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CONSTRAINTS ON A SYSTEM OF TWO NEUTRAL FERMIONS FROM COSMOLOGY

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ABSTRACT

Using the standard model of cosmology we study the evolution of the population of a coupled system of two neutral fermions in which the lighter one is stable. During the expansion their population can be frozen at a certain level which makes them contribute to the mass density of the universe. The details of the freezing depend crucially on the couplings and on the masses of these two fermions, so that, comparison with the measured mass density in the universe gives constraints on the parameters of the physical system we examine. We discuss in detail different configurations for the couplings among these fermions; in particular in the case of large mixing we obtain restrictive bounds on both masses. Our study is relevant to supersymmetric grand unified models which predict the occurence of light interacting neutral fermions, particularly Higgsinos.

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1:Introduction

Modern cosmology and the standard Big Bang model [1] provide us with an efficient framework to constrain proposed theories in high energy physics which are difficult to test in the laboratory [2]. In particular, this approach proved to be very fruitful concerning weakly interacting particles, such as neutrinos [3] and axions [4]: to say the least, in some occasion, cosmology strongly influenced a new approach to the problems [5].

Recently a very intense activity has been devoted to supersymmetry which, for aesthetical and technical reasons, appears to be a possible way to unify all interactions including gravity. Unification models based on supersymmetry possess a new rich spectrum of particles, which may not be out of reach of the forthcoming accelerators. In all models, each usual particle is expected to have a supersymmetric partner. Among all these newly proposed states, some are charged and/or colored (scalar partners of leptons and/or quarks), others are neutral colorless fermions (gauge and/or Higgs fermions). These gauginos and Higgsinos are similar to neutrinos, except perhaps for the photino, and as such may be submitted to a cosmological analysis.

In this paper we shall be concerned with the study of a coupled system of two massive neutral fermions which interact via the exchange of the neutral weak boson Z_0 (sometimes, we shall also use the name lepton for them). We shall try to obtain information on the parameters of this physical system, like mixing parameters and masses, using cosmological information. Let us first set the stage and briefly recall the analysis done by Lee and Weinberg [6] in the case of a stable heavy lepton.

Following the standard model, at very early times, the universe was very hot and dense so that most of the energy was carried by radiation and that matter appeared under the form of its fundamental constituents-leptons and quarks-. Contrary to usual physical systems which relax to equilibrium if one waits long enough, in the universe the shorter the time the larger the number of particle species which are in equilibrium. Under the extreme conditions described above, reactions rates are large and chemical equilibrium is achieved. However, as time flows the universe is expanding and its temperature drops, so that a particle of mass M, whose density is further diluted by the Boltzmann factor $\exp-(M/kT)$, becomes so rare that it ceases to interact and annihilate. Whatever the interaction cross section is, reaction rates are virtually vanishing because of the dilution. From this time on, the population of the concerned particle in a covolume is frozen to a given amount and contributes to the mass density in the universe. This freezing (quenching) phenomenon [6,7] has been studied and used to derive lower bounds on the masses of neutral leptons [6,8].

Consider, for instance, N to be such a particle which is coupled to the Z_0 and stable. The only reaction which can make the population evolve is annihilation $N+\overline{N}\to f+\overline{f}$, where f and $\bar{f}$ are light fermions coupled to the Z_0 as usual. As long as the temperature of the universe $T>M$ the inverse reaction $f+\bar{f}\to N+\bar{N}$ can occur at a sufficient rate to maintain the number of N's, which behave like a relativistic gas of particles. When T is below M, the process $f+\bar{f}\to N+\bar{N}$ slows down because of the decline of the available energy for the f's. Then one would expect that the N and the $\bar{N}$ continue to annihilate, leading them to extinction, along their equilibrium curve. In fact the N's will not completely disappear due to different phenomena: First as $T > G_F^{1/2}$ the interaction cross section $\sigma \simeq T^{-2}$, therefore the reactions are very rapid and maintain the equilibrium despite fast expansion. At a lower temperature, the production rates of N cannot compete with expansion and the mean free path of the N's becomes larger than the typical expansion length c/H (H is the Hubble constant). *Then the universe is transparent for these particles which* begin free expansion. Standard calculations [1] give for light weakly interacting particles a decoupling temperature $T_d \simeq O(1\ \text{Mev})$.

Second, if $M > T_d$ the N's will decouple much earlier than expected from the calculation of T_d. Indeed, in addition to the effect of expansion, their density is damped by the Boltzmann factor $\exp-(M/T)$ once $T < M$. They are now so rare that annihilation will eventually stop. This freezing does not happen suddenly, at some point $T_d < T < M$, the population of N's leaves the equilibrium curve (this is the freezing point) and little by little becomes stabilized at its frozen value. This value is much higher than if they had decoupled at T_d. The density number in a covolume stays constant, but the density is still diluted by expansion.

This illustrates the quenching which can happen to heavy neutrinos as was used by Lee and Weinberg [6]. They found that the present mass density of these particles depends on their mass M in the following way:

$$\rho_n \simeq C\,.[\,M(\text{Gev})\,]^{-1.83} \qquad (1-1)$$

We observe that the higher the mass M the lower the density of the fossilized N's. Thus a lower bound on M can be obtained if we require that ρ_n cannot exceed the presently measured mass density ρ. Using $\rho=2.10^{-29}\ \text{g/cm}^3$ it was found that a lower limit on M is 2 to 4 Gev. However, Gunn et al. [8] argued that such massive particles should bind around galaxies into halos. This can reduce the upper limit on ρ_n by an order of magnitude and raise the lower limit on M to 7-15 Gev.

We shall follow closely such an analysis in the case of two neutral weakly interacting fermions, one of them being unstable. In the next section we derive from general grounds the kinetic equations describing this physical system and we discuss in which approximation detailed balance applies. Section 3 is devoted to the solution of the obtained differential equations. We study different configurations for the couplings between our two fermions N_1 and N_2. This detailed discussion allows us to derive interesting constraints on the Higgsino sector of supersymmetric models in Section 4. Our conclusions are given in Section 5.

2:EVOLUTION EQUATIONS FOR A BINARY SYSTEM

In this section we derive the evolution equations for a system of two massive neutral fermions which interact among themselves and also with a thermal bath of light or massless fermions (quarks and leptons).
On general grounds [9-10] ,the problem of density number evolution of a particle species in the early universe-hot and dense- can be tackled using the transport equation of the kinetic theory of gases. For instance if we are interested in the evolution of the distribution of particle a_i we have:

$$\frac{df_{a_i}}{dt}=\frac{1}{2E_{a_i}}\sum_{i,j}\Big[\prod f_{b_i}.(1-f_{a_j}).|M(b_1,\dots,b_m\to a_1,\dots,a_n)|^2-\prod_{i,j} f_{a_j}(1-f_{b_i}).|M(a_1,\dots,a_n\to b_1,\dots,b_m)|^2\Big] \quad (2\text{-}1)$$

where the f's are the energy distribution functions of the fermions involved in the physical process. $M(b\longleftrightarrow a)$ is the transition amplitude for the reaction $b\longrightarrow a$ and the sum Σ runs over all processes which can create or destroy a_i and also includes phase space integrals for all the particles except a_i. The factors $(1-f_{a_j})$ and $(1-f_{b_i})$ account for the exclusion principle in the final state for the fermions a_j and b_i. In eq(2-1) we have assumed that particle momenta are uncorrelated , so that multiparticle distributions factorize and also that there are no spatial inhomogeneities i.e. no x dependence in these distributions. At equilibrium the distribution function for a fermion is the usual Fermi-Dirac distribution:

$$f^{eq}(E/kT)=f^{FD}(E/kT)=[1+\exp(E/kT)]^{-1} \quad (2\text{-}2)$$

We shall assume that the out of equilibrium distribution is still only a function of $e=E/kT$.
The number density is defined for one helicity state as:

$$n(T)=\int d^3p/(2\pi)^3\ f(E/kT) \quad (2\text{-}3)$$

From (2-1) one easily obtains the kinetic equation for $n(T)$ by integration,

$$\frac{dn(T)}{dt}+3.\frac{dR(T)}{dt}\frac{n(T)}{T^3}=\sum_{a,b}\int dLIPS(a,b)\Big[\prod_{i,j} f_{b_i}(1-f_{a_j}).|M(b\to a)|^2-\prod_{I,j}(1-f_{b_i})f_{a_j}.|M(a\to b)|^2\Big] \quad (2\text{-}4)$$

In (2-4) dLIPS(a,b) denotes the Lorentz invariant phase space volume element for ALL particles.

$$dLIPS(a,b)=\prod_{i,j} \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_j) \frac{d^3p_j}{(2\pi)^3 2E_j} \qquad (2\text{-}5)$$

The right hand side of (2-4) ,usually termed as the collision integral,simplifies greatly if we consider only 2-body and 3-body processes. Indeed it can then be expressed in terms of annihilation or scattering cross sections and decay rates.

