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The Photon Structure Function - Theory

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ABSTRACT

The theoretical status of the photon structure function is reviewed. Particular attention is paid to the hadronic mixing problem and the ability of perturbative QCD to make definitive predictions for the photon structure function.

1. Introduction.

Deep inelastic scattering provides a unique probe of the pointlike structure of matter. The structure of the photon has special interest due to its two component nature where it can interact directly through its pointlike couplings or indirectly through its hadronic component. Initial interest[1] in the photon structure function was based on the parton model. The parton model predicts that the virtual photon can interact directly with the target photon through the exchange of charged pointlike partons. The parton model prediction for the photon structure function becomes

$$F_2^{\gamma}(x,Q^2) = \langle e^4 \rangle \times \{P(x) \times \log(Q^2/m^2) + B(x)\} \quad (1)$$

where we see the sensitivity to the fourth moment of the parton charge and the nonscaling Q^2 dependence. The parton x distribution, $P(x)$, reflects the direct coupling to the photon. The parton mass sets the scale of the logarithm and reflects the infrared sensitivity of the parton structure function. In the following we will study the photon structure function within the context of the theory of perturbative quantum chromodynamics. We are particularly concerned with the separation of the direct pointlike couplings of the photon from the effects of the quark and gluon hadronic constituents.

2. Leading order QCD.

The application of perturbative QCD to the photon structure function is similar to its application to hadronic processes. The reaction can be factorized into a hard scattering cross-section of the constituents times their target probability,

$$F_2^{\gamma}(x,Q^2) = \sum_C F_{2C}^{\gamma}(Q^2) * \mathbf{A}_C \qquad (2)$$

The infrared sensitivity is absorbed in constituent probabilities, $\mathbf{A}_C$. The constituent cross-sections, $\mathbf{F}_{2C}$, are directly computed in perturbative QCD. Witten[2] was the first to observe that the proper treatment of the photon structure function requires that the photon be considered as its own constituent. Witten used operator product expansion and renormalization group methods to compute the hard scattering cross-sections. In leading order, the diagrams for quark and gluon production as shown in Figure 1 are summed to all orders. Of course only the hard scattering parts of these diagrams are correctly predicted by perturbative QCD. I emphasize the association of all Q^2 dependence with the hard scattering cross-sections rather than the Q^2 evolution of the parton distributions. The leading order results were also obtained using a wide variety of methods[3].

Quantum chromodynamics makes a unique prediction for the asymptotic behavior of the photon structure function. Witten presented the results for the moments of the structure function which are summarized by

$$\begin{aligned} M_n(Q^2) &= \int dx\, x^{n-2} \times F_2^{\gamma}(x,Q^2) \\ &= a_n/\alpha_s(Q^2) + b_n \qquad \text{(photon)} \\ &\quad + \sum_i [\alpha_s(Q^2)]^{d_{n,i}} \times (1 + \ldots) \times \mathbf{A}_{n,i} \qquad \text{(hadrons)} \end{aligned} \qquad (3)$$

where $d_{n,i} = \gamma^0_{n,i}/\beta_0 \geq 0$ are the hadronic anomalous dimensions. The

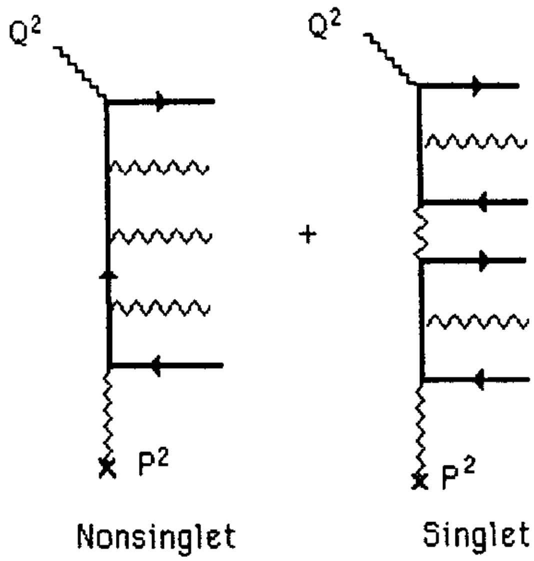

Figure 1. Leading order perturbative diagrams.

