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CLASSICAL SCALING LAWS OF A Z-PINCH WITH A COLD-MANTLE

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ABSTRACT

Classical scaling laws are deduced for the equilibrium relations between pinch current J, pinch radius a, axial density n_o and temperature T_o of linear Z-pinches having a finite length L, as well as for toroidal Z-pinches having a large aspect ratio. In both cases the radius a is found to increase almost linearly with the current J at a fixed density n_o, and the temperature T_o to increase with J at fixed values of the radius a. In principle, anomalous transport can be simulated in a first approximation by multiplying the transport coefficients by corresponding numerical factors.

At a fixed density n_o and a fixed external conductor current of an Extrap pinch, the radius a is found to increase more rapidly with J than the radius of the magnetic separatrix. Within the range of increasing pinch currents there are therefore three regimes of an Extrap system. For small J the system becomes unstable in the conventional way of unstabilized Z-pinches. For intermediate values of J, the magnetic surfaces become deformed by the external conductor field, and the constraints of this field combine with FLR and cold-mantle effects to provide a macroscopically stable state. Finally, for sufficiently large currents J, i.e. when the pinch radius a approaches and even tends to exceed the separatrix radius, the system is expected to become ballooning unstable in regions of "bad" field line curvature.

The present analysis thus provides relations between the basic pinch parameters which can be tested by experiments, and it also contributes to the understanding of Extrap stability in terms of increasing pinch currents.

1. Introduction

The plasma balance of a Z-pinch has earlier been analysed by several authors [1-10], in some cases including plasma-neutral gas interaction and aiming at Extrap geometry [4-10].

This report considers a pinch which is surrounded by a partially ionized cold-mantle, and where the hot plasma core is strongly impermeable to neutral gas. In such a system there are considerable radial heat losses by conduction from the fully ionized core to the cold-mantle and a surrounding wall [4,5,7,8,9,].The present study therefore differs from that earlier performed by Haines [1], who assumed the plasma to be thermally isolated at its boundary, e.g. by having a vacuum region between the plasma and the wall.

This paper aims at classical scaling laws for the basic plasma parameters, obtained from a simple analysis which excludes detailed investigations of the plasma profiles. In particular, such scaling laws become useful in a first estimate of the pinch radius and its relation to the separatrix radius in Extrap geometry. In its turn, such a relation becomes important to stability. In this connection possibly existing anomalous transport effects can, at least to some extent, be simulated by increasing the transport coefficients by corresponding numerical factors.

2. Assumptions

The following assumptions are made:

(i) A linear Z-pinch is considered, the length L of which is much larger than the radius a of its cross section. The limiting case $a/L \to 0$ is here used to simulate toroidal pinches of comparatively large aspect ratios.

(ii) The pinch is kept in a quasi-steady and macroscopically stable state, e.g. by using an Extrap confinement scheme. The plasma cross section of such a scheme is non-circular, but is here approximated by a circular shape. This does not change the main physical features of the present analysis on plasma equilibrium.

(iii) The plasma is strongly impermeable to neutral gas, as defined by the condition $\bar{n}a \gg 1/\sigma_{cf} \cong 5\times10^{18}\ m^{-2}$, where $\bar{n}$ is the average electron density and σ_{cf} the effective cross section of penetrating fast neutrals [11]. A fully developed cold-mantle system is then established as outlined in Fig.1, where a comparatively thin, cool and dense partially ionized boundary layer separates the fully ionized hot plasma core from its surroundings. Consequently, the thickness x_b of this boundary layer becomes much smaller than the pinch radius a. Further, the plasma temperature T_b and current density j_b at the "edge" $r = a-x_b$ of the same layer become much smaller than the maximum values T_o and j_o at the pinch axis, respectively. Finally, the plasma and neutral gas temperatures are approximated by a constant and low value $T \approx T_b$ in the boundary layer [11].

(iv) The present study is restricted to the fully ionized core defined by $r < r_b = a-x_b$. Thus, a detailed investigation of the rather complicated balance conditions of the partially ionized boundary layer is excluded from the analysis. Still the heat balance of the hot core can be treated to some detail for a strongly impermeable plasma where $x_b \ll a$ and $T_b \ll T_o$, $j_b \ll j_o$ in the model illustrated by Fig.1.