Having presented the general framework,let us now describe in more details the collision integrals relevant to our case. Let N_1 and N_2 denote two heavy neutral fermions such that $M_1>M_2$ and f be a generic notation for light fermions in thermal and chemical equilibrium. For the time being ,we do not refer to any specific model ,as we shall do in the next sections,but only assume that the interaction proceeds through the neutral weak current. Under this assumption, the reactions which change the number of N_1 and/or N_2 are the following:

$$N_1 \overline{N_1} \longrightarrow f\overline{f} \;,\; N_1 \overline{N_2} \longrightarrow f\overline{f},\; N_2 \overline{N_2} \longrightarrow f\overline{f} \qquad (2\text{-}6a)$$

$$\begin{array}{l} N_1 \overline{N_1} \longrightarrow N_2 \overline{N_1} \;,\; N_1 \overline{N_1} \longrightarrow N_2 \overline{N_2} \;,\; N_1 \overline{N_2} \longrightarrow N_2 \overline{N_2} \\ N_1 N_1 \longrightarrow N_2 N_1 \;,\; N_1 N_1 \longrightarrow N_2 N_2 \;,\; N_1 N_2 \longrightarrow N_2 N_2 \end{array} \qquad (2\text{-}6b)$$

$$N_1 f \longrightarrow N_2 f \;,\; N_1 \overline{f} \longrightarrow N_2 \overline{f} \qquad (2\text{-}6c)$$

$$N_1 \longrightarrow N_2 f \overline{f} \qquad (2\text{-}6d)$$

We have grouped the reactions into four categories which we discuss separately in the order given above. As long as equilibrium is maintained , the inverse reactions (T or CP transformed of (2-6)) occur at the same rate as the direct ones (2-6):we shall denote them by a bar over the label of the direct reaction label.

a/ $N_i \overline{N_j} \dashrightarrow f\overline{f}$ (i,j=1,2)

Since we consider our particle system in a regime where the universe temperature T is such that

$$M_i >> kT >> M_f \quad (i=1,2) \qquad (2\text{-}7)$$

the light fermions (f) are well above their decoupling temperature and are in thermal equilibrium. Then their energy levels are distributed following (2-2) .However as long as they participate to the interaction with N_1 and N_2 their energy is , in average, of the order of M_i.In such a circumstance the Fermi-Dirac distribution is ,to a good approximation , equivalent to the Maxwell-Boltzmann distribution i.e.

$$f^{eq}(e)=f^{FD}(e) \simeq f^{B}(e) = \exp(-E_f/kT) \ll 1 \qquad (2\text{-}8)$$

In the same way because of (2-7) and because the N_i's go out *of equilibrium* at a rather low temperature we can guess that $f(e_i) \ll 1$. Therefore we can forget the final state factor $\{1-f(e_i)\}$ and obtain for the collision integral ,denoted $A(i\bar{j} \longrightarrow f\bar{f})$:

$$A(i\bar{j}\to f\bar{f})=\int dLIPS(i\bar{j},f\bar{f}).|M(N_i\bar{N}_j\to f\bar{f})|^2.[f^0(e_f)f^0(e_{\bar{f}})-$$

$$- f(e_i)f(e_j)] \qquad (2\text{-}9)$$

In this equation we have explicitly assumed that the transition amplitudes are time reversal invariant ,which would not be the case if we had CP violating interactions. If we use energy conservation [10] :

$$f^{eq}(e_f).f^{eq}(e_{\bar{f}})=\exp-(E_f+E_{\bar{f}}/kT)=\exp-(E_i+E_j/kT)=f^B(e_i).f^B(e_j) \qquad (2\text{-}10)$$

we obtain:

$$A(i\bar{j}\dashrightarrow f\bar{f})= \int dLIPS.|M(N_i\bar{N}_j\to f\bar{f})|^2.\{f^B(e_i).f^B(e_j).f(e_i).f(e_j)\}$$

$$=\langle v.\sigma(i\bar{j}\to f\bar{f})\rangle.[n^0{}_i(T).n^0{}_j(T)-n_i(T).n_j(T)] \qquad (2\text{-}11)$$

where

$$\langle v.\sigma(i\bar{j}\to f\bar{f})\rangle= 1/4E_i.E_j.\int dLIPS(p_f,p_{\bar{f}}).|M(N_i\bar{N}_j\to f\bar{f})|^2 \qquad (2\text{-}12)$$

The brackets indicate that we have taken an average over the distributions of N_i and $\bar{N}_j$,this enables us to exhibit the number densities $n_i(T)$ and $n_j(T)$. We keep an index i and j on these quantities to remember their mass dependence. Note that we assume that all chemical potentials are zero i.e.,$n_i(T)=n_{\bar{T}}(T)$.

From now on we shall use the superscript 0 to label physical quantities at their equilibrium value. In the present case and unless specified ,in the sequel of this paper ,the quantity $\langle v.\sigma\rangle$ will be evaluated for heavy particles N_i at rest. We have calculated the collision integral for one channel $f\bar{f}$,ultimately we shall sum over all open channels and take $\Sigma_f\ A(i\bar{j}\to f\bar{f})$.

b/ $N_i\ \bar{N}_i \longrightarrow N_j\ \bar{N}_k$ $\quad (N_i\ N \longrightarrow N_j\ N_k)$

In this subsection we consider the processes which participate to the evolution of the neutral fermion N_1,which heavier than N_2 will decouple earlier, that is, at higher temperature. The collision integral reads:

$$B(i\bar{l}\rightarrow j\bar{k})=\int dLIPS.\,|M(N_i\bar{N_l}\rightarrow N_j\bar{N_k})|^2.[f(e_k).f(e_j)-f(e_l).f(e_i)] \tag{2-13}$$

This expression can be simplified to a form similar to (2-11). Indeed since $M_i+M_l \gtrsim M_j+M_k$ the direct channel b is always open ,whereas the inverse one is not. As the temperature of the universe drops reaction rates are slowing down because of the density dilution and the decline of the available energy. At some stage they will eventually stop and ,for instance the reaction (b) will stop later than the reaction ($\bar{b}$) [11] .In other words (b) ceases at a temperature $T=\Theta_b<\Theta_{\bar{b}}$, where Θ_r denotes the temperature at which the reaction (r) stops.For $T < \Theta_{\bar{b}}$ the equilibrium is no longer maintained in the channel under concern,therefore the temperature T_i at which N_i goes out of equilibrium should be lower than $Inf(\Theta_{\bar{r}})$ where $\bar{r}$ runs over all inverse reactions under concern.
Let us examine how the system evolves as the time goes on.

i) As $T > T_i$ all particles are still on their equilibrium distributions including the heaviest one,N_i,so we can write:

$$f(e_j).f(e_k)=f^0(e_j).f^0(e_k)=f^0(e_l).f^0(e_i) \tag{2-14}$$

which following the lines of subsection (a) gives for the collision integral:

$$B(i\bar{l}\rightarrow j\bar{k})= \langle v.\sigma(i\bar{l}\rightarrow j\bar{k})\rangle.[n^0_i(T).n^0_l(T)-n_i(T).n_l(T)] \tag{2-15}$$

ii) As $T < T_i$ all inverse reactions are virtually stopped and their contribution to (2-13) is negligible.

$$f(e_j).f(e_k)-f(e_l).f(e_i)\simeq -f(e_l).f(e_i) \tag{2-16}$$

In addition we expect that the number density of N_i will not decrease very fast because of the slowdown of the direct reactions. In particular we can safely say that $n_i(T<T_i) \gg n^0_i(T)$ since the equilibrium density is steeply decreasing. For these reasons we have to a good approximation

$$-f(e_l).f(e_i) \simeq f^0(e_l).f^0(e_i)-f(e_l).f(e_i) \tag{2-17}$$

This allows us to keep the same expression for $B(i\bar{l}\rightarrow j\bar{k})$ given in (2-15) in all the temperature range.

