

## A DIFFERENT INTERPRETATION OF THE NUCLEAR SHELL MODEL

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**ABSTRACT** In the first order approximation the nucleons are moving into a collective well extracted from the two-body N-N interaction. The nuclear shell model is explained by the structure of the first order solution of the Schrödinger equation. In the next step the two-body correlations generated by the N-N potential are introduced in the wave function.

The standard nuclear shell model emerged in the fifties from the study of the properties of the ground state of nuclei such as the discontinuities in binding energies, the pairing energy between two nucleons, the spin of odd nuclei ..., leading to the concept of "Magic Numbers".

It appeared soon that the ground and low lying excited states can be constructed, in the first approximation, from the Harmonic Oscillator (H.O.) states when a suitable spin-orbit interaction is introduced. As the harmonic oscillator is an independent particle model it has been concluded that each nucleon is moving into a common average field generated by the interaction of one particle with the others <sup>(1)</sup>. It is an independent particle model (IPM). But the H.O. is a confining potential preventing any nucleon to escape from the nucleus. Therefore another potential, like the Wood-Saxon well, vanishing at infinity has been substituted for the H.O. potential in order to allow nucleons to be separated from nuclei and generate a better charge density. Ultimately the

Hartree-Fock method has been used for generating an average field from a potential (e.g. Skyrme), adapted to the method, yielding good binding energies and densities for nuclei (2).

Our approach starts also from the H.O. Model but leads to another conclusion.

Let  $\vec{x}_i$  ( $i=1, \dots, A$ ) be the nucleon coordinates and  $\vec{X}$  the center of mass, the H.O. potential is proportional to

$$\sum_1^A x_1^2 = \sum_1^A (\vec{x}_i - \vec{X})^2 + AX^2 \quad (1)$$

Introducing the collective coordinate

$$r^2 = 2 \sum_1^A (\vec{x}_i - \vec{X})^2 = \frac{2}{A} \sum_{i,j>i} r_{ij}^2, \quad \vec{r}_{ij} = \vec{x}_i - \vec{x}_j \quad (2)$$

one deduces that the H.O. interaction is either a sum of independent particle interactions (the orthodoxe-interpretation) or a collective interaction of the collective translationally invariante coordinate  $r$ .

Here we investigate the consequence of this last interpretation. Let us consider the motions in which  $r$  is preserved, it can be the rotation of one particle around the center of mass  $\vec{X}$ , or the rotation of two particles around their own c.m. combined with the rotation of this two-body c.m., more over it can be the one of three particles around their c.m. and so one.

Assume that the part  $V_0(r)$  of  $V(r_{ij})$ , the N-N potential, invariant for such motions is predominante, then it can be written as :

$$V(r_{ij}) = V_0(r) + [V(r_{ij}) - V_0(r)] \quad (3)$$

where the last term, the residual interaction which contains the two-body correlations, is small. It vanishes identically for a H.O. according to (2). Using a decomposition of the wave function by pairs we write

$$\Psi(\vec{x}) = \sum_{i,j>i} \psi_{ij}(\vec{x}) \quad (4)$$

where  $\vec{x}$  stands for the set of particle coordinates  $(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_A)$ . We require that each partial wave is a solution of

$$\left\{ T + \frac{A(A-1)}{2} V_0(r) - E \right\} \psi_{ij} = -(V(r_{ij}) - V_0(r)) \sum_{k,l>k} \psi_{kl}(\vec{x}) \quad (5)$$

where  $T$  is the kinetic energy operator.

This equation is equivalent to the Schrödinger equation

$$\left( T + \sum_{i,j>i} V(r_{ij}) - E \right) \Psi(\vec{x}) = 0 \quad (6)$$

In the first order approximation, where one neglects the residual interaction, the solution is the product

$$\psi_{ij}(\vec{x}) = H_{[L]}(\vec{x}) r^{-(L+1)} u_n(r) \quad (7)$$

of a (homogeneous) harmonic polynomial  $H_{[L]}(\vec{x})$  and a function of the collective coordinate <sup>(8)</sup>. The polynomial is characterized by a set of (quantum) numbers  $[L]$  and  $n$  is the number of nodes of  $u_n(r)$ , the eigenfunction of the "hyper" radial equation

$$\frac{\hbar^2}{m} \left( \frac{d^2}{dr^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{r^2} \right) + \frac{A(A-1)}{2} V_0(r) - E_n \} u_n(r) = 0$$