(v) The radial profile of any plasma quantity $Q(r,z)$ is represented by the form

$$Q(r,z) = Q_{max} \cdot f_Q \qquad 0 \leq f_Q \leq 1 \tag{1}$$

where Q_{max} is the maximum value of $Q(r,z)$ for a fixed value of z, and f_Q is a dimensionless function of $\rho = r/a$ which describes the corresponding radial profile. According to Fig.1 we thus have $f_n=f_j=f_T=1$ and $f_B=0$ at $\rho = 0$, as well as $f_j=f_T=0$ and $f_b\cong 1$ at $\rho=1$. Scaling laws in a restricted and exact sense are obtained if there would exist sets of configurations having varying values of the amplitudes Q_{max} and <u>fixed</u> profile functions f_Q. The present analysis does not take detailed profile changes into account. It therefore provides an approximate method of relating varying values of the amplitudes Q_{max} with each other, as well as with the parameters obtained from integration over the profiles.

(vi) The present deductions are limited to a pure hydrogen plasma with the charge and mass numbers $Z=A=1$.

(vii) Classical transport is assumed to prevail. Anomalous transport can be included in a first crude approximation, by multiplying the relevant transport coefficients by corresponding numerical factors. Such transport is not treated in detail, but needs further investigation. Moreover, a rigorous analysis of such transport has to include modified analytical forms of the transport coefficients, as well as modified plasma profile shapes.

(viii) Bremsstrahlung and cyclotron radiation losses are neglected.

3. Heat Flow at the Edge of the Plasma Core

The heat conductivity of the fully ionized plasma in the directions across the magnetic field is given by

$$\lambda_\perp^i = \lambda_\perp^i + \lambda_\perp^e \tag{2}$$

where, according to Braginskij [12],

$$\lambda_\perp^i = \lambda_\perp^i(0)\left[(\omega_i\tau_i)^2 + 1.32\right] / \left[(\omega_i\tau_i)^4 + 2.7(\omega_i\tau_i)^2 + 0.68\right] \tag{3}$$

$$\lambda_\perp^i(0) = 2k^2 nT\tau_i/m_i = k_\lambda^i T^{5/2}/(\ell n\Lambda) \tag{4}$$

refer to the contribution from ions and

$$\lambda_\perp^e = \lambda_\perp^e(0)\left[(\omega_e\tau_e)^2 + 2.57\right] / \left[(\omega_e\tau_e)^4 + 14.8(\omega_e\tau_e)^2 + 3.77\right] \tag{5}$$

$$\lambda_\perp^e(0) = 4.66k^2 nT\tau_e/m_e = k_\lambda^e T^{5/2}/(\ell n\Lambda) \tag{6}$$

from electrons. In eqs. (3)-(6) the collision times are further given by

$$\nu_s = 1/\tau_s = k_T^s n(\ell n\Lambda)/T^{3/2} \qquad (s = i,e) \tag{7}$$

where $\ell n\Lambda$ stands for the Coulomb logarithm, ω_i, ω_e are the gyro frequencies, $k_T^i = 5.9 \times 10^{-8}$, $k_\lambda^e = 3.6 \times 10^{-6}$, $k_\lambda^i = 3.9 \times 10^{-12}$, $k_\lambda^e = 2.7 \times 10^{-10}$, and SI-units are used throughout this paper.

The heat conductivity at the "border" $r = r_b$ of the fully ionized core in Fig.1 will be of special interest to the discussions of this paper. Indicating quantities at $r = r_b$ by subscript $(_b)$, we have

$$(\omega_s\tau_s)_b = k_b^s T_b^{3/2}(n_o T_o)^{1/2}/n_b(\ell n\Lambda) \qquad (s = i,e) \qquad (8)$$

where

$$k_b^s = ek_B/m_s k_\tau^s \qquad (s = i,e) \qquad (9)$$

and

$$k_B = (2\mu_o kF_J/F_p)^{1/2} \qquad (10)$$

In eq. (10) the quantities F_J and F_p are dimensionless profile factors of order unity, being connected with the current and pressure distributions, as shown later in Section 5 of this paper. Thus, the coefficient k_B links the magnetic field B_b in Fig.1 to the values n_o and T_o at the axis, i.e. through the radial balance of forces and in analogy with the Bennett relation. As an example we put $F_p = 0.25$, $F_J = 0.5$, $\ell n\Lambda = 10$, $T_b = 3x10^4$ K, $n_b = n_o/3$, $T_o \leqslant 3 \times 10^5$ K. Then $n_o \geqslant 10^{22}$ m^{-3} yields $(\omega_i\tau_i)_b \leqslant 0.12$ and $(\omega_e\tau_e)_b \leqslant 3.5$.