c/ $N_1\ f \longrightarrow N_2\ f$, $N_1\ \bar{f} \longrightarrow N_2\ \bar{f}$

These reactions represent the scattering of N_1's on the light fermions which, in the temperature domain of interest do not go out of equilibrium.
If the line of reasoning used in the previous case is still valid we cannot ,without care, factor out the cross section.Rather one should calculate the integral of $v.\sigma$ over the f^0 of the light fermions. We prove in appendix A that

it is a very good approximation to use $\langle v.\sigma \rangle$ calculated for an incoming energy squared $s=M_1.(M_1+2\eta kT)$, where $\eta=3.15$ corresponds to the average energy for a fermion

$$\langle E_f \rangle = 7/2.\zeta(4)/\zeta(3).kT = \eta kT \simeq 3.15.kT \qquad (2\text{-}18)$$

d/ $N_1 \longrightarrow N_2\, f\bar{f}$

The contribution of this decay channel can be derived as above. Here as soon as $T<T_1$ the 3-body fusion $N_2\, f\, \bar{f} \longrightarrow N_1$ becomes very unlikely and we can write for all temperatures with the same approximation as in (2-17) the collision integral:

$$D(1\rightarrow 2f\bar{f}) = \langle \Gamma(N_1\rightarrow N_2 f\bar{f}) \rangle \,.\, [n^0_1(T) - n_1(T)] \qquad (2\text{-}19)$$

We note that this contribution is linear in the number density contrary to all others.
Let us finally consider the reaction which concerns N_2, namely;

e/ $N_2\, \overline{N_2} \longrightarrow f\bar{f}$

As T is below T_1 the population of N_2 is maintained in equilibrium by this reaction. The freezing temperature T_2 of the N_2's is then close to the stopping temperature Θ. We, once again, can write for this annihilation channel:

$$A(2\bar{2}\rightarrow f\bar{f}) = \langle v\sigma(2\bar{2}\rightarrow f\bar{f}) \rangle \,.\, [\, n^0_2(T)^2 - n_2(T)^2 \,] \qquad (2\text{-}20)$$

This is just a particular case of (2-11).

Collecting all the terms we can obtain the kinetic equation for $n_1(T)$ and $n_2(T)$:

$$\frac{dn_1(T)}{dt} + 3.\frac{dR(t)}{dt}\frac{n_1(T)}{T^3} = \Sigma\,[\, A(1\bar{1}\rightarrow f\bar{f}) + A(1\bar{2}\rightarrow f\bar{f}) + D(1\rightarrow 2f\bar{f})\,] + \Sigma\, B(1i\rightarrow jk) + \Sigma\, B(1f\rightarrow 2f) \qquad (2\text{-}21a)$$

$$\frac{dn_2(T)}{dt} + 3.\frac{dR(t)}{dt}\frac{n_2(T)}{T^3} = \Sigma\,[\, A(2\bar{2}\rightarrow f\bar{f}) + A(1\bar{2}\rightarrow f\bar{f}) - D(1\rightarrow 2f\bar{f})\,] - \Sigma\, B(1i\rightarrow jk) - \Sigma\, B(1f\rightarrow 2f) \qquad (2\text{-}21b)$$

where the sums are over particle and antiparticle labels. The second term in the left hand side of the equations is related to the expansion of the universe; if instead of $n(T)$ we define $f(T)=n(T)/T^3$ -not to be confused with $f(e)$- , which corresponds to the number density for a covolume , this term disappears. We obtain :

$$\frac{1}{T^3}.\frac{df_1(T)}{dt} = \Sigma \{<v.\sigma(1\overline{1}\rightarrow f\overline{f})> [f^o_1(T)^2-f_1(T)^2] + \\ + <v.\sigma(1\overline{2}\rightarrow f\overline{f})>.[f^o_1(T).f^o_2(T)-f_1(T).f_2(T)]\} \\ +\Sigma'2' <v.\sigma(1i\rightarrow jk)>.[f^o_1(T).f^o_i(T)-f_1(T).f_i(T)] \\ +\{ \Sigma <\Gamma(1\rightarrow 2f\overline{f})+\Sigma<v.\sigma(1f\rightarrow 2f)>.n^o(T)\}.[f^o_1(T)-f_1(T)]/T^3 \tag{2-22a}$$

$$\frac{1}{T^3}.\frac{df_2(T)}{dt} = \Sigma \{<v.\sigma(2\overline{2}\rightarrow f\overline{f})> [f^o_2(T)^2-f_2(T)^2] + \\ + <v.\sigma(1\overline{2}\rightarrow f\overline{f})>.[f^o_1(T).f^o_2(T)-f_1(T).f_2(T)]\} \\ -\Sigma'2' <v.\sigma(1i\rightarrow jk)>.[f^o_1(T).f^o_i(T)-f_1(T).f_i(T)] \\ -\{ \Sigma <\Gamma(1\rightarrow 2f\overline{f})+\Sigma<v.\sigma(1f\rightarrow 2f)>.n^o(T)\}.[f^o_1(T)-f_1(T)]/T^3 \tag{2-22b}$$

where $n^o(T)$ is the equilibrium density of the light fermions which is

$$n^o(T)=\frac{1}{\pi^2}\,\frac{3}{4}\,\zeta(3).T^3 \tag{2-23}$$

for each helicity state if we neglect the mass.Let us note that in this case $f^o(T)=n^o(T)/T^3$ is a constant. The factor '2' takes into account the fact that reaction $N_1N_1\longrightarrow N_2N_2$ suppresses 2 N_1's at once.

3.SOLUTION OF THE KINETIC EQUATIONS AND DISCUSSION.

In this Section,we will solve the system of equations established in Section 2.In order to do that,we first have to calculate the cross-sections associated to processes (2.6).Let us assume that the coupling of our binary system to the Z_0 gauge boson takes the general following form :

$$M_z\sqrt{(G_f/\sqrt{2})}.(\overline{N_1}\ \overline{N_2}).\gamma\mu(1-\gamma_5).\begin{bmatrix} b & c \\ c & b \end{bmatrix}.\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}\ Z\mu \tag{3-1}$$

We will first review the different reactions arising from such a Lagrangian whereas the corresponding cross-sections are given in Appendix B.
A first class of processes couples the system to the light fermion matter.More precisely,we have :

-- annihilation (Figs.1a-b-c)

$$N_1\,\overline{N_1} \longrightarrow f\,\overline{f} \tag{a}$$

$$N_1\,\overline{N_2} \longrightarrow f\,\overline{f} \quad \text{and} \quad N_2\,\overline{N_1} \longrightarrow f\,\overline{f} \tag{b}$$

$$N_2\,\overline{N_2} \longrightarrow f\,\overline{f} \tag{c}$$

These are the only ones present in the case of stable neutrinos [6,8].They are consequently the only reactions responsible for the violation of the total number n_1+n_2 .

-- decay (Fig.1d)

$$N_1 \longrightarrow N_2 \; f \; \overline{f} \qquad (d)$$

that leaves N_2 as the only stable particle of the binary system.

-- the scattering of heavy "neutrinos" upon light matter (Fig.1e)

$$N_1 \; f(\text{or } \overline{f}) \longrightarrow N_2 \; f(\text{or } \overline{f}) \qquad (e)$$

which will prove to introduce noticeable modifications with the one lepton case.
A second class is provided by processes internal to the binary system :

-- annihilation (Fig.1f-g-h)

$$N_1 \; \overline{N_1} \longrightarrow N_2 \; \overline{N_2} \qquad (f)$$

$$N_1 \; \overline{N_1} \longrightarrow N_2 \; \overline{N_1} \qquad (g)$$

$$N_1 \; \overline{N_2} \longrightarrow N_2 \; \overline{N_2} \qquad (h)$$

-- scattering (Fig.1i-j-k)

$$N_1 \; N_1 \longrightarrow N_1 \; N_2 \qquad (i)$$

$$N_1 \; N_1 \longrightarrow N_2 \; N_2 \qquad (j)$$

$$N_1 \; N_2 \longrightarrow N_2 \; N_2 \qquad (k)$$

We did not compute the exact behaviour of the cross-section for the decay process $N_1 \longrightarrow N_2 \; N_2 \; \overline{N_2}$ assuming that we could take it into account in the effective number of decay channels for process (d).
The evolution equations for n_1 and n_2 have been derived in Section 2 (eqs.2-22).Processes (a) to (c) and (f) to (k) yield terms quadratic in the number densities n_1 or n_2 whereas the presence of the decay (d) and the scattering upon light matter (e) gives rise to linear terms which will prove to play an important role in the evolution of the number densities.We prefer to use in eqs(2-22) the variable

$$x = T/M_1 \qquad (3\text{-}2)$$

where the temperature T is related to the time by the following relation ,valid in the radiation dominated era:

$$t = \sqrt{\frac{45}{32\pi^3 G}} \cdot \frac{1}{\sqrt{N_f}} \cdot \frac{1}{T^2} = \frac{K}{\sqrt{N_f}} \cdot \frac{1}{T^2} \qquad (3\text{-}3)$$