hadronic part has exactly the same structure as hadronic deep inelastic scattering. In leading order, asymptotic freedom predicts that the effective strong coupling should vanish for large Q^2 as $\alpha_s(Q^2) \to 4\pi/\beta_0 \times \log(Q^2/\Lambda^2) \to 0$. Hence the photon component of the structure function in Eq. 3 dominates asymptotically over the hadronic components and the moments have the behavior,

$$M_n(Q^2) \to a_n \times (\beta_0/4\pi) \times \log(Q^2/\Lambda^2). \quad (4)$$

The coefficients a_n are computed in perturbative QCD and yield the nonparton but stiff x distribution shown in Figure 2. The result of Eq. 4 predicts the ultimate asymptotic behavior of the moments as the anomalous dimensions given in Eq. 3 imply that all other corrections to the moments are logarithmically suppressed.

3. Higher order QCD.

The higher order corrections to the moments can also be computed in perturbative QCD. The next corrections to the photon component are O(1) in an α_s expansion and will continue to aymptotically dominate over the

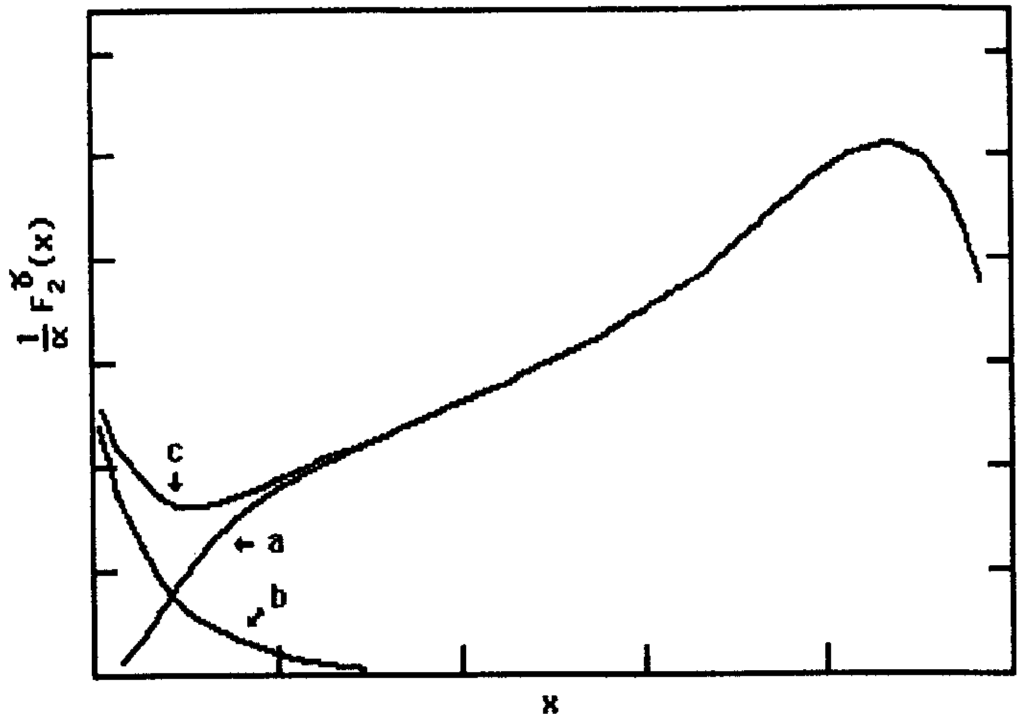

Figure 2. Leading order structure function. **a)** Valence component. **b)** Sea component. **c)** Total.

hadronic components due to the positivity of the hadronic anomalous dimensions with the exception of the second moment where the anomalous dimension can vanish. The coefficients, b_n, were computed[4] for $n > 2$ and combined with the higher order corrections to α_s to determine the photon component of the structure function through next leading order. These corrections are required for a significant determination of the QCD scale, Λ_{QCD}, from this process.