For strongly impermeable pinches where a is of the order of 10^{-2} m and $n_o > 10^{22}$ m^{-3}, it is thus seen that $(\omega_i\tau_i)_b$ becomes much smaller than unity, and $(\omega_e\tau_e)_b$ less than or of the order of unity. The transverse heat conductivity at the "edge" $r = r_b$ is therefore not too far from the limiting value $\lambda_\perp(0) = \lambda_\perp^i(0) + \lambda_\perp^e(0)$ which is being approached at small $(\omega_s\tau_s)_b \ll 1$.

4. Comparison between Longitudinal and Transverse Heat Losses

Following Tendler [8] we now compare the transverse and longitudinal heat losses of the fully ionized plasma core. The transverse power losses through heat conduction in the radial direction become

$$\Lambda_{\perp} \cong 4\pi a L \lambda_{\perp b} |dT/dr|_b \tag{11}$$

at the edge $r = r_b$. In a pinch where $L \gg a$ these losses become much larger than those from axial heat conduction.

There is no axial magnetic field component and no axial mass flow in the present system. Therefore the main axial heat losses are due to the particle transport by the axial current density. The corresponding power loss becomes [5]

$$\Lambda_{\parallel} \cong 5\pi (k/e) a^2 j_o T_o F_{\parallel} \tag{12}$$

where $F_{\parallel}$ is a dimensionless profile factor of order unity. Eqs. (11) and (12) combine to

$$\theta_L \equiv \Lambda_{\parallel}/\Lambda_{\perp} \cong \left[5kF_{\parallel} j_o T_o / 4e\lambda_{\perp b} |dT/dr|_b\right](a/L) \tag{13}$$

Two limiting cases will be considered in the following sections, as defined by eq. (13):

(I) For finite and not too small values of a/L we obtain $\theta_L \gg 1$, and the longitudinal losses will have a major influence on the heat balance. An example is given by a relatively short linear pinch where the total pinch current $J = 2 \times 10^4$ A, $a = 0.01$ m, $L = 0.1$ m, $j_o = 10^8$ A/m, $n_o = 10^{22}$ m^{-3}, $T_o = 3 \times 10^5$ K, $T_b = 3 \times 10^4$ K, $|dT/dr|_b \cong 10^7$ K/m, $F_{\parallel} = 1/6$, $F_J = 2F_p = 0.5$, and $n_b = 3 \times 10^{21}$ m^{-3}. Then $\lambda_{\perp}^i(0) = 6.1\text{x}10^{-2}$, $\lambda_{\perp}^e(0) = 4.2$, $(\omega_i \tau_i)_b = 0.13$, $(\omega_e \tau_e)_b = 3.8$, $\lambda_{\perp b}^i = 0.11$, $\lambda_{\perp b}^e = 0.17$ and $\lambda_{\perp b} = 0.28$, from which $\theta_L \cong 20 \gg 1$.

(II) When a/L is chosen small enough for $\theta_L \ll 1$, such as for very long linear pinches or toroidal pinches (with a/L = 0), only the transverse losses have to be taken into account.

5. Longitudinal Loss Dominated Case

We first turn to case I where $\theta_L >> 1$. This case has been considered earlier [5]. In a cylindrical frame (r,φ,z) with z along the pinch axis, $z = 0$ at the anode and $z = L$ at the cathode, the temperature distribution along the pinch axis $r = 0$ was found to have the form (see also Ref. [1])

$$T_o(z) \cong T_{oo}\left[1-(z/L)\right]^{2/5} \tag{14}$$

$$T_{oo} = (ek_\eta L j_o/k)^{2/5} \qquad k_\eta = 129(\ell n\Lambda) \tag{15}$$

This relation results from a balance between Ohmic heating and heat losses due to particles escaping to the end electrodes, i.e. when the transverse heat losses and the radiation losses from the fully ionized plasma are neglected, as well as the ion temperature in the anode sheath.