In (3-3) G is the gravitationnal constant ,Nf counts the number of degrees of freedom effective at the temperature T (if nF [nB] is the fermionic [bosonic] number of helicity states, Nf=7/16.nF+1/2.nB). Finally ,if temperatures and masses are expressed in Mev,K=0.26 10^{22} Mev ($\hbar=c=k=1$). Using the notations defined in the appendix B for cross sections ,the evolution equations now read :

$$\frac{df_1}{dx} = \frac{2KM_1}{\sqrt{Nf}} \{[f_1(x)^2-f^0{}_1(x)^2][\langle\sigma_a v\rangle+\langle\sigma_f v\rangle+\langle\sigma_g v\rangle+\langle\sigma_i v\rangle+2\langle\sigma_j v\rangle]$$

$$+[f_1(x)f_2(x)-f^0{}_1(x)f^0{}_2(x)][\langle\sigma_b v\rangle + \langle\sigma_h v\rangle + \langle\sigma_k v\rangle]$$

$$+ [f_1(x)-f^0{}_1(x)][\Gamma + n^0{}_f(x).\langle\sigma_e v\rangle]/ (M_1.x)^3\} \qquad (3\text{-}4a)$$

$$\frac{df_2}{dx} = \frac{2KM_1}{\sqrt{Nf}} \{[f_1(x)^2-f^0{}_1(x)^2][- \langle\sigma_f v\rangle - \langle\sigma_g v\rangle - \langle\sigma_i v\rangle - 2\langle\sigma_j v\rangle]$$

$$+[f_2(x)^2-f^0{}_2(x)^2].\langle\sigma_c v\rangle +$$

$$+[f_1(x)f_2(x)-f^0{}_1(x)f^0{}_2(x)][\langle\sigma_b v\rangle - \langle\sigma_h v\rangle - \langle\sigma_k v\rangle]$$

$$- [f_1(x)-f^0{}_1(x)][\Gamma + n^0{}_f(x).\langle\sigma_e v\rangle]/ (M_1.x)^3\} \qquad (3\text{-}4b)$$

where $n^0{}_f(x)$ is given in eq(2-23).
We emphasize the fact that f_1 refers to the population of N_1 fermions (and not to the antifermions $\overline{N_1}$). Since we assumed CP conservation and that the chemical potentials vanish:

$$f_{N_1} = f_{\overline{N_1}} = f_1 \qquad ; \qquad f_{N_2} = f_{\overline{N_2}} = f_2 \qquad (3\text{-}5)$$

As already mentionned, the only processes that violate the total number n_1+n_2 are the annihilations into light matter(a-b-c). Therefore ,as one can check

$$\frac{d(f_1+f_2)}{dx} = \frac{2K.M_1}{\sqrt{Nf}} \{[f_1(x)^2-f^0{}_1(x)^2]\langle\sigma_a v\rangle + [f_2(x)^2-f^0{}_2(x)^2]\langle\sigma_c v\rangle$$

$$+2.[f_1(x)f_2(x)-f^0{}_1(x)f^0{}_2(x)].\langle\sigma_b v\rangle \} \qquad (3\text{-}6)$$

We will now discuss the solutions of the system (3-4) going all the way from a diagonal type of coupling (b=1,c=0) to an antidiagonal coupling (b=0,c=1) and investigating intermediate possibilities. The discussion will therefore be based on the values for the b and c parameters and on the mass gap between the two heavy fermions:

$$\Delta M = M_1 - M_2 = M_1.(1 - \alpha) \qquad (3\text{-}7)$$

3-1: The quasi diagonal coupling

In the diagonal case (c=0,b=1), we are left with two stable particles which interact with light matter through annihilation processes a) and c). The analysis of Lee and Weinberg [6] can therefore be readily generalised to this

mixture of two heavy neutrinos to give a bound on the lower mass M_2. These bounds still depend on the mass gap. We have two extreme solutions:
-if $M_1 >> M_2$ the present mass density is mainly due to N_2 and is equal to [6] :

$$\rho_2 = (4.2\ 10^{-28}\ g/cm^3).(M_2\ Gev)^{-1.85}.(N_a/\sqrt{Nf})^{-0.85} \qquad (3-8)$$

where N_a is the number of annihilation channels(N_a =14.6 for a mass of 5 Gev). *We obtain a lower bound on M_2 from the* requirement that

$$\rho_2 < \Omega.\rho_c.(H/H_0)^2 \qquad (3-9)$$

where H_0 = 50. Km/s/Mpc and ρ_c = 5. 10^{-30} g/cm^3 is the critical density [2] .Taking Ω=1 gives an absolute lower bound on M_2 since it assumes that the N_2 fermions fill the whole missing mass in the universe.We can also,following Gunn et al. [8],consider that the heavy neutrinos collapsed around galaxies forming the so-called galactic halo,in which case Ω=0.1.The limits on M_2 reads :

$$M_2 > 4.5\ Gev.\ (\Omega^{-1})^{0.54}\ .(H_0/H)^{1.08} \qquad (3-10)$$

that *is for Ω=0.1*

$$M_1 >> M_2 > 15\ Gev\ .(H_0/H)^{1.08} \qquad (3-11)$$

-if $M_1 = M_2$,the present energy of these leptons is just twice that of each particle.Therefore (See eq(3-8)) the lower bound is raised by a factor $2^{0.54}$ and we obtain for Ω=0.1,

$$M_1 = M_2 > 22\ Gev.\ (H_0/H)^{1.08} \qquad (3-12)$$

We now allow for a small departure from this diagonal case by giving the non diagonal term a small value $\varepsilon << 1$.The decay process d) and the scattering process e) on the light fermions play now a crucial role in the evolution and will lead to a rather different situation. If the mass gap is important,the scattering is suppressed but the decay is determinant.This situation has been studied in detail in ref [12],where we have derived *limits on the mass M_1* and on the lifetime τ (which is related to the parameter ε).In Fig.2-a we give the evolution of n_1 and n_2 for ΔM=500 Mev and $\varepsilon^2=10^{-8}$ .Clearly N_1 leptons disappear completely after some time of the order of their lifetime τ.
For a much smaller mass gap (ΔM=10 Mev) ,the lifetime becomes larger than the age of the universe at the time of quenching and the decay does not play a significant role.For $\varepsilon^2=10^{-8}$ (Fig.2-b),we are still in a case similar to the pure diagonal one :each heavy lepton behaves as if it was alone and we recover the limit (3-12).But already for $\varepsilon^2=10^{-6}$ (fig.3-a),the scattering process e) starts playing an important role.Basically,as long as there are important amounts of light fermions and of N_1 leptons,the transformation of N_1 into N_2 occurs.But the expansion of the

universe and the decrease of the number density n_1 generate the freezing of n_1. After some time and before the decay becomes important (for $\Delta M=10$ Mev, $\tau=20$ minutes), both N_1 and N_2 populations are fossilized. When we increase ε, the role of the scattering process e) becomes more and more crucial and the frozen value of n_1/T^3 decreases. From Fig.3-b for example we see that the frozen value is down by more than one order of magnitude when ε^2 is changed from 10^{-6} to 4.10^{-6}.

Therefore as soon as one departs from the diagonal case with equal masses ($\Delta M \neq 0$ or $\varepsilon \neq 0$), we have the elimination of the N_1 leptons and we recover the limit of Lee and Weinberg(3-11):

$$M_2 > 15 \text{ Gev} \, . \, (H_0/H)^{1.08} \qquad (3\text{-}13)$$

Due to the relaxation, when the decay or the scattering becomes important, the n_1 curve is pushed toward its equilibrium value n^0_1. When these processes play a crucial role, N_1 remains in chemical equilibrium because very fast reactions connect it with light matter.