The moments can be directly compared to data or inverted to give directly the structure function. For moderate x, the higher order corrections do not dramatically alter the shape of the x distribution but do provide the overall scale of the structure function. The results are shown in Figure 3. For small x, the higher order prediction of the photon component breaks down as it appears to predict a negative cross-section. As emphasized by Duke and Owens[5], this effect is due to mixing with the hadronic component which can not be suppressed at small x. The effect is

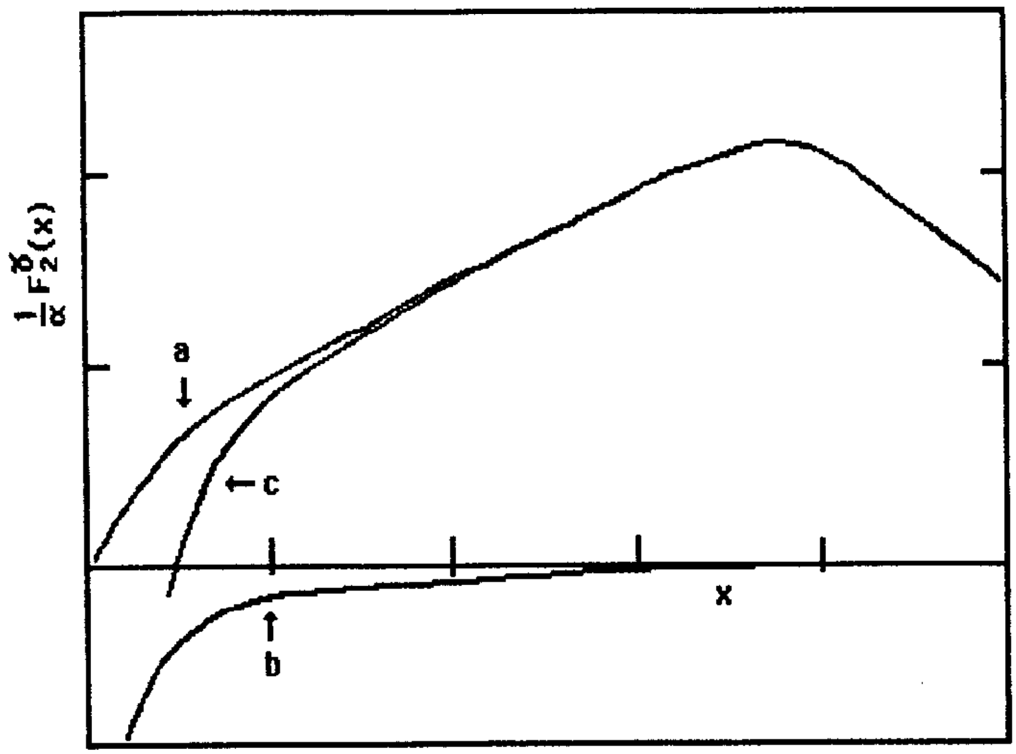

Figure 3. Higher order structure function. **a)** Valence component. **b)** Sea component. **c)** Total.

best seen by the separation of the theoretical prediction into the valence component, $\sim[\langle e^4\rangle-\langle e^2\rangle^2]$, and the sea component, $\sim\langle e^2\rangle^2$. The negative terms appear only in the sea component.

4. Mixing singularities.

The negative contribution to the sea component of the structure function arises from a pole in the b_n coefficient at $n = 2$, $b_n \to b/(n-2)$. When inverted this pole generates a singularity at $x = 0$,

$$F_2^{sea}(x,Q^2) \to -(1/x). \qquad (5)$$

This singularity arises from the mixing of the photon and hadronic components and can be seen from the evolution equations for the structure functions[4,6]. The evolution equations generate terms in the form

$$\left\{1-[\alpha_S(Q'^2)/\alpha_S(Q^2)]^{d_{n-}}\right\}/d_{n-} \tag{6}$$

where the singlet anomalous dimension can vanish, $d_{n-} \to (n-2) \to 0$. While the expression in Eq. 6 is nonsingular even at $n = 2$, the existence of such terms can produce the singular terms in the photon component. This behavior of the anomalous dimensions produces nonuniform evolution in Q^2 as $x \to 0$ since the "hadronic" component of Eq. 6 will dominate for $n < 2$ and the "photon" component will dominate for $n > 2$. Hence the singular terms found by Duke and Owens[5] in the sea component of the photon structure function is spurious and must be cancelled by similar singularities in the hadronic component,

$$A_{n-} \times [\alpha_S(Q^2)]^{d_{n-}} \to (1/(n-2)) \times [\alpha_S]^{d_{n-}}. \tag{7}$$

This cancellation was shown[7] to occur for the virtual photon structure function. In this case the target photon is taken to be highly virtual and the entire amplitude is calculable in perturbative QCD. The coefficient A_{n-} can be computed exactly and does contain the pole expected form Eq. 7. We conclude that the "hadronic" component may not necessarily be ignored even for real photons as the coefficients may be enhanced due to poles even though the terms are suppressed by powers of α_S for $n > 2$.