With the notation defined by eq. (1), and for $x_b << a$, the total pinch current becomes

$$J \cong 2\pi a^2 j_o \int_0^1 \rho f_j(\rho)d\rho \equiv 2\pi a^2 j_o F_J \tag{16}$$

where F_J is a dimensionless factor being somewhat smaller than unity. The magnetic field and the current density are related by

$$B_b \cong B_a = \mu_o J/2\pi a = \mu_o a j_o F_J \tag{17}$$

Further, integration of the radial balance of forces with respect to r yields

$$nT = n_o T_o - (a/2k)\int_0^\rho jB d\rho \tag{18}$$

In the model of Fig.1 we have $n_b T_b << n_o T_o$. At $\rho = 1$ a pressure balance relation

$$n_o T_o \cong (a j_o B_b/2k) \int_o^1 f_j f_B d\rho \equiv a j_o B_b F_p/2k \qquad (19)$$

is therefore obtained where F_p is a dimensionless factor being somewhat smaller than unity. Combination of eqs. (16)-(19) yields

$$n_o T_o = \mu_o F_p J^2/8\pi^2 k F_J a^2 \qquad (20)$$

which is equivalent to the Bennett relation. Eq. (20) also corresponds to the condition that the beta value defined by $4\mu_o n_o k T_o/B_b^2$ is nearly equal to unity.

It has earlier been shown that $\partial(nT)/\partial z = 0$ for a linear Z-pinch [5] . Since $T_o(z)$ is a slow function of z according to eq. (14), we can put $T_o \cong T_{oo}$ along a considerable part of the pinch length. With this as a first approximation, eqs. (15), (16) and (20) combine to the relations

$$a = C_{a\parallel}(F_p^5/F_J^3)^{1/6} J^{4/3}/n_o^{5/6} L^{1/3} \qquad C_{a\parallel} = (\mu_o^5/5632\pi^8 e^2 k^3 k_\eta^2)^{1/6} \qquad (21)$$

$$T_o = C_{T\parallel}(1/F_p)^{2/3}(L n_o/J)^{2/3} \qquad C_{T\parallel} = (4\pi e k_\eta/\mu_o)^{2/3} \qquad (22)$$

We notice that the pinch radius relation (21) has earlier been deduced by the author [6] , and later in an equivalent form by Tendler [9]. Here $C_{a\parallel} \cong 3.1 \times 10^{10}$ and $C_{T\parallel} \cong 1.37 \times 10^{-6}$ in SI-units. The obtained results imply that the pinch radius increases almost linearly with the pinch current J, is almost inversely proportional to the axial plasma density n_o, and decreases very slowly with increasing pinch lengths L. Further, the axial temperature increases with the pinch length L and the density n_o, but decreases at increasing pinch currents J. This latter behaviour is simply due to the fact that the average current density $J/\pi a^2 \propto 1/J^{5/3}$ and the Ohmic heating power decrease at increasing pinch currents J. Keeping instead the pinch radius constant, by varying the density

$n_o \propto J^{8/5}$, we would have a temperature $T_o \propto J^{2/5}$, thus increasing with J. For a fixed pinch radius eqs. (21) and (22) then yield

$$T_o = C_{T\parallel}(1/F_p)^{2/3}\left[C_{a\parallel}(F_p^5/F_J^3)/\bar{a}\right]^{4/5}\cdot(J\cdot L)^{2/5} \tag{23}$$

A numerical example is finally chosen with $F_p = 0.25$, $F_J = 0.5$, $J = 10^4$A, $n_o = 3 \times 10^{21}\ \text{m}^{-3}$ and $L = 0.1$ m. Then eqs. (21) and (22) yield a pinch radius $a \cong 8 \times 10^{-3}$ m and an axial temperature $T_o \cong 3 \times 10^5$ K, being not too far from the data measured and estimated in earlier Extrap pin experiments [13].

6. Transverse Loss Dominated Case

We now turn to case II where $\theta_L << 1$. The analysis is limited to conditions where there is no radial flux of particles within the plasma core, i.e. the neutral density n_n is neglected for $r \lesssim r_b$, and no matter is injected into the core by external means [10]. The plasma heat balance equation then reduces to

$$-\frac{1}{r}\frac{d}{dr}(r\lambda_\perp \frac{dT}{dr}) = k_\eta j^2/T^{3/2} \tag{24}$$

Integrating this equation over the plasma core, the notation of eq. (1) yields

$$4\pi^2(\ell n\Lambda)F_J^2\lambda_{\perp b}T_o^{5/2}a^2/k_\eta J^2 = F_\lambda \tag{25}$$

$$F_\lambda = \int_0^1 \left[f_j^2(\ell n\Lambda)\rho/f_T^{3/2}(-df_T/d\rho)_b\right]d\rho \tag{26}$$

when use has been made of eq. (17).