$$\mathcal{L} = L + \frac{3A}{2} - 3 \quad (8)$$

where  $L$  is the degree of the harmonic polynomial. In this solution the state is defined by the harmonic polynomial  $H_{[L]}(\vec{x})$  which must exhibit the symmetry required by the particle statistic (bosons or fermions) in any exchange of two particles. It is independent on the collective potential  $V_0(r)$ , which determines only the shape of  $u_n(r)$ . The ground state is reached when  $\mathcal{L}$ , i.e. the degree  $L$  of the polynomial is the smallest (then the repulsive centrifugal barrier is the weakest) and when  $u(r)$  has not node ( $n=0$ ).

For nucleons  $H_{[L]}(x)$  must be an antisymmetric polynomial. In order to construct harmonic polynomials of low degree corresponding either to ground or low lying excited states we introduce the homogeneous polynomials of degree  $2n+l$

$$\varphi_{n\ell jm}(\vec{x}_i) = x_i^{2n+l} \psi_{\ell jm}(\omega_i), \quad \vec{x}_i(x_i, \omega_i), \quad (9)$$

where  $\psi_{\ell jm}(\omega_i)$  stands, as usual, for the spherical harmonics  $Y_{\ell}^m(\omega_i)$  coupled to the nucleon spin to produce an individual total angular momentum  $j$  with projection  $m$ .

Then we construct the Slater determinant

$$H_{[L]}(\vec{x}) = |\dots \varphi_{0\ell jm}(\vec{x}_i) \varphi_{1\ell jm}(\vec{x}_i) \dots \varphi_{n\ell jm}(\vec{x}_i) \dots| \quad (10)$$

where for any fixed  $l, j, m$  the number  $n$  is used without any hole from 0 to the maximal value  $n_{l, j, m}$  chosen independently for each set  $l, j, m$ . A homogeneous polynomial constructed according to this rule is a harmonic polynomial (4). When it is multiplied by  $\prod_{i=1}^A e^{-\frac{X_i^2}{b}} = e^{-\frac{r^2}{2b^2}} e^{-\frac{Ax^2}{b^2}}$ , one obtains a H.O. Slater determinant constructed with the individual H.O. eigenstates  $\psi_{n, l, j, m}^{H.O.}(x_i)$ . The harmonic polynomials of low degree are constructed with the  $n, l, j, m$  quantum numbers in following the rule used for filling the ground or low lying states H.O. Slater determinants.

The discontinuities in separation energy of the last particle after a closed shell is generated by a sudden increase in kinetic energy. When one more nucleon is introduced in the lowest unfilled shell of a nucleus the increase of  $\varphi$  is  $l_m + \frac{3}{2}$  (see eq.(6)) where  $l_m$  is the highest orbital allowed in the shell (e.g.  $l_m = 0$  in 1s,  $l_m = 1$  in 1p,  $l_m = 2$  in 2s, 1d shells)(4).

The increase in the centrifugal barrier is then

$$[(2l_m+3)l + (l_m+\frac{3}{2})(l_m+\frac{5}{2})] \hbar^2 / m r^2$$

where  $l_m$  conserves the same value in a shell. But  $l_m$  becomes  $l_m + 1$  for the next nucleon included after the closure of a shell and the kinetic energy undergoes a sudden increase which is not balanced by a similar decrease in potential energy then generating a drop in the separation energy of the last particle, hence explaining the discontinuities after  ${}^4\text{He}$ ,  ${}^{16}\text{O}$  and  ${}^{40}\text{Ca}$ . This mechanism is mixed up beyond the (2s, 1d) shell by the spin-orbit effect of the nuclear force. The spin orbit operator  $l_{ij} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)$  does not change the degree of the polynomial  $H_{[L]}(x)$  but selects which one of the harmonic polynomials brings the largest binding and must be chosen to describe the ground state.

