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M.B.Voloshin

QUANTUM DECAY RATE OF METASTABLE PHASE IN (1+1) DIMENSIONS

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By means of the effective action method the rate of quantum decay of false vacuum in (1+1) dimensions is expressed in a form of a universal closed formula, whose validity does not rely on semiclassical expansion.

It is well known [1] that the probability W of quantum tunneling from a metastable phase ϕ_+ to a lower phase ϕ_- of a real scalar field ϕ with the Lagrangian*)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\phi) \tag{1}$$

where the potential $V(\phi)$ has local minima at ϕ_+ and ϕ_- (Fig. 1), is given by the path integral in the Euclidean version of the theory

$$W = 2\, Im\ ln \left\{ \int [\mathcal{D}\phi] \exp(-S) \right\} \tag{2}$$

with S being the Euclidean action corresponding to the Lagrangian (1). The path integral runs over all configurations of the field ϕ in the Euclidean space-time, which approach ϕ_+ at infinity. The configuration which contributes to W and on which S is stationary is the so-called bounce [1], i.e. a round-shaped bubble of the lower phase ϕ_- surrounded by

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*) We use the system of units $\hbar = v = 1$, where v is the characteristic velocity in the problem, i.e. the speed of propagation of infitesimally short-wave exitations of the field ϕ.

the phase ϕ_+ . In the case when the minimal of the masses of exitations in the phases $\phi_\pm$ satisfies the condition $mR \gg 1$ where R is the radius of the bubble, the critical value of R and the value of S at the stationary point are expressed only in terms of μ and ε where μ is the surface tension of the boundary between the phases and ε is the difference of energy density in the phases, $\varepsilon = \varepsilon(\phi_+) - \varepsilon(\phi_-)$. In the (1+1)-dimensional theory which is considered here $R = \mu/\varepsilon$ and $S_0 = \frac{\pi\mu^2}{\varepsilon}$ (in this (1+1)dimensional case μ is sometimes called the soliton mass).

The same condition $mR \gg 1$ ensures that the spectrum of fluctuations of ϕ around the bounce configuration consists of two distinctively different parts: 1) low-energy fluctuations, corresponding to eigenvalues of $\delta^2 S/\delta\phi^2$ of the order of R^{-2} and 2) high-energy fluctuations with eigenvalues $O(m^2)$. The first part corresponds to fluctuations of the shape of the bubble and is described by the following low-energy effective action

$$S_{eff} = \mu L - \varepsilon A \tag{3}$$

where L is the length of the bubble boundary, and A is the area of the bubble. The high-energy spectrum corresponds to exitations of the field ϕ in the bulk of the phases $\phi_\pm$ and to fluctuations of the profile of the field ϕ inside the bubble boundary (whose thickness is $O(m^{-1})$).

Let us first consider the functional integration over the high-energy degrees of freedom, i.e. we integrate over all configurations of the field ϕ which correspond to fixed shape of the bubble. According to the well known decoupling theorem /2/ the result of such integration reduces in the low-energy sector to renormalization of the parameters of the low energy theory (i.e. μ and ε in the case considered), provided that the low-energy theory itself is renormalizable. This is certainly the case for the theory with the action (3) since it is equivalent (at least perturbatively) to quantum mechanics of one degree of freedom. Therefore the functional integral in eq. (2) is reduced to path integral over shapes of bubble with the action (3) in which μ and ε are the renormalized parameters. (These quantities recieve no renormalization in the llow-energy sector and one can easily invent "gedanken" experiments to measure them).

To calculate the latter integral we introduce parametrization of the shape of the bubble boundary in the polar coordinates (ρ , α), in which the action (3) has the form

$$S_{eff} = \int_0^{2\pi} \left(\mu \sqrt{\dot{\rho}^2 + \rho^2} - \frac{1}{2} \varepsilon \rho^2 \right) d\alpha \qquad (4)$$

where the dot denotes derivative with respect to the angular parameter α which plays the role of (periodic) time. To determine the measure in the path integral we employ the formulation of the integral in the phase space, i.e. over the coordinates ρ and the canonical momenta p . Standard transformations result in the following expression

$$Z = \int \prod \frac{dp\, d\rho}{2\pi} \exp\left\{ - \int_0^{2\pi} \left(p\dot{\rho} + \sqrt{\mu^2 - p^2}\, \rho - \frac{1}{2} \varepsilon \rho^2 \right) d\alpha \right\} \qquad (5)$$