With $\lambda_{\perp b}$ given by expressions (2)-(10) in terms of (n_o, T_o, n_b, T_b), it is thus seen that eq. (25) and the Bennett-type relation (20) combine to two equations for T_o and a as functions of J and n_o, i.e. when n_b/n_o and T_b are considered as given parameters.

When there is a high-density cold-mantle, $(\omega_i\tau_i)_b << 1$ and $(\omega_e\tau_e)_b$ has values not too far from unity. Then $\lambda^e_{\perp b}/\lambda^e_{\perp b}(0)$ varies slowly with $(\omega_e\tau_e)_b$ and becomes of the order of unity. Consequently

$$\lambda_{\perp b} \cong k_\lambda T_b^{5/2}/(\ell n\Lambda) \qquad k_\lambda = k_\lambda^i + k_\lambda^e \tag{27}$$

and eq. (25) reduces to

$$(2\pi F_J a/J)^2(k_\lambda/k_\eta)(T_bT_o)^{5/2} \cong F_\lambda \tag{28}$$

Combination with eq. (20) yields

$$a = C_{a\perp}(1/F_J^{1/2}F_\lambda)^{1/3}F_p^{5/6}J/n_o^{5/6} \qquad C_{a\perp} = (k_\lambda/k_\eta)^{1/3}\cdot(\mu_o T_b/2k)^{5/6}/2\pi \quad (29)$$

$$T_o = C_{T\perp}(F_\lambda/F_pF_J)^{2/3}n_o^{2/3} \qquad C_{T\perp} = (2kk_\eta/\mu_o k_\lambda T_b^{5/2})^{2/3} \quad (30)$$

where $C_{a\perp} \cong 4.3 \times 10^{12}$ and $C_{T\perp} \cong 0.64 \times 10^{-10}$ in SI-units when $T_b = 3 \times 10^4$ K. At a fixed pinch radius we further have

$$T_o = C_{T\perp}(F_\lambda/F_J)^{2/3}\left[C_{a\perp}(1/F_J^{1/2}F_\lambda)^{1/3}\ (J/a)\right]^{4/5} \quad (31)$$

From eqs. (29)-(31) is thus seen that the pinch radius increases linearly with J at a fixed density n_o, and that the temperature at the axis increases with J when the pinch radius is kept constant. This behaviour is similar but not identical to that of case I represented by eqs. (21)-(23).

A numerical example of case II is finally given by $n_o = 10^{22}\ m^{-3}$, $J = 10^5$ A and $F_p = 0.25$, $F_J = 0.5$, $F_\lambda = 3$. Then eqs. (27)-(30) and (2)-(8) yield $T_o \cong 1.8 \times 10^6$ K and $a = 1.9 \times 10^{-2}$ m. For these data $(\omega_e\tau_e)_b \cong 5$ and $(\omega_i\tau_i)_b \cong 0.2$.

7. The Magnetic Separatrix

At a fixed density n_o, the pinch radius has been found to increase with the pinch current as given by

$$a = a_c(J/J_c)^{\alpha} \tag{32}$$

Here $\alpha = 4/3$ and 1 in the longitudinal and transverse loss dominated cases I and II, respectively, and (a_c, J_c) are characteristic constants defined by eqs. (21) and (29).

In linear Extrap geometry, with an octupole external conductor field $\underline{B}_v$, and in corresponding toroidal Extrap systems with large aspect ratios, the axial distance of a zero line of the magnetic separatrix becomes

$$r_m = a_v(J/4J_v)^{1/4} \tag{33}$$

Here J_v is the current in each of the external rod conductors being situated at a distance a_v from the axis.

According to eqs. (32) and (33), the pinch radius thus becomes equal to the characteristic separatrix radius r_m when the pinch current has increased to the value

$$J_m \approx \left[a_v J_c^{\alpha}/\sqrt{2}J_v^{1/4}a_c\right]^{4/(4\alpha-1)} \tag{34}$$

A further increase of J beyond J_m would lead to a pinch radius larger than r_m, but this is not reconcilable with the present plasma balance equations. It would lead to strongly enhanced losses at the pinch boundary, and to substantial changes in the plasma profiles. We finally observe that J_m can be increased by increasing the rod distance a_v.