5. Regularization.

The singularities discussed in the previous section require that the simple separation of the photon and hadron components be modified. Much of the predictive power of perturbative QCD may be retained through the proper regularization of these singularities[8]. The basic point is that the singular terms produce a large effect in the sea distribution at small x. However, except for the singularity, the sea component is expected to be small. Therefore, any reasonable regularization will cancel the singularity and leave a remaining small sea component. Antoniadis and Grunberg[9] have made an explicit construction of the regularized structure function. The method first involves the explicit separation of the singular terms. Then they introduce a new parameter, λ, to tune the strength of the induced terms. The resulting structure function is nonsingular,

$$b_n = b_n^{reg} + b/(n-2), \qquad [\, t = \lambda \times \alpha_s(Q^2) \,],$$

$$M^{\gamma}{}_n^{reg} = a_n/\alpha_s(Q^2) + b_n^{reg} + [b/(n-2)] \times \{1-[t]^{d_{n-}}\}. \qquad (8)$$

For reasonable values of λ, the singular terms are reduced to a true higher order correction with sensitivity to λ only at small x. Their results are shown in Figure 4. The procedure used here is by no means unique but other methods will yield similar results.

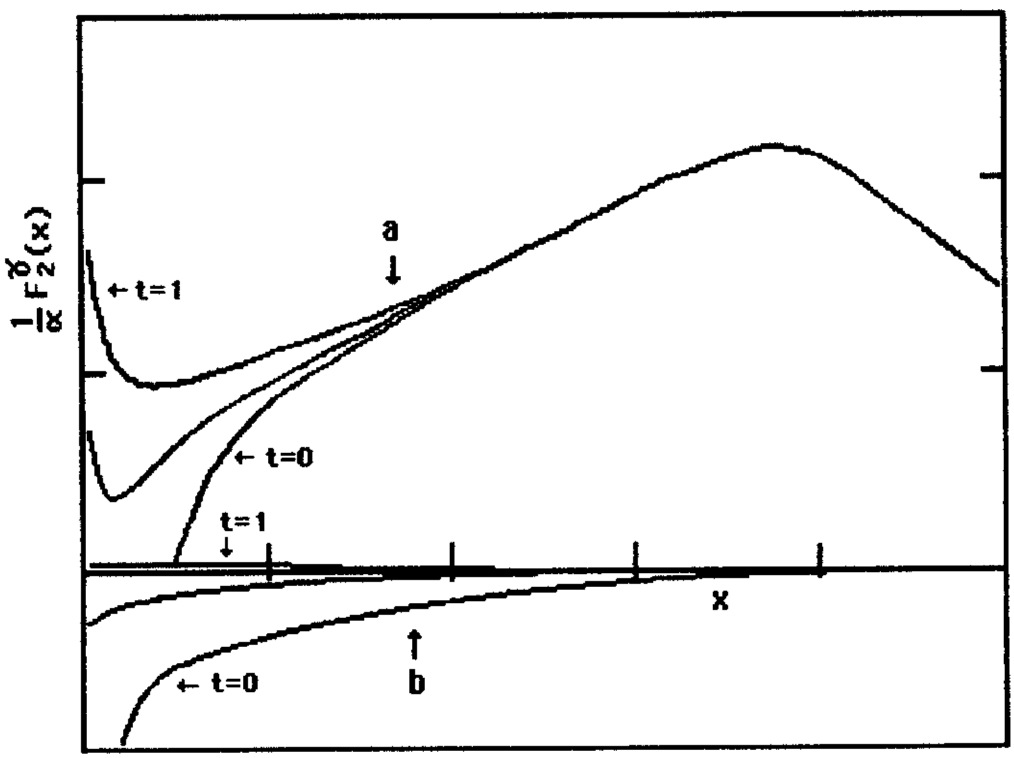

Figure 4. Regularized structure functions. **a**) Total contribution for various t values. **b**) Sea component.

6. Higher order singularities.

We have discussed the singularities and the regularization of the next leading contributions to the photon structure function. G. Rossi[10] has made a systematic study of singularities induced by higher order corrections. He finds that the mixings generated in higher orders produce poles which move to larger values of n. Poles at larger values of n correspondto more singular x distributions,

$$M_n \to 1/(n-n_0) \;\Rightarrow\; F_2(x) \to 1/x^{n_0-1}. \qquad (9)$$

These poles are a further reflection of the nonanalytic behavior at small x. The singularities must be cancelled by similar singularities in the "hadronic" terms. The poles can be discussed from the perspective of the previous section with suitable regularization methods. When the singularities are cancelled, the higher order corrections will be reduced to higher order except at very small x where perturbative QCD breaks down.