3-2: The "democratic" coupling

We consider the intermediate case where b=c=1. If $M_1=M_2$, the cross sections of reactions (a), (b) and (c) turn out to be equal. We therefore obtain from (3-6) the evolution equation for the total number density.

$$\frac{d(f_1+f_2)}{dx} = \frac{2K}{\sqrt{N_f}} . M_1 \langle \sigma_1 v \rangle . [(f_1+f_2)^2-(f^0_1+f^0_2)^2] \qquad (3\text{-}14)$$

Clearly the solution is

$$f_1(x) = f_2(x) = f(x)/2 \qquad (3\text{-}15)$$

where $f(x)$ is the solution given by Lee and Weinberg. Despite the fact that we have two stable heavy neutrinos the present mass density is the same as if we had only one particle, and we still have the lower bound (3-11). On the other hand when $M_1 \neq M_2$, even for a very small mass gap the decay d) and the scattering e) will play a crucial role and quicken dramatically the disapearance of N_1. We are thus rapidly left with a unique stable heavy lepton N_2 and the Lee-Weinberg analysis still applies. Thus in both cases

$$M_2 > 15 \text{ Gev} \, . \, (H_0/H)^{1.08} \qquad (3\text{-}16)$$

3-3: The anti-diagonal coupling

We finally study the case c=1 and $b=\varepsilon' \ll 1$. Since the coupling of N_1 to N_2 is the strong one, the decay d) and the scattering on light fermions e) will play the proeminent role and tend to eliminate N_1 as the temperature drops below M_1. Therefore, even after the decoupling of the N_1 population, the number density of N_1 will stay close to its equilibrium value because every excess would decay or be scattered off. During the quenching of the N_2 population, we can therefore assume that :

$$n_1 \simeq n^0_1 \ll n_2 \qquad (3\text{-}17)$$

Under these conditions, eq(3-6) supplies us with an evolution equation for f_2 :

$$\frac{df_2}{dx} = \frac{2K}{\sqrt{N_f}} . M_1 \left\{ [f_2(x)^2 - f^0_2(x)^2] <\sigma_c v> + 2 f^0_1(x) [f_2(x) - f^0_2(x)] <\sigma_b v> \right\} \qquad (3\text{-}18)$$

Thus only annihilation processes influence the evolution of n_2: a slow one (proportionnal to ε'^2) involving only N_2 and a fast one annihilating N_2 on N_1.

We first discuss the case $\varepsilon'=0$. Results are shown in Fig.4, for M_2=5Gev and various mass gaps(ΔM=0.5,1,2,5,10 Gev). We observe that the larger the mass gap the more numerous the present N_2 population is. Moreover, as compared to the previous cases discussed above, the N_2 density (even for a rather low mass gap of ΔM=500 Mev) is much higher and actually exceeds the upper limit allowed by eq(3-9). The physical interpretation of this phenomenon is rather simple: the only reaction that will allow N_2 to disappear is the annihilation process b) ($N_2\overline{N}_1 \to f\overline{f}$) but at the time of quenching there are so few N_1 leptons left (because of decay and scattering) that this annihilation is slowed down noticeably and even stopped for large enough mass gaps. We can therefore not only put a limit upon the mass of the N_2 lepton but also on the mass gap. The upper bound on ΔM is estimated very conservatively to be :

$$\Delta M = M_1 - M_2 \leq 0.1 \ M_2 \qquad (3\text{-}19)$$

As for the limit upon M_2, since we have a mixture of quasi degenerate heavy leptons, we recover the limit (3-12).

$$M_2 > 22 \text{ Gev} . (H_0/H)^{1.08} \qquad (3\text{-}20)$$

We then turn to the case $\varepsilon' \neq 0$. Examples are shown for M_2=5 Gev and for various mass gaps in Fig.5. For mass gaps up to 2 Gev nothing is changed. The mixing interaction b) dominates and the upper limit upon the mass density of the N_2 population is exceeded even for ΔM=500 Mev. For very large mass gaps (ΔM=10 Gev), we observe that the present mass density of the surviving N_2 does not depend on ΔM and decreases as ε' increases; physically, for a large mass gap, annihilation b) is suppressed whereas annihilation c) ($N_2\overline{N_2} \to f\overline{f}$) acts with an extra factor ε'^2 exactly as if the mass M^2 was renormalised to $\varepsilon'^{2/3}.M_2$.

Finally we give the mass bounds that arise from our analysis.

Either

$$M_2 > 15 \text{ Gev} . \varepsilon'^{-2/3} . (H_0/H)^{1.08} \qquad (3\text{-}21)$$

and the mass M_1 is unconstrained ($M_1 > M_2$), or M_2 is smaller than this limit and the mass gap is also bounded

$$M_2 > 22 \text{ Gev} . (H_0/H)^{1.08} \quad ; \quad \Delta M/M_2 < 0.1 \qquad (3\text{-}22)$$

4.A PHENOMENOLOGICAL MODEL.

The supersymmetric grand unified theories provide us with models where the above results apply in a very direct way. In a simple SU(5) model for example, it is necessary [13] to double the 5 of Higgs scalars and introduce a 5 and a $\overline{5}$ scalar superfields (respectively H_1 and H_2). Their spin 1/2 components, the Higgsinos $\tilde{H}_1$ and $\tilde{H}_2$, therefore form a binary system of the kind studied in the previous section. Realistic theories are somewhat more intricate. In the model of J.Ellis, L.E.Ibanez and G.G.Ross [14], in the case where the photino is not the lightest particle, we have to consider a system of three particles : $\tilde{H}_1$, $\tilde{H}_2$ and the fermionic component $\tilde{Z}$ of an extra chiral superfield which transforms as a singlet under SU(5). The mass eigenstates are [14] (neglecting a small admixture of gauginos):

$$\tilde{A}^0 = \frac{v_1\tilde{H}_1{}^0 - v_2\tilde{H}_2{}^0}{v}$$
$$\tilde{S}^0_\pm = \frac{1}{\sqrt{2}}\left(\frac{v_2\tilde{H}_1{}^0 \pm v_1\tilde{H}_2{}^0}{v} + \tilde{Z}\right) \qquad (4.1)$$

where v_1 and v_2 are the vacuum expectation values of superfields H_1 and H_2 ($v^2 = v_1{}^2 + v_2{}^2$).
These three neutral fermions couple to the Z through the coupling :

$$\frac{M_Z}{\sqrt{2}}\sqrt{\frac{G_F}{\sqrt{2}}}\,Z\mu\,(\overline{\tilde{S}^0_+\tilde{S}^0_-\tilde{A}^0})\begin{bmatrix} \frac{v_2{}^2-v_1{}^2}{2v^2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} & \frac{2v_1v_2}{v^2}\begin{bmatrix}1\\ 1\end{bmatrix} \\ \frac{2v_1v_2}{v^2}\begin{bmatrix}1 & 1\end{bmatrix} & \frac{v_1{}^2-v_2{}^2}{v^2}\end{bmatrix}\gamma\mu(1-\gamma_5)\begin{bmatrix}\tilde{S}^0_+\\ \tilde{S}^0_-\\ \tilde{A}^0\end{bmatrix} \qquad (4\text{-}2)$$

Let us suppose that the $\tilde{A}^0$ fermion is the heaviest ; the $\tilde{S}^0_-$ is the lightest and , unless forbidden by a suitable choice of parameters the $\tilde{A}^0$ and $\tilde{S}^0_+$ will decay into it . Following the results of Section 3, we can outline two different situations :

4-1: The asymmetric vacuum.

We set for example

$$v_2 \simeq v \qquad \text{and} \qquad v_1 = \varepsilon v \qquad (4\text{-}3)$$

If $\varepsilon \neq 0$, $\tilde{A}^0$ and $\tilde{S}^0_+$ will decay rapidly into $\tilde{S}^0_-$ and we recover the bound (3-13) of Lee and Weinberg (accounting for a difference of a factor $2\sqrt{2}$ between couplings in Eq(3-1) and Eq(4-2) and the fact that we deal with Majorana spinors)

$$\text{mass of } \tilde{S}^0_- > 30 \text{ Gev} . \; (H_0/H)^{1.08} \qquad (4\text{-}4)$$

This certainly is a fairly high value for the mass of $\tilde{S}^0_-$.
If $\varepsilon = 0$, the $\tilde{A}^0$ does not decay and its presence would still raise the bound .