As an example the values of case I adopted at the end of Section 5 are choosen, i.e. $F_p = 0.25$, $F_J = 0.5$, $n_o = 3x10^{21}\ m^{-3}$, $L = 0.1\ m$, $\alpha = 4/3$, and $a_c/J_c^{\alpha} = 3.7x10^{-8}$. With $a_v = 3x10^{-2}$ m and $J_v = 3x10^4$ A the limiting current of eq. (34) then becomes $J_m = 1.9x10^4$ A.

8. Stability Considerations

Starting from the deduced relations between the pinch radius a and the plasma current J at a given density n_c, we now turn to some considerations about Extrap stability. In a high-beta system, such as the Z-pinch, the density n_b at $r = r_b$ becomes approximately equal to the neutral blanket density n_{na} of Fig.1 [10,11]. Having a large vacuum vessel with constant neutral filling density n_{na}, it is thus justified to assume the axial density n_c as a nearly constant parameter, being of the order of the filling density n_{na}. It should further be observed that, at a fixed external conductor current J_v, the pinch radius increases more rapidly with J than the separatrix radius r_m, as shown by eqs. (21), (29) and (33). Consequently, when considering gradually increasing pinch currents J which start from a small value, the stability situation can roughly be outlined in the following way which is at least in qualitative agreement with experiments [4,13,14]:

(i) At low currents J and small radii a the field $\underline{B}_v$ from the external conductors becomes very weak within the entire plasma volume, because $B_v \propto (r/a)^3$ near the axis at $r = 0$. The pinch then obeys conventional stability theory, thus becoming unstable to kink and sausage modes (Fig.2a).

(ii) As the current J and radius a increase, the outer layers of the plasma approach the magnetic separatrix and become influenced by the constraints imposed by the external conductor field $\underline{B}_v$. Then the cross sections of the magnetic surfaces of these outer layers become non--circular, tending towards a "diamond-shape" [4]. These constraints, in combination with finite Larmor radius effects, and possibly also in combination with temperature profile shaping in the partially ionized boundary layer, lead to a macroscopically stable plasma. Also the formation of "good" regions of negative field line curvature in the outer layers should contribute to stability [14,4] (Fig.2b).

(ii)When there is a further increase in J and a, the pinch radius approaches the radius of the separatrix, and finally tends to exceed the latter. Within this parameter range, the regions of "bad" field line curvature in the outer plasma layers become increasingly "loaded" by the plasma pressure gradient, and sooner or later they therefore turn "ballooning" unstable. The detailed conditions for the development of this instability, as well as its dependence on the plasma density, are given elsewhere [14] (Fig.2c).

9. Concluding Remarks

The present discussion on the Z-pinch scaling laws is important in several respects:

(i) It provides a first hint to more rigorous and detailed self-consistent calculations of the plasma and neutral gas parameter relations.

(ii) It predicts a simple relation between the pinch radius, total current, density, and temperature which can be tested by experiments.

(iii) It contributes to the understanding of the observed Extrap stability and its limits. Nevertheless an extended stability analysis is needed which takes into account the coupled particle, momentum and heat balance of a dissipative plasma state.

(iv) An extension of the present analysis to include anomalous transport and impurity radiation should modify the corresponding scaling laws, in a way which also should become possible to test by experiments.

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Figure Caption

Fig.1. Outline of the profiles of the plasma and neutral gas densities n, n_n, the current density j, the magnetic field strength B, and the plasma temperature T of a strongly impermeable Extrap pinch. Here r_w denotes the wall radius, r_m the average radius of the magnetic separatrix in Extrap geometry, a the pinch radius, and x_b the thickness of the partially ionized boundary layer.