It favors the high spins and is mainly responsible for the feature of the sequence of the nuclear ground states after the (2s, 1d)

shells . At least two collective motions are explained by our model : i) the breathing mode which is exactly described by the excited states of the radial motion (see eq.(8)) (6), (7) ; ii) the giant dipole resonance which originates from the translational invariance of the harmonic polynomial describing the ground state. Indeed the degree of the antisymmetric polynomial  $H_{[L_m]}(x)$  describing the ground state is minimal, therefore any symmetric differential operator decreasing the degree of a homogeneous polynomial gives zero on  $H_{[L_m]}(x)$ . In particular

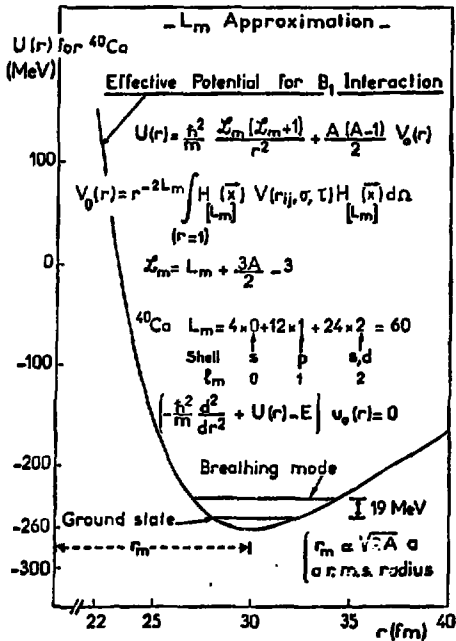
$$\sum_{i=1}^A (i \pm \tau_z(i)) \vec{\nabla}_i H_{[L_m]}(\vec{x}) = 0$$

shows the translational invariance of  $H_{[L_m]}(x)$  for any motion of either the neutrons or the protons center of mass. The relative motion of the neutrons versus protons center of masses can be factorized in agreement with the Goldhaber and Teller picture of the giant dipole resonance (8).

The potential between the two center of mass can be calculated from the charge form factors and leads to dipole excitation energies in agreement with experimental data (9). The collective potential in eq.(8)

$$V_0(r) = r^{-2L} \int_{(r=1)} H_{[L]}^*(\vec{x}) V(r_{ij}, \sigma, \tau) H_{[L]}(\vec{x}) d\Omega$$

is the average taken over the unit hypersphere ( $r = 1$ ) of the N-N potential. It takes into account the effect of the exchange operators on the harmonic polynomial describing the state. To illustrate the first order approximation we have drawn in fig.1 the effective potential  $U(r)$ , including the kinetic centrifugal barrier, obtained with the Brink-Boeker B1 potential<sup>(10)</sup> for <sup>40</sup>Ca.



It looks like a molecular potential: the ground state wave function is concentrated near the bottom of the well, the next state is the breathing mode standing 19 MeV above, in approximate agreement with the experimental energy  $80 A^{-1/3}$  MeV. The position  $r_m$  of the minimum of  $U(r)$  is related to the r.m.s. radius  $a$  by

$$r_m = \langle r^2 \rangle^{1/2} = \sqrt{ZA} \langle (\vec{x}_i - \vec{x})^2 \rangle^{1/2} = \sqrt{ZA} a \quad (11)$$

Then a look at  $U(r)$  provides an easy estimate of both the total binding energy and the size of the nucleus.

For introducing the two-body correlations we assume that the residual interaction in (5) is small. We substitute the first order solution  $H_{[L]}(\vec{x}) \phi(r)$  for  $\psi_{k1}$  in (5) generating the right member  $A(A-1)/2 (V(r_{ij}) - V_0(r)) H_{[L]}(\vec{x}) \phi(r)$ . The partial wave  $\psi_{ij}$  becomes the product (11)

$$\psi_{ij}(\vec{x}) = H_{[L]}(\vec{x}) F(r_{ij}, r) \quad (12)$$

of a harmonic polynomial characterizing the state and a function  $F(r_{ij}, r)$  describing the correlations generated by the two-body potential  $V(r_{ij})$ . This function is a solution of

$$\begin{aligned} (T + \frac{A(A-1)}{2} V_0(r) - E) H_{[L]}(\vec{x}) F(r_{ij}, r) \\ = -(V(r_{ij}) - V_0(r)) H_{[L]}(\vec{x}) \sum_{k, \ell > k} F(r_{k\ell}, r) \end{aligned} \tag{13}$$