4-2: *The symmetric vacuum.*

We now study the case

$$v_1 \simeq v_2 \quad \text{and} \quad (v_2 - v_1)/v = \varepsilon' \ll 1 \tag{4-5}$$

The coupling matrix in Eq(4-2) now reads :

$$\begin{bmatrix} \frac{\varepsilon'}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \end{bmatrix} & -\varepsilon'\sqrt{2} \end{bmatrix} \tag{4-6}$$

This is precisely a coupling of the antidiagonal form (Sect.3-3) . Because the $\tilde{S}^0$ fermions can annihilate only on the $\tilde{A}^0$ still present at the time of their quenching , their number density is untolerably high unless $\tilde{A}^0$ is almost degenerate to them . We therefore can bound all the masses of the system :

$$\frac{\text{mass of } \tilde{A}^0 - \text{mass of } \tilde{S}^0}{\text{mass of } \tilde{S}^0} < 0.1 \tag{4-7}$$

and (extrapolating the result of Eq(3-20) to the case of three Majorana particles)

$$\text{mass of } \tilde{S}^0 > 27 \text{ Gev } (H_0/H)^{1.08} \tag{4-8}$$

As the masses can be expressed in terms of several parameters of the model [14], Eqs(4-7) and (4-8) put rather stringent bounds on these parameters . In particular, Eq(4-7) which relates a priori uncorrelated parameters constrains them in a somewhat unnatural way .
One should finally note that the situation of a symmetric vacuum , where our study gives the most severe constraints , is the one encountered when grand unified theories are coupled to N=1 supergravity [15].

5.CONCLUSION.

We think that it is important to study the evolution of the densities of a heavy stable neutral fermion and an heavier one that decays into it as the evolution of a coupled system. In that respect, not only the decay but also the scattering upon light particles prove to play an important role , because both of them yield linear terms in the evolution equations . Thanks to them, as soon as one departs from a diagonal coupling (Sect.3-1 and 3-2), one recovers for the lighter one the well-known limit given by Lee and Weinberg. However, we showed that in the case of a (quasi-)antidiagonal coupling (Sect.3-3), we can constrain the masses of the whole system. *In particular the masses of the two neutral fermions* must be equal within 10%. This enabled us to give constraints on the Higgsino masses of supersymmetric grand unified models (or models coupled to N=1 supergravity).

Appendix A

In this appendix, we justify our assumption made in section 2 where for reactions (2-6c) we stated

$$\int d^3p_f/(2\pi)^3 . f^0(e_f) . v\sigma(N_1 f \to N_2 f) = \langle v\sigma \rangle . n^0_f(T) \qquad \text{(A-1)}$$

where $\langle v\sigma \rangle$ is calculated for an average energy of the light fermion $\langle E_f \rangle \sim 3.15 . kT$.
We prove our assumption in the case where $f^0(e_f)$ is a Maxwell-Boltzmann distribution to do the calculation analytically. We have:

$$\langle \sigma . v \rangle = \frac{G_f^2}{24\pi} s . \left(1 - \frac{M_2^2}{s}\right)^2 . \left[8 + \frac{(M_1 M_2)^2}{s^2} + \frac{M_1^2 + M_2^2}{s} \right] \qquad \text{(A-2)}$$

where $s = M_1^2 + 2 . M_1 . E_f = M_1^2 . t$ i.e. $t = 1 + 2E_f/M_1$. In this case the left-hand side of (A-1) takes the following form:

$$I = A . \exp(\mu) . \int dt . \exp-(\mu t)(t-1)^2 t^{-3}(t-\alpha^2)^2 [8t^2 + 2\alpha^2 + t(1+\alpha^2)]$$

where $A = G_f^2/(48\pi^3) . M_1^5 . \Delta(T)/8$, $\Delta(T)$ is defined in appendix B , $\alpha = M_2/M_1$ and $\mu = 1/2x = M_1/2T$. The integrals in (A-2) can be performed analytically, the result contains the exponential integral function

$$E_1(\mu) = \int dt/t \, \exp-(\mu t) \qquad \text{(A-3)}$$

However we are in a regime where $T < M_1$ i.e. μ is large, so that we can use the following expansion:

$$E_1(\mu) \simeq \exp-(\mu)/\mu . [1 - 1/\mu + 2/\mu^2 - 6/\mu^3 \ \ldots] \qquad \text{(A-4)}$$

Collecting all the terms we find that the terms in μ^{-1} and μ^{-2} vanish , so we are left with

$$I = 6 . A . [8 . (1-\alpha^6)/\mu^4 + (3 - 5\alpha^2 + \alpha^4 + \alpha^6)/\mu^3] + O(\mu^{-5}) \qquad \text{(A-5)}$$

This expression has to be compared with the right-hand side of (A-1)
In the case of a Maxwell-Boltzmann distribution,

$$n_0(T) = T^3/\pi^2 = 1/(8\pi^3) \, M_1^3/\mu^3 \qquad \text{(A-6)}$$

$$\langle E \rangle = 3T \text{ i.e. } s = M_1^2 . \, (1+3/\mu) \qquad \text{(A-7)}$$

Then if one calculates :

$$\langle \sigma v \rangle n_0(T) = 2 . A/\mu^3 . (1+3/\mu)^{-3} (1 - \alpha^2 + 3/\mu)^2 [8(1+3/\mu)^2 + 2\alpha^2 + (1+\alpha^2)(1+3/\mu)] \qquad \text{(A-8)}$$

which expanded in μ coincides with (A-5).
One can check that

$$I = \langle \sigma v \rangle . \, n_0(T) . [1 + O(\mu^{-2})] \, . \qquad \text{(A-9)}$$

Let us mention that higher order terms are all proportional to α^4 so that the accuracy improves for $M_1 \gg M_2$ and the two expressions coincide analytically for $\alpha = 0$.

Appendix B

In this appendix we give the cross sections for all the processes which are discussed in the paper. The relevant graphs are given in Fig.1. We first review the notations that we use.
We take the couplings of the light fermions to the neutral Z_0 gauge boson to be of the form :

$$L_{ffz} = M_z/\sqrt{2}.\sqrt{(G_f/\sqrt{2})}.\bar{f}.\gamma\mu.(v_f-a_f\gamma_5).f.Z\mu \qquad \text{(B-1)}$$

The number of channels open for a specific process where the available energy is $\sqrt{s}$, will be denoted by:

$$\Delta(\sqrt{s}/2) = \Sigma_f\ (v_f{}^2 + a_f{}^2)/2 \qquad \text{(B-2)}$$

where the sum runs over particles (not their antiparticles) for which $m_f<\sqrt{s}/2$. Finally we will use

$$\alpha = M_2/M_1 \qquad \text{(B-3)}$$

The cross sections for the annihilation into light matter are :

a) $N_1\overline{N_1}\to f\bar{f}$ $\qquad \langle\sigma_a v\rangle = G_f{}^2/(2\pi)M_1{}^2b^2\Delta(M_1)$ (B-4)

b) $N_1\overline{N_2}\to f\bar{f}$ $\qquad \langle\sigma_b v\rangle=G_f{}^2/(8\pi)M_1{}^2(1+\alpha)^2c^2\Delta(M_1(1+\alpha)/2)$ (B-5)

c) $N_2\overline{N_2}\to f\bar{f}$ $\qquad \langle\sigma_c v\rangle=G_f{}^2/(2\pi)M_1{}^2\alpha^2b^2\Delta(\alpha M_1)$ (B-6)

The decay width for the process d) $N_1\to N_2f\bar{f}$ is :

$$\Gamma=\tau^{-1}=G_f{}^2/(384\pi^3).M_1{}^5c^2\Delta(M_1(1-\alpha)/2).f(\alpha) \qquad \text{(B-7)}$$

where

$$f(\alpha)=1-8\alpha^2-12\alpha^4\ln\alpha^2+8\alpha^6-\alpha^8 \qquad \text{(B-8)}$$