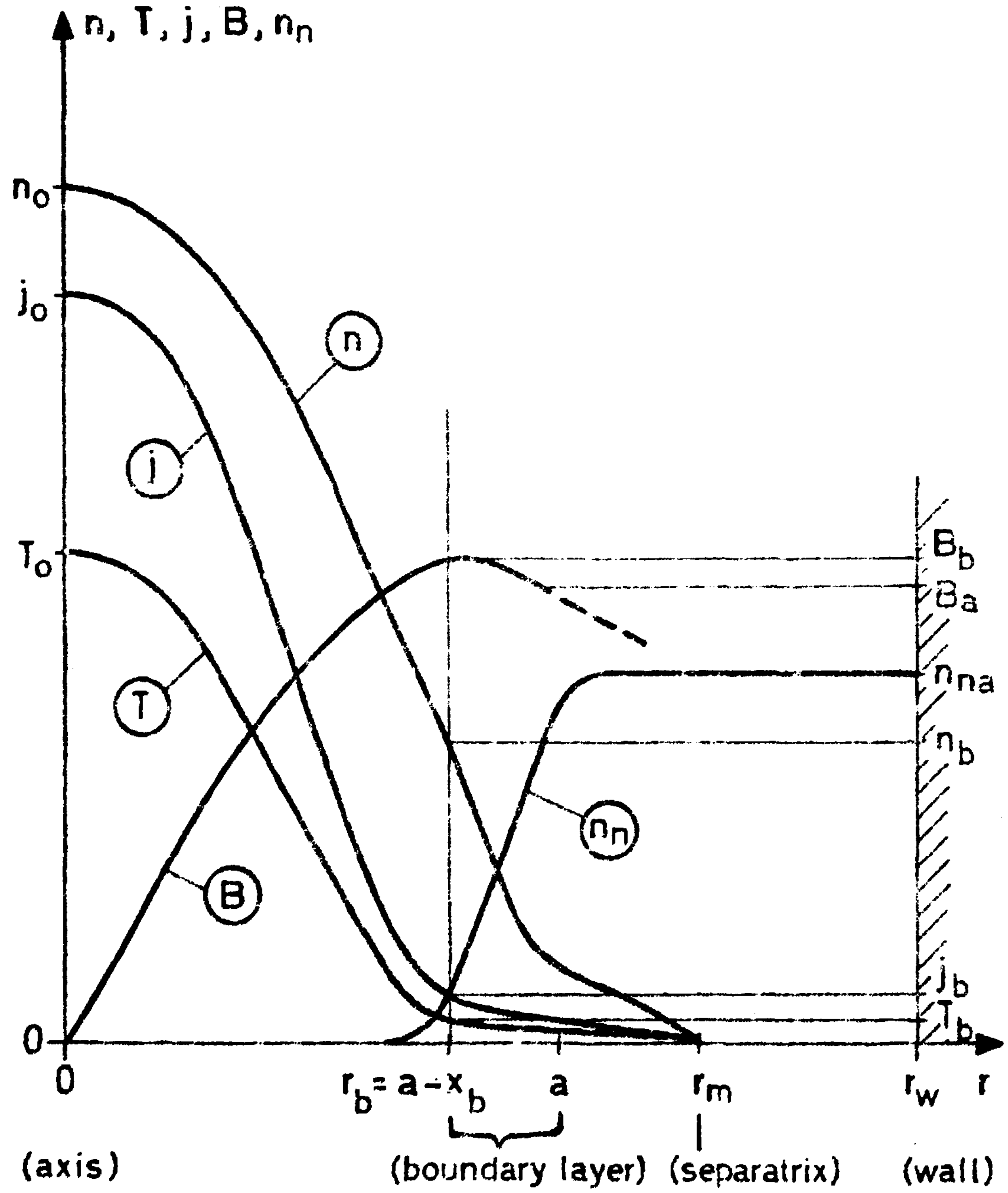
Fig.2. Outline of the three regimes of an Extrap pinch, with respect to stability, on a scale of gradually increasing pinch currents J and radii a:

(a) Unstable regime of small currents.

(b) Stable regime of intermediate currents.

