FR8501136OC

5. Annual conference on modeling and simulation
Pittsburgh, PA (USA) 19-20 Apr 1984
CEA-CONF--7550

# USE OF HANKEL TRANSFORM FOR AXIALLY SYMMETRIC PROBLEMS APPLICATION OF FINITE HANKEL TRANSFORM TO THE SCHRODINGER EQUATION

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ABSTRACT

We present here a numerical solution of the Schrodinger equation with axial symmetry based on the Finite Hankel Transform, in view to solve light propagation problems.

Both forms of the Hankel Transform remove the laplacian term, but the Finite Hankel Transform is better suited to numerical calculation because it deals with a finite interval. Moreover, this transform is well connected to integral conservation laws. For large Fresnel numbers, the computer program simulating non linear light propagation has been found to perform better than a direct code.

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SUMMARY

In many physical problems that involve axial symmetry and the Laplacian operator (1), (2), (3), the Hankel transform is useful because the Laplacian term $\nabla^2_\rho$ is handled analytically simply in the transformed space.

This may lead to analytical solutions, and for computational treatment one can take advantage from the fact there is no more discretization of $\nabla^2_\rho$ and no more stability criterion in $\delta z/\delta^2_\rho$.

Moreover, the Hankel transform has the physical significance to be the Fourier transform relative to the space variables x,y. (The dual variable k is then $\sqrt{k_x^2 + k_y^2}$ ).

Usually, physical quantities vanish for $\rho > \rho$ max. However the faster the origin function decreases to zero at infinity, the more slowly the decrease of the transformed function. This causes great difficulties in numerical treatment of infinite Hankel transform when we need to go back to the physical space. Use of the so-called finite Hankel transform, which is a series expansion using the zeros of the Bessel function $J_0$, can solve this problem.

The integral $\int_0^\infty |f(\rho)|^2 \rho d\rho$ has a physical significance and is usually related to the system energy; it is well known that if $\overline{f}$ (k) is the Hankel transform, we have $\int_0^\infty |\overline{f}(k)| k dk = \int_0^\infty |f(\rho)|^2 \rho d\rho$ (energy - conservation law). Finite Hankel transforms are computed with a special quadrature formula using the zeros of $J_0$ for integrals of the type $\int_0^\infty g(\rho)\rho d\rho \sim \int_0^{\rho max} g(\rho)\rho d\rho$; together with energy conservation this formula provides a convenient check for the number of zeros to be taken.

A computer program has been written on a CRAY-1, that simulates the propagation of a resonant laser beam in a two-level atomic vapor, using dimensionless variables defined in (4) and 100 zeros of $J_0$. The algorithm is well suited to vectorial hardware and for large values of the coefficient of $\nabla^2_\rho$ the program is found to be better than a direct code that discretizes $\nabla^2_\rho$ with uniform grids. It is presently applied to the problem of superfluorescence and non-linear propagation.

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# UTILISATION DE LA TRANSFORMEE DE HANKEL POUR DES PROBLEMES A SYMETRIE AXIALE : APPLICATION DE LA TRANSFORMEE DE HANKEL FINIE A L'EQUATION DE SCHRODINGER

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RESUME

*Nous présentons ici une méthode de résolution numérique de l'équation de Schrödinger en symétrie axiale, basée sur la transformée de Hankel finie, aux fins d'application à des problèmes de propagation laser.*

*Les deux formes de la transformée de Hankel éliminent le laplacien, mais la transformée finie est mieux adaptée aux calculs numériques comme elle s'applique à un intervalle finie. De plus, cette transformation est élégamment reliée aux lois de conservation intégrales. Pour de grands nombres de Fresnel, un tel programme simulant la propagation non-linéaire de la lumière laser a de meilleurs performances qu'un code direct.*