For solving this equation we introduce, beside the collective coordinate  $r$ , a "correlation" coordinate  $z$  related to the relative coordinate  $r_{ij}$  by

$$r_{ij} = r \cos \phi, \quad z = \cos 2\phi = 2r_{ij}^2/r^2 - 1 \tag{14}$$

Then we define the set of polynomials  $P_K^{[L]}(z)$  such that the product

$$H_{[L]}(x) r^{2K} P_K^{[L]}(z) = r^{L+2K} P_{L+2K}^{[L]}(\Omega_{ij}) \tag{15}$$

be a harmonic polynomial of degree  $L + 2K$  <sup>(12)</sup>. It must fulfill the orthonormal condition

$$r^{-2L} \int_{(r=1)} H_{[L]}^*(\vec{x}) P_K^{[L]}(z) H_{[L]}(\vec{x}) P_{K'}^{[L]}(z) d\Omega = \delta_{KK'} \tag{16}$$



where the integral is taken over the surface of the hypersphere ( $r = 1$ ). The surface element over the hypersphere ( $r = 1$ )  $d\Omega$  is separated in two terms :

$$d\Omega = d\Omega_1 \cdot 2^{-D/2} (1-z)^{\frac{3A-4}{2}} (1+z)^{1/2} dz$$

where  $\Omega_1$  stands for all the surface coordinates but  $z$ . Integrating (16) over all variables  $\Omega_1$  leads to the orthonormal conditions

$$\int_{-1}^1 W_{[L]}(z) P_K^{[L]}(z) P_{K'}^{[L]}(z) dz = \delta_{KK'} \quad (17)$$

where  $P_K^{[L]}(z)$  are polynomials associated with the weight function

$$W_{[L]}(z) = 2^{-D/2} (1-z)^{\frac{3A-4}{2}} (1+z)^{1/2} r^{-2L} \int_{(r=1)} |H_{[L]}(\vec{x})|^2 d\Omega_1$$

The weight function is the product (12)

$$W_{[L]}(z) = (1-z)^{L-2\ell_m-1} (1+z)^{1/2} \rho_{[L]}(z) \quad (18)$$

where  $\ell_m$  is the largest orbital in the last shell and  $\rho_{[L]}(z)$  is a polynomial of degree  $2\ell_m$  with zeroes out of the range  $-1 \leq z \leq 1$ . The polynomials associated with such a weight function are obtained from the Christoffel's formula (1). We reduce eq.(13) to a two variables integro-differential equation by operating a projection of this equation on the space spanned by  $r_{ij}$ . For this purpose we introduce the projection function

$$f_{[L]}(z, z') = W_{[L]}(z') \sum_{K=0}^{\infty} \left[ \sum_{k,l \neq (i,j) <}^{[L]} \left| \begin{matrix} [L] \\ L+2K(\Omega_{ij}) \end{matrix} \right| \begin{matrix} [L] \\ L+2K(\Omega_{kl}) \end{matrix} \right] \\ \times P_K^{[L]}(z) P_K^{[L]}(z')$$

(19) .

where the sum over the pairs occurs for  $(k,l) \neq (i,j)$ . The brackets stand for the overlapping of polynomials related to different pairs. The sum over  $k,l$  decreases as  $(A-2)/2^{2K-1}$  ( $K \geq 2$ ) for increasing  $K$  <sup>(13)</sup> so that only a few terms (e.g.  $K < 7$ ) are sufficient to obtain a very accurate projection function even for large  $A$  (e.g.  $A < 10^3$ ). The two first terms are  $A(A-1)/2-1$  for  $K = 0$  and  $-1$  for  $K = 1$ . We write  $F(r_{ij}, r)$  as a function of the two independent variables  $z$  and  $r$  :

$$F(r_{ij}, r) = P(z, r) \quad (20)$$

The wave function for fermions is the antisymmetric combination

$$\Psi_{[L]}(\vec{x}) = H_{[L]}(\vec{x}) r^{-(L+1)} \sum_{i,j > i} P(2r_{ij}^2/r^2-1, r) \quad (21)$$

where  $P(z, r)$  is a solution of the integro differential equation <sup>(14)</sup>

$$\begin{aligned}
& \left\{ \frac{\hbar^2}{m} \left[ -\frac{\partial^2}{\partial r^2} + \frac{\ell(\ell+1)}{r^2} - \frac{4}{r^2} \frac{1}{W_{[L]}(z)} \frac{\partial}{\partial z} (1-z^2) W_{[L]}(z) \frac{\partial}{\partial z} \right] \right. \\
& \left. + \frac{A(A-1)}{2} V_0(r) - E \right\} P(z,r) \\
& = -[V(r\sqrt{\frac{1+z}{2}}) - V_0(r)] \{ P(z,r) + \int_{-1}^1 P_{[L]}(z,z') P(z',r) dz' \}
\end{aligned} \tag{22}$$

