than the bunch duration Δt (broad-band PU with bandwidth much greater than bunch frequency $W \gg 1/\Delta t = f_B$), all this power is concentrated only in a time interval of bunch duration $\Delta t \ll T_0$ and the <u>peak pulsed power</u> is enhanced over the <u>average power</u> by the bunching factor $B = T_0/\Delta t = 1/(f_0 . \Delta t)$ so that

$$\hat{P}_W = [P_W]_{peak} = B \cdot [P_W]_{av.} = \frac{|Z'_{PU}|^2}{R_{in}} \cdot Ne^2 \langle A^2 \rangle W f_B \qquad \text{(A5)}$$

where $f_B = 1/\Delta t$ is the bunch frequency [see Fig. 5(b)]. In the frequency domain we observe that all the betatron bands within a frequency interval of f_B are strongly correlated due to the bunched nature of the beam and add up coherently rather than incoherently in mean square for the peak power[11]. However, bands separated by more than or equal to f_B, add up incoherently and the situation is similar to a continuous coasting beam.



<u>Fig. 5(a)</u>



<u>Fig. 5(b)</u>

For a 3 dB noise figure preamplifier, the thermal noise power in a bandwidth W is simply 2 kT.W so that the peak pulsed Schottky signal to thermal noise power ratio, for overlapping betatron bands is:

$$\frac{1}{U} = \frac{\hat{P}_W}{2\ kT \cdot W} = \frac{(N \cdot B)\ e^2 f_0 \langle A^2 \rangle}{2\ kT\ R_{in}} \ |Z'_{PU}|^2 = \frac{N_{eff}\ e^2 f_0 \langle A^2 \rangle}{2\ kT\ R_{in}} \ |Z'_{PU}|^2 \qquad (A6)$$

where $N_{eff.}$ = (N.B) is the effective number of particles in an equivalent continuous coasting beam description.

(B) If the PU rise-time is much longer than the bunch duration Δt (narrow-band PU with bandwidth much less than the bunch frequency $f_0 << W << f_B = 1/\Delta t$), the peak power from $[(n_\ell/2)]$ coherent (+) betatron bands and $(n_\ell/2)$ coherent (-) betatron bands is:

$$\hat{P}_W = 2\ P_{(n\pm Q)} \cdot \left(\frac{n_\ell}{2}\right)^2 = [P_W]_{av.} \cdot \frac{n_\ell}{2} = [P_W]_{av.} \cdot \frac{T_0}{(\Delta t)_W} = \frac{|Z'_{PU}|^2}{R_{in}}\ \frac{N}{2}\ (ef_0)^2\ \langle A^2 \rangle \frac{n_\ell^2}{2} \qquad (A7)$$

where $(\Delta t)_W = 1/W$ is the effective bunch length in time as seen by the PU. For a set of M such identical PUs as above, but centred on frequencies spaced by the bunch frequency f_B (Fig. 6), the peak power is given by:

$$\hat{P}_W = P_{(n\pm Q)} \cdot \frac{n_\ell^2}{2} \cdot M = [P_W]_{av.} \cdot \frac{n_\ell}{2} \cdot M = \frac{|Z'_{PU}|^2}{R_{in}} \cdot \frac{N}{2}\ (ef_0)^2\ \langle A^2 \rangle \frac{n_\ell^2}{2}\ M \qquad (A8)$$

The peak Schottky signal to thermal noise power ratio for the whole system is then:

$$\frac{1}{U} = \frac{\left(N \cdot \frac{n_\ell}{2}\right) e^2 f_0 \langle A^2 \rangle}{2\ kT\ R_{in}} \ |Z'_{PU}|^2 = \frac{(N_{eff})_W\ e^2 f_0\ \langle A^2 \rangle}{2\ kT\ R_{in}} \ |Z'_{PU}|^2 \qquad (A9)$$

where

$$(N_{eff})_W = N(n_\ell/2) = N \cdot [T_0/(\Delta t)_W].$$

For non-overlapping betatron bands, Schottky power stays the same but the thermal power increases with harmonic number since betatron bands spread out as we go high in frequency. Thermal noise power P_{th} in a bandwidth W with no overlapping bands is:

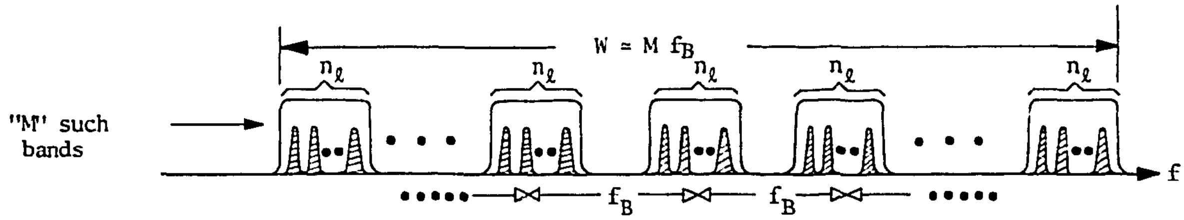


Fig. 6

$$P_{th} = 2kT \cdot 2 \sum_{n_1}^{n_2} n \cdot \Delta f \simeq 4kT\, \Delta n \cdot \bar{n} \cdot \Delta f \tag{A10}$$

where $\bar{n} = \frac{1}{2}(n_1 + n_2)$ and $\Delta n = n_2 - n_1 = W/f_0$ are the average and the range respectively of the revolution harmonic number in the bandwidth and $\Delta f \simeq 2\eta \cdot (\Delta p/p) \cdot f_0$ is the revolution frequency spread in the beam in the equivalent continuous coasting beam language. Instead of (A6), we then have

$$\frac{1}{U} = \frac{(N \cdot B)\, e^2 f_0^2\, \langle A^2 \rangle}{4kT \cdot \Delta f \cdot \bar{n}\, R_{in}} \, |Z'_{PU}|^2 \tag{A11}$$

and similarly for (A9).

2. Kicker power

The total gain per frequency line can be defined as

$$\alpha(\Omega) = \frac{(\beta_K \beta_{PU})^{\frac{1}{2}}}{2} \cdot \frac{1}{\beta E} \left[Z'_{PU} \cdot g_A \cdot K_\perp \right] (\Omega) \tag{A12}$$

where β_K, β_{PU} are the usual lattice β-functions at the kicker and PU respectively, β the relativistic velocity factor, E the energy, g_A the pure electronic gain of the amplifiers and $K_\perp$ the kicker sensitivity defined in terms of the transverse kick

$$\Delta X'(\Omega) = \frac{U_0}{\beta E} K_\perp (\Omega) \tag{A13}$$

experienced by a particle in a single pass through the kicker driven by $U_K = U_0 \exp(i\ \Omega t)$. Assuming there is no thermal noise $U = 0$ and perfect mixing $\mathcal{H} = 1$ corresponding to complete betatron band overlap (i.e. beam Schottky noise is almost white), the mean square change in betatron amplitude per unit of time due to Schottky noise heating is:

$$\frac{\langle \Delta A^2 \rangle}{T_0} = \frac{\alpha^2 [d]^2_{RMS}}{T_0} = \alpha^2 f_0 \left[\frac{N}{2} (ef_0)^2 \ \langle A^2 \rangle \right] \cdot \frac{n_\ell^2}{2} M \qquad \text{(A14)}$$

and the change in betatron amplitude squared per unit of time due to coherent cooling force is

$$\frac{\Delta A^2}{T_0} = 2\,A \cdot \frac{\Delta A}{T_0} = - \frac{\alpha (ef_0)\ A^2}{T_0} n_\ell M = - \alpha f_0 (ef_0)\ A^2\ n_\ell M \qquad \text{(A15)}$$