Let us note that for $M_1\simeq M_2$

$$f(\alpha)=2/5.(1-\alpha^2)^5 + O[(1-\alpha^2)^6] \qquad \text{(B-9)}$$

whose effect is to slow down the decay in a drastic way.
For the scattering on light matter, we have

e) N_1 f(or $\bar{f}$) $\to$ N_2 f(or $\bar{f}$)

$$\langle\sigma_e v\rangle=G_f{}^2/(12\pi)M_1{}^2c^2(1-\alpha^2+2\eta x)^2[(1+2\eta x)(9+16\eta x)+\alpha^2(3+2\eta x)]$$

$$.[\Delta(M_1x)-3/2]/[1+2\eta x]^3 \qquad \text{(B-10)}$$

where ηT is the average energy of the light fermions (η is defined in eq(2-18)).
The cross sections for processes that involve only heavy fields read :

f) $N_1\overline{N_1}\to N_2\overline{N_2}$ $\qquad \langle\sigma_f v\rangle=G_f{}^2/\pi(b^2+c^2)^2M_1{}^2\sqrt{(1-\alpha^2)}$ (B-11)

g) $N_1\overline{N_1} \to N_2\overline{N_1}$

$$\langle\sigma_g v\rangle = G_f{}^2/(16\pi)b^2c^2M_1{}^2(3+\alpha^2)(5-\alpha^2)\sqrt{(9-\alpha^2)(1-\alpha^2)} \qquad \text{(B-12)}$$

h) $N_1\overline{N_2} \to N_2\overline{N_2}$ $\quad \langle\sigma_h v\rangle = G_f{}^2/\pi b^2c^2M_1{}^2(1+\alpha)\sqrt{(1-\alpha)(1+3\alpha)} \qquad$ (B-13)

i) $N_1N_1 \to N_1N_2$

$$\langle\sigma_i v\rangle = G_f{}^2/(2\pi)b^2c^2M_1{}^2(3-\alpha^2)\sqrt{(1-\alpha^2)(9-\alpha^2)} \qquad \text{(B-14)}$$

j) $N_1N_1 \to N_2N_2$ $\quad \langle\sigma_j v\rangle = G_f{}^2/\pi c^4M_1{}^22(2-\alpha^2)\sqrt{(1-\alpha^2)} \qquad$ (B-15)

k) $N_2N_1 \to N_2N_2$

$$\langle\sigma_k v\rangle = G_f{}^2/\pi b^2c^2M_1{}^2(1+2\alpha-\alpha^2)\sqrt{(1-\alpha)(1+3\alpha)}/(1+\alpha) \qquad \text{(B-16)}$$

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FIGURE CAPTIONS

Fig.1 : Processes leading to a change in the number of N_1 and N_2 via the exchange of a neutral weak gauge boson.

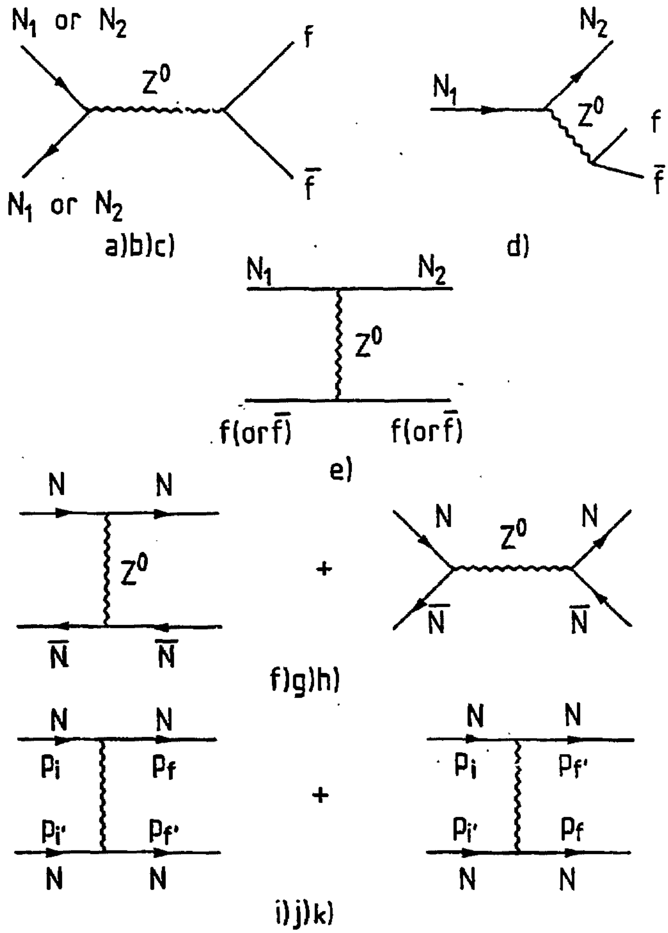
a)b)c) annihilation into light fermions
d) decay e) scattering upon light matter
f)g)h) annihilation into N_1 or N_2
i)j)k) scattering of N_1 upon N_1 or N_2.

Fig.2 : Evolution of the densities n_1 and n_2 in the quasi-diagonal case for M_2=5.0 Gev,$\varepsilon^2=10^{-6}$ and a) ΔM=500Mev b) ΔM=10 Mev . The steep curves (which are placed on top of each other in case b)) are the equilibrium densities. τ is the lifetime of fermion N_1.

Fig.3 : Evolution of the densities n_1 and n_2 in the quasi-diagonal case for M_2=5.0 Gev , ΔM=10 Mev and a) $\varepsilon^2=10^{-6}$ b) $\varepsilon^2=4.10^{-6}$.

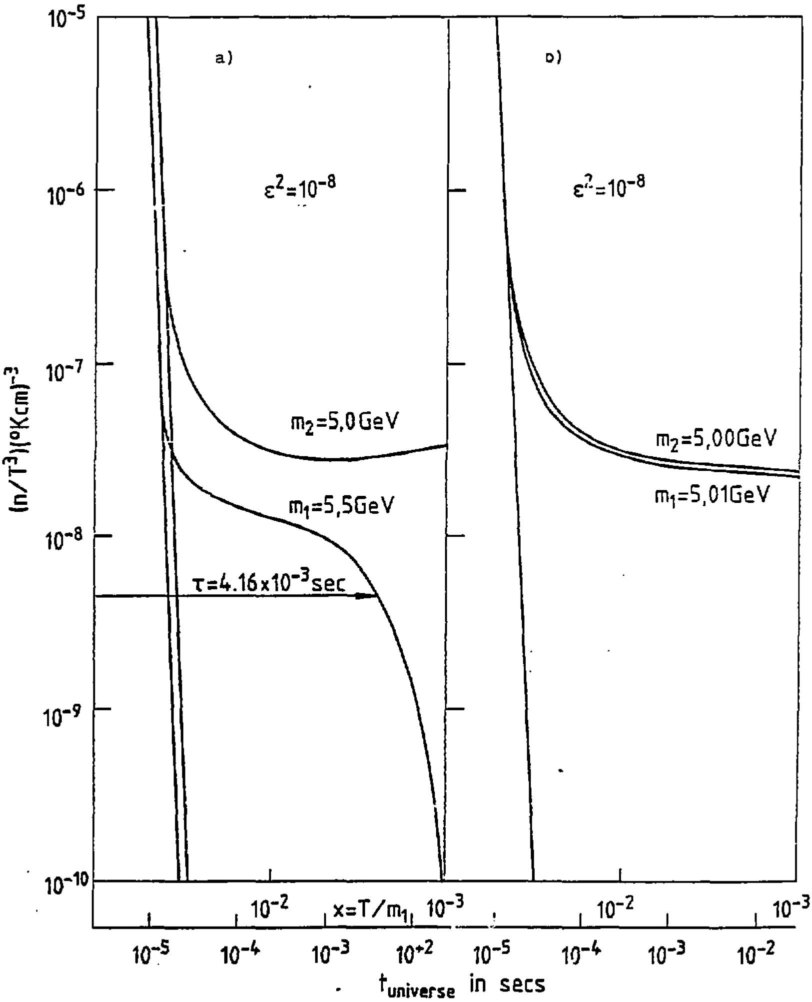
Fig.4 : Evolution of the density n_2 in the case of an antidiagonal coupling (ε'=0) for M_2=5 Gev and M_1=5.5,6,7,10 and 15 Gev.

Fig.5 : Evolution of the density n_2 in the case of a quasi-antidiagonal coupling ($\varepsilon'^2=10^{-4}$) for M_2=5 Gev and M_1=5.5,6,7,10 and 15 Gev.