This equation has been solved by using the polynomial expansion

$$P(z,r) = \sum_{K=0}^{\infty} P_K^{[L]}(z) u_K(r) \tag{23}$$

and by projecting (22) on the  $P_K^{[L]}(z)$  basis. The partial waves  $u_K(r)$  are a solution of a system of coupled second order differential equations in  $r$ . When the two-body potential  $V(r_{ij})$  contains exchange operators, the weight functions generated by these operators must be taken into account. Essentially the  $P_{[L]}(z)$  polynomial is modified and eq.(22) must be changed accordingly.

For bosons in the ground state the symmetrical harmonic polynomial  $H_{[L_m]}(\vec{x})$  describing the state is constructed with all particles in the  $1s$  state, therefore it is a constant ( $L_m = \ell_m = 0$ ) and the polynomials  $P_K^{[0]}(z)$  associated with the weight function (18) are the suitably normalized Jacobi polynomials  $P_K^{3A/2-4, \frac{3}{2}}(z)$ . The ground state of the trinucleon system and  $^4\text{He}$  are constructed as for a bosons system when the potential is purely central and the mixed symmetry state generated by the difference between triplet and singlet even potentials is disregarded.

In Table 1 is shown a comparison between the binding energies obtained by integrating (22) for various nuclear potentials (15-18) and by the best variational calculations taking the two-body correlations into account with Jastrow cores (19) eventually combined with a Montecarlo method (20) and with Yakubovsky equations (37).

The potentials contain a repulsive core which can be strong as in the S3 Afnan-Tang (20) and S4 Eikemeier-Hachenbroich (21) potentials for which the ratio between the strenght of repulsion and attraction is ten. The soft core Volkov (22) potential is also used for comparison .

Table 1 Ground state binding energies in MeV.

Potential	V	S4	S3
$^3\text{He}$ (14)(15)	8.47	7.03	6.66
Var. (9)	8.45	7.03	6.61
MonteCarlo(20)		7.00	6.56
Yak.(37)	8.44		6.40
$^4\text{He}$ (17)	30.4	27.9	26.0
Variational(9)	30.3	28.2	26.5
MonteCarlo (20)	30±.1	27.7±.1	26.4±.1
Yak.(37)	30.2		25.5

From the values displayed in Table 1 it turns out that our results are as good as those obtained with the best variational calculations and therefore that the wave function (21) contains a good description of the two-body correlations.

Calculations with realistic interactions for the same nuclei are also in agreement with other calculations performed by solving Fadeev or Yakubovsky equations (23-25).

Eq.(22) is general and can be applied to any nucleus as soon as the "state"  $H_{IL}(x)$  is defined. In order to estimate the contribution of the two-body correlations in the ground states of  $^{16}O$  and  $^{40}Ca$  we solved (22) with the Brink-Boeker B1 potential (10) and compared our results (27) with those obtained applying the Hartree-Fock method (28).

	$E_0$	$E_{HF}^0$	$E_{HF}^c$	$\Delta E_{HF}$	$\Delta E_{HF}$	$E_{HF}$	$E_{exp.}$	$E_{HH}$
$^{16}O$	106.5	113	152.1	6.5	45.6	99.6	127.7	138.3
$^{40}Ca$	323.4	337	468.1	13.6	144.7	264.4	342	388.1

In Table 2,  $E_0$ ,  $E_{HF}^0$  and  $E_{HF}^c$  are respectively the first order, Hartree-Fock and Harmonic polynomial binding energies without Coulomb in Mev.  $\Delta E_{HF}$  and  $\Delta E_{HH}$  are the corresponding increase in binding energy with respect to the first order.  $E_{HF}$  and  $E_{HH}$  contain the effect of the coulomb potential and are compared with  $E_{exp.}$  the experimental binding energy.