The overall cooling rate can then be written as (for $U = 0$):

$$\frac{1}{\tau} = \frac{1}{A^2} \frac{dA^2}{dt} = \frac{W}{N_{eff.}} \left[2g - g^2 \right] \qquad \text{(A16)}$$

where

$$g = \alpha \cdot e\ f_0 \cdot N \frac{n_\ell}{2} = \frac{\left[\beta_K \beta_{PU} \right]^{\frac{1}{2}}}{2\beta E} \left[Z'_{PU} \cdot g_A \cdot K \right] (ef_0) \cdot \left(N \frac{n_\ell}{2} \right) \qquad \text{(A17)}$$

and $W = M\ f_B$ is the total bandwidth. For optimum cooling, the optimum gain g_{opt} for individual samples processed by the system should be unity:

$$g_{opt.} = 1 = \frac{(\beta_K \beta_{PU})^{\frac{1}{2}}}{2\beta E} \left[Z'_{PU} \cdot g_A \cdot K_\perp \right] (ef_0) \left(N \frac{n_\ell}{2} \right) \qquad \text{(A18)}$$

corresponding to $1/\tau_{opt.} = W/N_{eff.}$. The kicker power (peak) is then given by [from (A18) and (A8)]:

$$\hat{P}_K = |g_A|^2 \cdot \hat{P}_W = \frac{4\beta^2 E^2}{N_{eff}.} \cdot \frac{1}{\beta_K \beta_P} \cdot \frac{1}{K_\perp^2} \langle A^2 \rangle \ \frac{W}{R_{in}\ f_0} \qquad \text{(A19)}$$

assuming negligible thermal noise $U \approx 0$.

For the longitudinal current signal, an essential difference is

$$< I_n > \sim e\, f_o N \neq 0$$

and more importantly the power in the n^{th} revolution band is

$$\langle |I_n|^2 \rangle = e^2 f_0^2 [\underbrace{N}_{\text{Schottky}} + \underbrace{O(N^2)}_{\text{Coherent longitudinal line}}]$$

Thus the central lines corresponding to zeroth synchrotron harmonics, add up coherently [$O(N^2)$] as opposed to the non-zero synchrotron harmonics which add up incoherently (Schottky noise power per band $\propto N$) in the mean square. It is therefore implicit in any bunched beam cooling scheme that the central coherent longitudinal lines, undesirable for purposes of cooling incoherent betatron or longitudinal motion, be removed by suitable filtering.

REFERENCES

1. D. Möhl. G. Petrucci, L. Thorndahl, S. van der Meer, Physics and Technique of stochastic cooling, CERN/PS/AA 79-23.

2. S. Chattopadhyay, On stochastic cooling of bunches in the colliding beam mode in high energy $p\bar{p}$ storage rings, IEEE Trans. on Nucl.Science, Vol. NS 30, No. 4, August 1983, p. 2334.

3. D. Boussard, A 100 MHz RF system in the SPS? CERN/SPS/ARF Note 83-83.

4. J. Gareyte, Performance of the SPS as a proton-antiproton collider, 12th International Conference on High-Energy Accelerators, Fermilab, August 1983.

5. D. Boussard, T.P.R. Linnecar, RF noise performance of the CERN proton-antiproton collider, 12th International Conference on High-Energy Accelerators, Fermilab, August 1983.

6. B. Autin, Fast betatron cooling in an antiproton accumulator, CERN/PS/AA 82-20.

7. T. Linnecar, W. Scandale, A transverse Schottky noise detector for bunched proton beams, CERN SPS/81-5.

8. Hewlett Packard Note 11729B-1. Phase noise characterization of microwave oscillators.

9. L. Brillouin, Science and Information Theory, Academic Press Inc., Publishers, 1956.

10. D. Slepian, Some comments on Fourier Analysis, Uncertainty and Modeling, SIAM Review, Vol. 25, No. 3, July 1983, p.379 - 393.

11. S. Chattopadhyay, On Stochastic cooling of bunched beams from fluctuation and kinetic theory, Lawrence Berkeley Laboratory Report, LBL-14826, September 1982.