Since the integration over ϱ is Gaussian it is convenient to integrate out the coordinates ϱ rather than the momenta p. After doing this one finds

$$\mathcal{Z} = \mathcal{N} \int \prod dp \, \exp\left\{ -\frac{\pi\mu^2}{\varepsilon} - \int_0^{2\pi} \frac{\dot{p}^2 - p^2}{2\varepsilon} d\alpha + \int_{p(0)}^{p(0)} \frac{\sqrt{\mu^2 - p^2}}{\varepsilon} dp \right\} \quad (6)$$

($\mathcal{N}$ is well known p -independent normalization factor). The latter integral in the exponent obviously counts the munber of turns in the complex p -plane around the cut which goes from $p = -\mu$ to $p = +\mu$. Paths $p(\alpha)$ with zero number of such turns correspond to one-bubble configurations. Let us denote the integral over such paths as $\mathcal{Z}_1$. The complete path integral $\mathcal{Z}$ is given by /1/ $\mathcal{Z} = \exp \mathcal{Z}_1$. Therefore evaluation of the decay rate (eq. (2)) amounts to calculation of $\mathcal{Z}_1$, i.e. the same path integral as in eq. (6) but with the latter integral in the exponent set equal to zero.

From eq. (6) it is obvious that $\mathcal{Z}_1$ is related to the statistical sum for a harmonic oscillator with the frequency $\omega^2 = -1$ at the temperature $T = (2\pi)^{-1}$. These values of ω and T correspond to the situation of a kinetic focus and the statistical sum is divergent. The divergency is caused by two zero modes $p = a \cos \alpha$ and $p = a \sin \alpha$ which arise due to translational symmetry. At this point one should recall that the finite quantity is not W itself but rather the probability of formation of a critical bubble per unit length and unit time: $w = d^2 W / dx\,dt$. To regularize the infinity brought in by the translational group and thus to calculate w we slightly spoil the translational symmetry by introducing a factor depending on the position x_μ of the center-of-gravity of the bubble: $d^2W/dx\,dt = w \exp(-\lambda x_\mu^2)$ so that $W = w\pi/\lambda$, where λ is infinitesimal regularizing parameter. Using the explicit expression for the zero modes in the p-space one can readily relate their amplitudes to displacement of the bubble as a whole and thus verify that the λ-regularization is equivalent to the shift of the oscillator frequency from $\omega^2 = -1$ to $\omega^2 = -1 + 2\lambda/(\pi\varepsilon)$. With this modified frequency one finds $\mathrm{Im}\, Z_1 = \frac{\varepsilon}{4\lambda} \exp\left(-\frac{\pi\mu^2}{\varepsilon}\right)$. (One should also take into account that integration over the amplitude of the negative mode p = const goes along the imaginary axis over one half of the Gaussian peak /3,1/, which gives additional factor 1/2 in $\mathrm{Im}\, Z_1$). Summarizing the previous discussion we find that the parameter λ cancells in the expression for w which has the form

$$d^2W/dx\,dt = \frac{\varepsilon}{2\pi} \exp(-\pi\mu^2/\varepsilon) \qquad (7)$$

and constitutes the final result of our calculation.

The only approximation made in derivation of eq. (7) was the thin wall approximation $mR = m\mu/\varepsilon \gg 1$. As far as another dimensionless parameter ε/μ^2 is concerned, eq. (7) is exact in this parameter. In the special case when the potential $V(\phi)$ is a polinomial of the fourth power in ϕ the recently calculated /4/ one-loop result for w coincides with eq. (7) when the result of ref. 4 is expressed in terms of renormalized μ and ε . What is stated by eq. (7) is that all higher order terms amount merely to renormalization of the same parameters and that there is no series in powers of ε/μ^2 .

Eq. (7) can also be applied when the temperature is below the critical one: $T < T_c = \frac{\varepsilon}{2\mu}$. The relative magnitude of temperature corrections to eq. (7) in this region is proportional to $\exp\{-const\cdot\varepsilon\,(T^{-1} - T_c^{-1})\}$.

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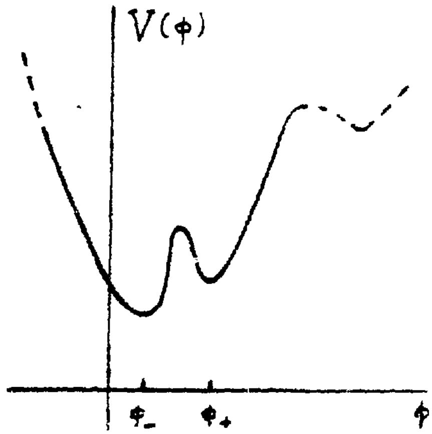

V(φ)
φ₋
φ₊
φ

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М.Б.Волошин

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