The potential B1 which already overbind  $^4He$  (29) has the same effect on  $^{16}O$  and  $^{40}Ca$ . The increase of binding energy originating from the correlations in  $^{16}O$  and  $^{40}Ca$  are respectively 7 and 10 times larger than the one obtained applying the Hartree-Fock method. The overlap  $\langle \psi_{HH} | \psi_{HF} \rangle$  between H.F. and H.H. solutions are respectively .9 and .85 for  $^{16}O$  and  $^{40}Ca$ . The H.F. solution cannot be a reliable solution if one believes that the Rayleigh-Ritz variational principle provides a test for the quality of a ground state wave function

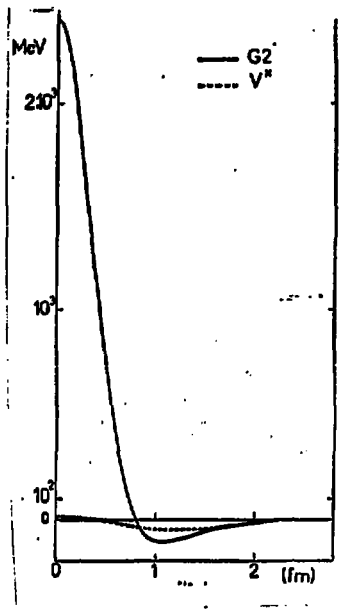
We assumed in our derivation of eq.(13) that the residual interaction is small. If it is true the first term ( $K=0$ ), independent of  $z$  the correlation variable, must be predominant in the expansion of  $P(z,r)$  on the  $P_{K=0}^{(L)}(z)$  basis. We found that for the analyzed V, S3 and S4 potentials, the norm of the partial wave  $u_0(r)$  in (23) are respectively .99, .98 and .99 for  $^3H$  and .99, .96 and .98 for  $^4He$  ground states. When  $V_0(r)$  is dominant it can be used to calculate the excited state energies. We did it for  $^6Li$  in the first order approximation (30).

The excited states are described in the L.S. scheme by the coupling of the two 1p shell nucleons. We choose for simplicity i) two phenomenological potentials S 1<sup>(20)</sup> and S 4<sup>(21)</sup> adjusted without tensor force to the <sup>1</sup>S<sub>0</sub> and <sup>3</sup>S<sub>1</sub> phase shifts and ii) the central part of the realistic GPDT<sup>(31)</sup> potential. The excitation energies obtained for the various states are displayed in Table 3 in MeV. The 1<sup>+</sup>, 2<sup>+</sup> and 3<sup>+</sup> states, degenerated without spin orbit force, are compared with the centroid of the experimental energies<sup>(32)</sup>.

J <sup>π</sup>	T	ℓ	S4	S1	GPDT	Exp.
(g.s.)1 <sup>+</sup>	0	0	0	0	0	0
0 <sup>+</sup>	1	0	5.64	4.69	3.49	3.56
3 <sup>+</sup> 2 <sup>+</sup> 1 <sup>+</sup>	0	2	2.90	3.18	4.19	3.60
2 <sup>+</sup>	1	2	6.37	5.32	5.06	5.36
r.m.s. radius			2.13	2.12	2.55	2.33±.1 fm

T and ℓ refer to the total isospin and orbital momentum. The r.m.s. radius of the ground state is given in the last row.

We cannot expect a better agreement with the experimental data from a truncated interaction without tensor and spin-orbit force. Now we come to the most important question : does our wave function well describe the two-body correlations. To answer we solved the



Tritium with two potentials giving both the experimental binding energy and r.m.s. radius. One potential ( $V_x$ ) has a very weak repulsive core and the other (G2) a very strong one (17) (see fig.2). The one body density (continuous line) and two-body correlation function (dashed line) are shown on fig.3a and 3b respectively for the  $V_x$  and G2 potentials. Whatever is the strength of the repulsion the one body density is flat near the origin but the two-body correlation function has a hole in the center generated by the potential core. The hole is very deep, and goes to zero at the origin for the G2 potential, as expected for a very strong repul-