7. Evolution.

A different perspective[11] concerning the application of perturbative QCD to the photon structure function was presented to this conferenceby Drees. This work concludes that only evolution of the structure functions can be computed due to the mixing singularities in next and higher order. The dominant photon component can not be isolated from the hadron component. The treatment of the photon structure function is reduced to that of the hadronic structure functions where one can only predict evolution of the structure function from one value of Q^2 to higher values of Q^2. This is very difficult to exploit in the case of the photon structure function. It requires knowledge of three distribution functions q_{NS}, q_S, and G at one value of Q_0^2 or measurements at three values of Q^2 to determine the full Q^2 dependence (note there are no sum rules in this case). In this procedure we lose all sensitivity to Λ_{QCD}.

In practice one must make an ansatz, at one Q^2, to relate the singlet quark and the singlet gluon distributions to the nonsinglet quark distribution. Drees et al choose the following relations,

$$\Sigma^{\gamma}(x,Q_0^2) = \{\langle e^2\rangle/[\langle e^4\rangle-\langle e^2\rangle^2]\}\times q_{NS}(x,Q^2),$$

$$G^{\gamma}(x,Q_0^2) = \{2/\beta_0\}\times P^0{}_{gq} * \Sigma^{\gamma}. \tag{10}$$

In order to test the evolution predictions, they let $Q_0^2 = 1$ GeV^2 and evolve to fit the data at $Q^2 = 5$ GeV^2 which determines the valence quark distribution, $q_{NS}(x,Q_0^2)$. Their predictions for the photon structure function at higher Q^2 is shown in Fiqure 5. They compare the leading order evolution and higher order evolution and find little difference. They find no sensitivity to Λ_{QCD} as expected in this approach. With their parameterization, they find only a slow approach to the asymptotic structure function.

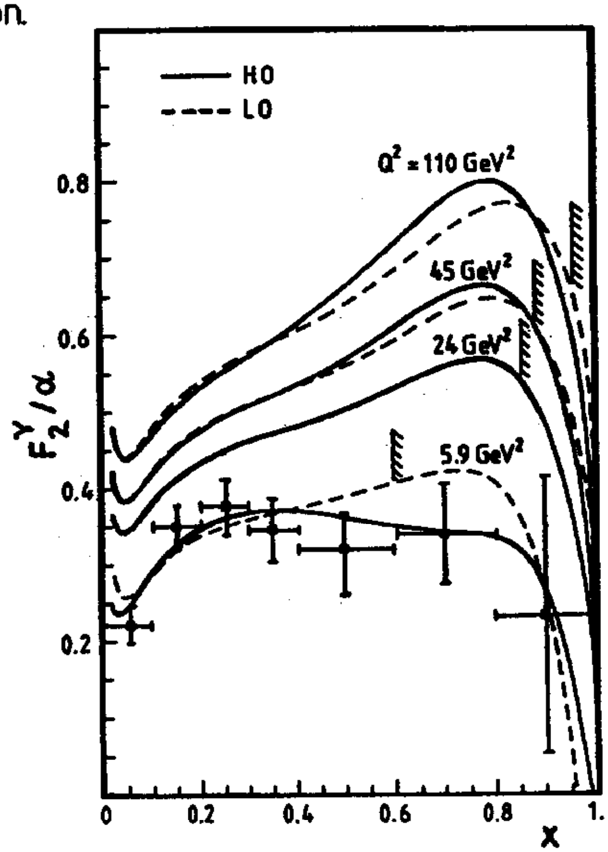

Figure 5. Comparison of leading and higher order evolution of the photon structure function.

We may try to analyze the results of Drees et al by comparing their results with the results of Antoniadis et al. We may compare the asymptotic forms for the fitted moments,

$$M_n^{\gamma} = a_n/\alpha_s(Q^2) + b_n + \Sigma_{NS,+,-} A^{\gamma}_{n,i} \times [\alpha_s(Q^2)]^{d_{n,i}} . \qquad (11)$$

The two approaches must agree on the values of a_n and b_n. The coefficient A_{n-} must have the same n=2 singularities. However the nonsingular parts of the $A_{n,i}$ can be expected to differ as well as the choice of Λ_{QCD}. We see that the two approaches differ only in those terms which are not calculable in perturbative QCD. Whether the photon component or the hadron component dominates depends on the fitting procedure.