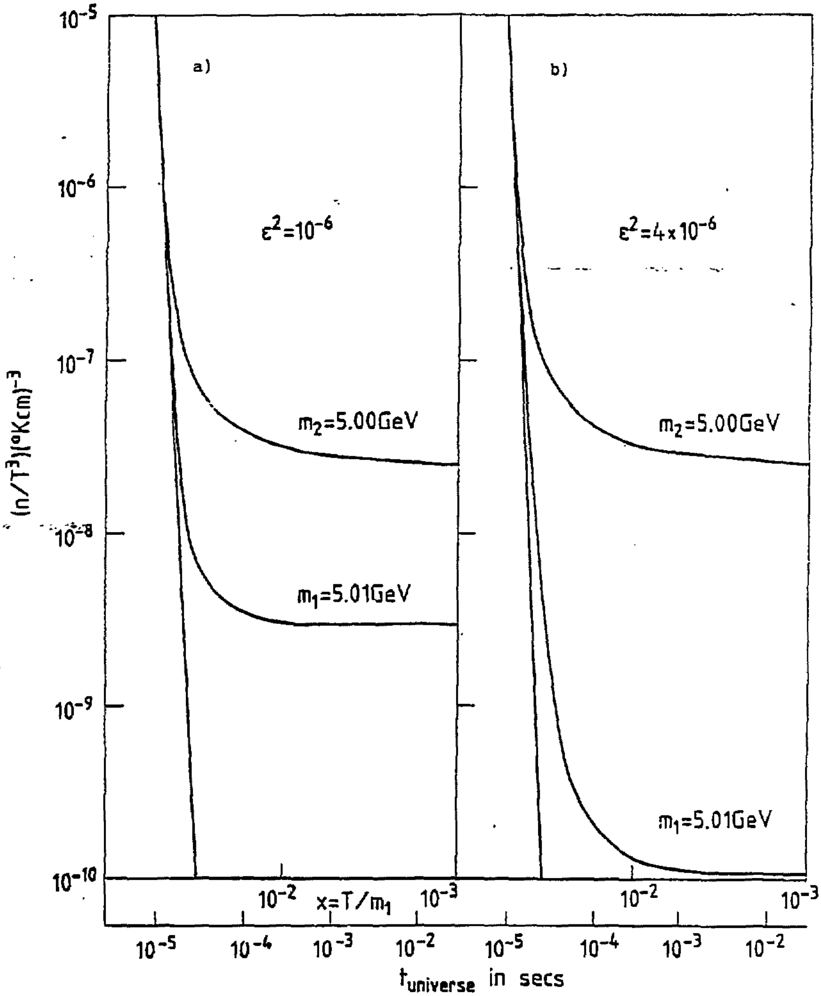
N1 or N2
Z0
f
f̄
N1 or N2
a)b)c)
N2
N1
Z0
f
f̄
d)
N1
N2
Z0
f(or f̄)
f(or f̄)
e)
N
N
Z0
N̄
N̄
+
N
Z0
N
N̄
N̄
f)g)h)
N
N
pi
pf
pi'
pf'
N
N
+
N
N
pi
pf'
pi'
pf
N
N
i)j)k)

Fig. 1



a)
b)
ε²=10⁻⁸
ε²=10⁻⁸
m₂=5,0GeV
m₁=5,5GeV
τ=4.16x10⁻³sec
m₂=5,00GeV
m₁=5,01GeV
(n/T³)[(°Kcm)⁻³]
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻²
x=T/m₁
10⁻³
10⁻²
10⁻³
10⁻⁵
10⁻⁴
10⁻³
10⁺²
10⁻⁵
10⁻⁴
10⁻³
10⁻²
t_universe in secs

Fig. 2



a)
b)
$\varepsilon^2=10^{-6}$
$\varepsilon^2=4\times10^{-6}$
$m_2=5.00$GeV
$m_1=5.01$GeV
$m_2=5.00$GeV
$m_1=5.01$GeV
$(n/T^3)(°Kcm)^{-3}$
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-2} $x=T/m_1$ 10^{-3}
10^{-2} 10^{-3}
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-5} 10^{-4} 10^{-3} 10^{-2}
$t_{universe}$ in secs

Fig. 3

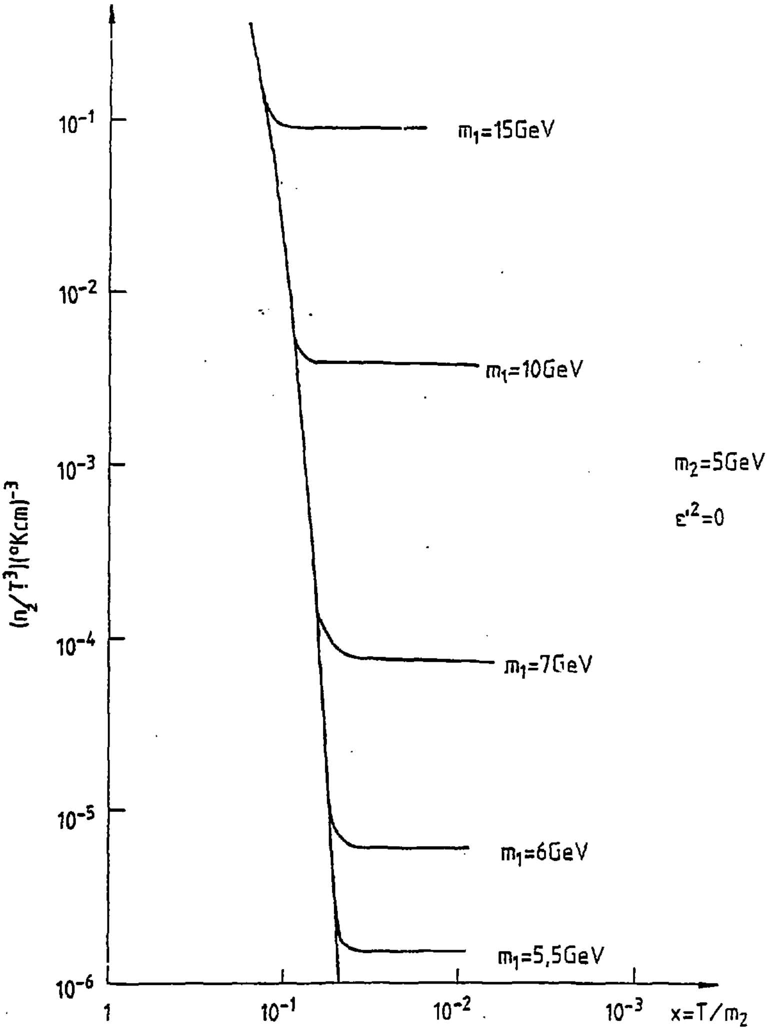

m_1=15GeV
m_1=10GeV
m_1=7GeV
m_1=6GeV
m_1=5,5GeV
m_2=5GeV
ε'^2=0
$(n_2/T^3)(({}^\circ K cm)^{-3}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
1
10^{-1}
10^{-2}
10^{-3}
$x=T/m_2$

Fig. 4

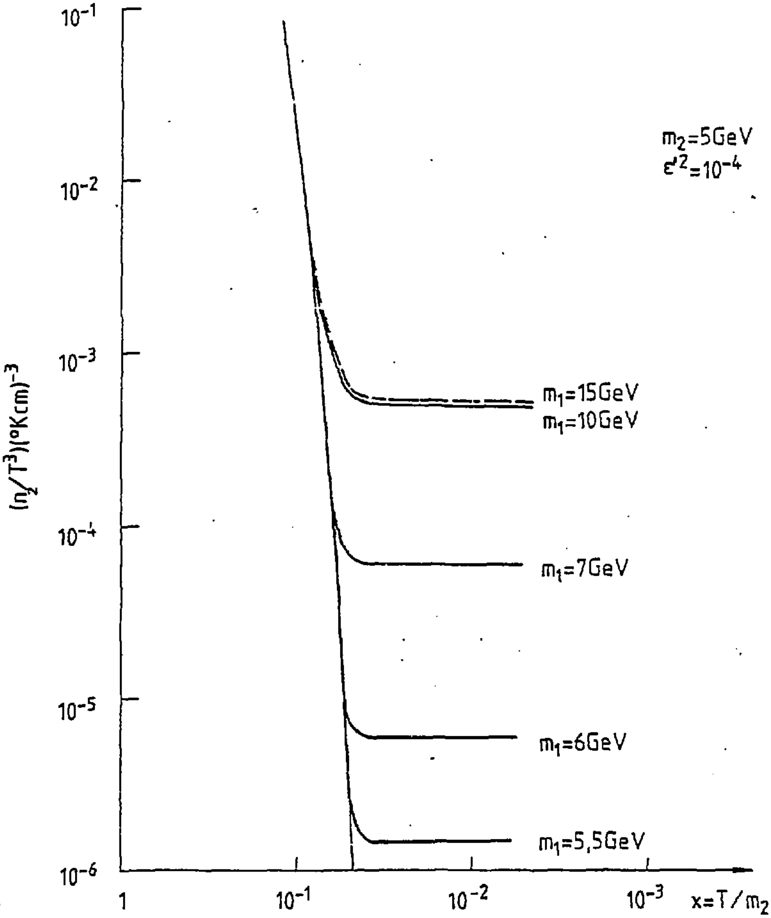

10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$(n_2/T^3)(°K cm)^{-3}$
$m_2=5GeV$
$\epsilon'^2=10^{-4}$
$m_1=15GeV$
$m_1=10GeV$
$m_1=7GeV$
$m_1=6GeV$
$m_1=5,5GeV$
1
10^{-1}
10^{-2}
10^{-3}
$x=T/m_2$

Fig. 5