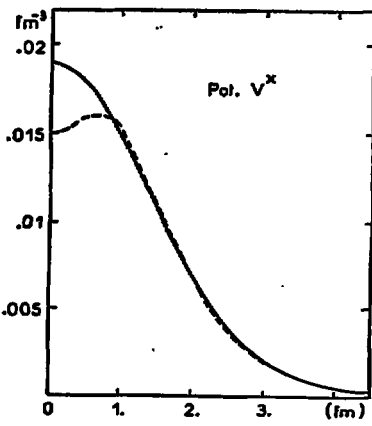


Fig. 3a

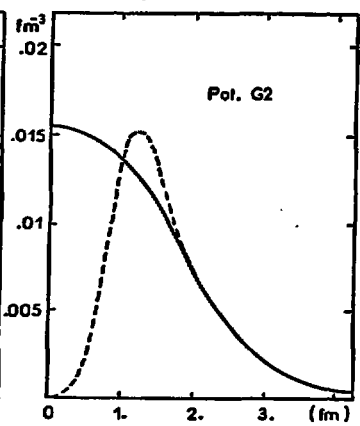


Fig. 3b

sive core potential. There is a last question we would like to comment about : Does the energy spectrum determine the interaction ? Assume that for a nucleus an overall good agreement between theoretical and experimental energy spectrum of excited states is reached with an independent particle model (particles moving in a well). Does it prove that the associated wave functions are good ? Take a three-particle system with a two-body interaction  $V(r_{ij})$ . We eliminate the c.m. by using the Jacobi coordinates  $\vec{x}_2 - \vec{x}_1 = \vec{\xi}_1$  and  $\sqrt{3}(\vec{x}_3 - \vec{X}) = \vec{\xi}_2$ , where  $X$  is the center of mass. The wave function  $\psi(\vec{\xi}_1, \vec{\xi}_2)$  is a solution of the Schrödinger equation

$$\left\{ -\frac{\hbar^2}{m} (\nabla_{\xi_1}^2 + \nabla_{\xi_2}^2) + \sum_{i,j>i} V(r_{ij}) - E \right\} \psi(\vec{\xi}_1, \vec{\xi}_2) = 0$$

Let us now exchange  $\vec{\xi}_1$  and  $\vec{\xi}_2$ , then the sets  $(\vec{x}_i - \vec{x}_j)$  and  $\sqrt{3}(\vec{x}_k - \vec{X})$  are also exchanged (33) in such a way that  $\psi(\vec{\xi}_2, \vec{\xi}_1)$  becomes a solution of

$$\left\{ -\frac{\hbar^2}{m} (\nabla_{\xi_1}^2 + \nabla_{\xi_2}^2) + \sum_{i,j} V(\sqrt{3}(\vec{x}_k - \vec{X})) - E \right\} \psi(\vec{\xi}_2, \vec{\xi}_1) = 0$$

Then  $\psi(\vec{\xi}_1, \vec{\xi}_2)$  and  $\psi(\vec{\xi}_2, \vec{\xi}_1)$  have the same energy spectrum but one-body density and two-body correlation functions are exchanged (i.e. the densities in fig.3 are exchanged). An agreement with the energy spectrum is not sufficient to accredit the wave functions generated from a well for particles interacting by pairs.

From our analysis emerge a new picture of the nucleus in which the wave function is the product of an antisymmetrized harmonic polynomial defining the state, which exhibit an independent particle behavior, and a symmetrised function describing the two-body correlations generated by the interaction. Therefore, most of the particle-hole configurations are not "states" in our scheme, except when they are constructed according to the rule leading to a harmonic polynomial (see eq.(10)). Our wave function is cleared from



spurious center of mass motions when the state is described by a harmonic polynomial of minimal degree. It can be eliminated for low degree polynomial <sup>(34)</sup>. Besides the monopole and dipole giant resonances other collective states can be described by our model. For instance, the interacting boson approximation (IBA) belongs to our scheme because the low-lying IB excited states are constructed by coupling the nucleons in the last unfilled shell. The total kinetic energy is fixed by the degree of the harmonic polynomial independent of the coupling between the nucleons and by the size of the nucleus.

The excitation energy proceeds from the effect of the coupling of the nucleons on the N-N interaction. It must be only slightly sensitive to the two-body correlations in order to preserve the agreements already reached with experimental data.

The utilisation of the harmonic polynomial method requires the knowledge of the weight function (18) which is obtained from the Fourier transform of the two-body correlation function calculated with the first order H.O. wave function deduced from (10). The Fourier transform can be calculated by using the Talmi coefficients and the Talmi integrals <sup>(35)</sup>.

Our conclusion is still the one of our first paper using the harmonic polynomial method for solving the few-body problem <sup>(36)</sup> : "In this formalism extended to the many body problem, which gives a dominant central potential, lies probably the justification of the success of the shell model".

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