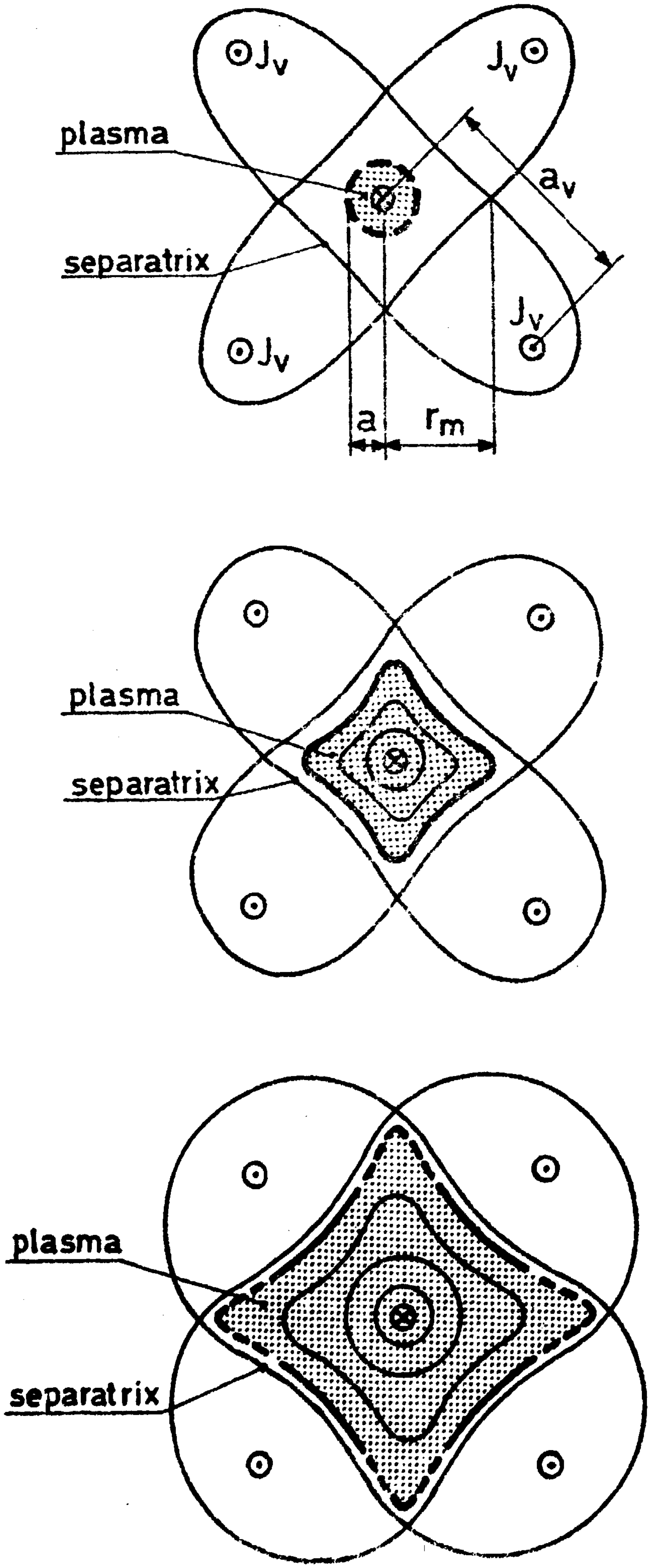
(c) Unstable regime of large currents.

Fig. 1



n, T, j, B, n_n
n_o
j_o
T_o
0
n
j
T
B
n_n
B_b
B_a
n_{na}
n_b
j_b
T_b
0
$r_b = a - x_b$
a
r_m
r_w
r
(axis)
(boundary layer)
(separatrix)
(wall)

Fig. 2



(a) small currents J unstable

(b) intermediate currents J stable

(c) large currents J unstable

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Classical scaling laws are deduced for the equilibrium relations between pinch current J, pinch radius a, axial density n_o and temperature T_o of linear Z-pinches having a finite length L, as well as for toroidal Z-pinches having a large aspect ratio. In both cases the radius a is found to increase almost linearly with the current J at a fixed density n_o, and the temperature T_o to increase with J at fixed values of the radius a. In principle, anomalous transport can be simulated in a first approximation by multiplying the transport coefficients by corresponding numerical factors.

At a fixed density n_c and a fixed external conductor current of an Extrap pinch, the radius a is found to increase more rapidly with J than the radius of the magnetic separatrix. Within the range of increasing pinch currents there are therefore three regimes of an Extrap system. For small J the system becomes unstable in the conventional way of unstabilized Z-pinches. For intermediate values of J, the magnetic surfaces become deformed by the external conductor field, and the constraints of this field combine with FLR and cold-mantle effects to provide a macroscopically stable state. Finally, for sufficiently large currents J, i.e. when the pinch radius a approaches and even tends to exceed the separatrix radius, the system is expected to become ballooning unstable in regions of "bad" field line curvature.

The present analysis thus provides relations between the basic pinch parameters which can be tested by experiments, and it also contributes to the understanding of Extrap stability in terms of increasing pinch currents.

Key words: Z-pinches, Extrap, plasma balance, scaling laws, stability.