Assuming that both approaches can fit the data, we can address the sensitivity to the determination of Λ_{QCD}. The nonsingular hadronic terms of Drees et al can imitate the dependence on Λ^2 only if they have the correct x distribution over the fitted range of Q^2. The Λ^2 values are related by

$$a_n \times (\beta_0/4\pi) \times \log(\Lambda^2_{AG})$$
$$= a_n \times (\beta_0/4\pi) \times \log(\Lambda^2_{DGGR}) - \Sigma\, \Delta A_{n,i} \times (\alpha_s)^{d_{n,i}} \qquad (12)$$

where only the nonsingular parts contribute to $\Delta A_{n,i}$. If the values of Λ^2 differ in the two fits, then the hadronic terms must have the pointlike structure of the x distribution dictated by the coefficient, a_n. However the true hadronic part of the structure function is expected to have an x distribution similar to typical hadronic structure functions and not the stiff x distribution of the pointlike contribution from a_n. I conclude that the evolution approach advocated by Drees et al is quite conservative. It chooses to iqnore our ability to directly compute the large photon component which may dominate the entire cross-section.

8. Conclusions.

In this talk I have only briefly discussed the fundamental QCD analysis of the photon structure function as it has been extensively presented in the literature. Instead I have focussed on the questions related to the hadronic mixing problem. My basic conclusion is that a large pointlike photon component can dominate the photon structure function with a calculable dependence on Λ^2_{QCD}. The mixing singularities discovered in the perturbative analysis must be properly treated. However the analysis of Antoniadis et al does provide a reasonable prescription for regularizing the perturbative singularities but requires the introduction of a new parameter, λ. The structure function is sensitive to the value of λ only at small x. Of course the size of the true hadronic component is not calculable in perturbative QCD. However since this hadronic component is not expected to have the pointlike x distribution, the data can be used to determine whether a large hadronic component is required. If the structure function is pointlike, then we can presume the photon component dominates and use the moderate x range to determine Λ^2_{QCD}.

In fits to the data, a vector meson dominance (VMD) contribution is usually included with the hadronic x distribution. Although only a small effect, this hadronic component should include the expected Q^2 dependence. The λ parameter of Antoniadis et al produces a VMD-like effect with the correct Q^2 dependence.

Strictly perturbative analysis can not be used further to resolve the structure of the hadronic components. Diagram calculations are sensitive to the wrong, perturbative infrared dependence which neglects all confinement effects. In the future we must look to nonperturbative estimates of the hadronic coefficients, $A_{n,i}$. Perhaps the QCD lattice industry can be induced to study the appropriate matrix elements and determine whether anomalously large hadronic components contribute to the photon structure function.

Acknowledgements

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COMMENTS

S. BRODSKY: As you have emphasized, the mass dependence in the QPM result for $F_2^{\gamma} \sim \log(Q^2/m^2)$ is replaced by the QCD scale Λ^2 in the all-order calculation. For heavy quarks $m^2 >> \Lambda^2$, the mass dependence can not be neglected. Thus shouldn't we include higher dimension operators in the analysis to recover the mass dependence, including the vacuum expectation values which give large consistent quark masses? Neglecting these contributions could affect the determination of $\Lambda_{\overline{MS}}$ from F_2^{γ}.

J.H. FIELD: For the charm quark contribution to F_2, the quark parton prediction is normally used as the mass scale is set not by Λ_{QCD} but by the charm quark mass. If however Λ_{QCD} is as small as 100 MeV and the lightest constituent quark mass is ~300 MeV, is to be expected that even for the light quarks, the quark mass will set the energy scale, not Λ_{QCD}. In fact existing data are fitted equally well by the QPM with conventional constituent quark masses or by asymptotic QCD predictions with Λ as an adjustable parameter.

K. GRASSIE: How do you obtain the typical behavior of structure functions as predicted by QCD, namely an increase of F_2 at large x and decrease at small x if you change Λ^2 or Q^2 appropriately? This behavior is predicted by the A_i terms which have been neglected in the semi-asymptotic solution of Antoniadis and Grunberg.

WINSTON KO: If we measure the longitudinal structure function, F_L, which is expected to not have a Q^2 dependence, would it be useful to determine the Q^2 - independent term of F_2?