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RARE DECAYS OF NEUTRAL π AND η

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The decays of pseudoscalar neutral mesons π°, η° have provided a crucial test of fundamental principles. $\pi^\circ \to 2\gamma$, the main branch, has been investigated in the late '60s in the context of current algebra and, following a classic paper by Alder[1] explaining the null result obtained via soft pion techniques, the decay rate calculated from the singular triangle diagram is in excellent agreement with the experiment if we assume that the fundamental fermions (quarks) of the strong interaction come in three coloured species. This is still one of the most convincing arguments for our tricolour world.*

This successful theory of the triangle anomalies can be extended to treat the off-mass shell $\pi^\circ \to \gamma\gamma$ vertex. This is usually done in two different ways:

- quark loop relativistic calculation in the limit of zero pion mass
- hard pion techniques (Weinberg and Schnitzer[2])

Earlier evaluations in the '60s were based on vector dominance model (VDM). Recently Bramon et al.[3] have shown the consistency between the vector dominance approach and the quark loop model in what they called the Q^2 duality: for a particular choice of the quark masses, the VDM behaviour of the form factor for pseudoscalar mesons can be mimicked by quark loops.

At the other end of the spectrum, rare leptonic decays of the neutral pseudoscalar mesons are of great interest because of the information they may reveal about neutral currents or other "exotic" interactions between leptons and quarks (by exotic we mean non-electromagnetic). I shall discuss recent information obtained on the $\pi^\circ \to e^+e^-$ decay, which has disturbed the assurance of the theorists.

Also since the self-conjugate pseudoscalar mesons like π° and η cannot couple to an odd number of photons, $\pi^\circ \to 3\gamma$ has been studied to place limits on charge conjugation invariance. A recent LAMPF experiment[4] has set an upper limit of 1.5×10^{-6} for the branching ratio $\pi^\circ \to 3\gamma / \pi^\circ \to 2\gamma$ and this implies that the ratio of charge violating to charge conserving amplitude is less than 0.26.

The π° meson being the lightest of the known hadrons is stable vis a vis strong interactions and decays mainly through electromagnetic processes:

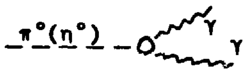
$\pi^\circ \to \gamma\gamma$	98.8%	1)
$\pi^\circ \to \gamma e^+e^-$	1.15 10^{-2}	2)
$\pi^\circ \to e^+e^-e^+e^-$	3.32 10^{-5}	3)
$\pi^\circ \to e^+e^-$	1.7 10^{-7}	4)

In the first approximation, the $\pi^\circ(\eta_0)$ couples to 2 photons and estimates for these decays can be obtained from purely electro-

*Although the agreement is remarkable, it should be noted that only recent measurements using the Primakoff effect are compatible. An independent confirmation is called for.

magnetic interaction; the effect of π° structure is absorbed in a form factor. In reaction (1), being a two-body reaction one cannot extract information on the π° structure.

The electromagnetic form factor of the $\pi^\circ(\eta)$ represents the real part of the simplest electromagnetic vertex for their decay.



when one or both of the photons are off their mass-shell.

I shall now discuss the recent ideas about the 2 photon form factor of the pseudoscalar meson, π° in particular, with the intent of presenting recent results obtained at LAMPF and CERN and future experiments in preparation at TRIUMF.

In general, the amplitude for the $\pi^\circ \to \gamma^* \gamma^*$ vertex can be written as:

$$\langle \gamma(k_1)\gamma(k_2)|T|\pi^\circ\rangle = \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^\mu(k_1)\, \varepsilon^\nu(k_2)\, k_1^\alpha\, k_2^\beta\, F(k_1^2,k_2^2)$$

where $F(k_1^2,k_2^2)$ is the form factor normalised to the real two photon decay rate

$$\sigma(\pi^\circ \to \gamma\gamma) = \frac{\alpha^2 \pi}{4}\, m_{\pi^\circ}^2\, |F(0,0)|^2 \; .$$

Dalitz, Kroll and Wada[5] have calculated the distribution of the lepton pairs produced in reaction (2) for the case where one assumes a contact interaction $\{F(k_1^2,k_2^2)=1\}$, later modified to take into account a possible form factor parametrized by

$$F(k_1^2,0) = 1 + a\, \frac{k_1}{m_V}^{2} + \ldots$$

where a is called the slope parameter of the π° form factor.

In the $\pi^\circ \to e^+e^-\gamma$ decay, the effect of the structure of the π° can be seen by looking at the rate for events corresponding to a large k_1, the photon invariant mass. The integrated rate is very insensitive to the structure effect and has been recently verified at LAMPF[6] where the decay rate was found to be $R_{e^+e^-\gamma} = 1.160 \pm 0.047\ 10^{-2}$ compared to $R_{theo} = 1.19\ 10^{-2}$ in good agreement with the Dalitz predictions based on purely QED effects. The slope parameter a has been estimated through various models and Table I presents a summary of the predictions.

Recently Pitch and Bernabeu[7] and Ametller, Bergström, Bramon and Masso[8] have shown that in a quark loop model the form factor can be expressed as

$$F(k_1^2,0) = 1 + \frac{m_\pi^2 + k_1^2}{12\, m_q^2} + 0\left(\frac{m_\pi}{m_q}\right)^4$$

$$\frac{F(k_1^2,0)}{F(0,0)} = 1 + \frac{k_1^2}{12\, m_q^2} + 0\left(\frac{m_\pi}{m_q}\right)^4$$

Table I Theoretical prediction for $\pi^\circ e^+e^-\gamma$ form factor slope

Author	Prediction for a	Comments
	$\pi^\circ\gamma\gamma$ Theory - Vector Dominance Model	
S.D. Drell[a]		
Berman & Geffen[b]	0.031	ρ-exchange, unsubtracted dispersion relation
Gell-Mann & Zachariasen[c]	0.0304	VDM with SU(3)
Geffen & Yound[d]	±0.06	once-subtracted dispersion relation
Barton & Smith[e]	0.046	once-subtracted dispersion relation
Mukherjee[f]		many solutions depending on sign ambiguities
	$\pi^\circ\gamma\gamma$ Theory with Triangle Anomalies	
Young[g]	$0.01<\|a\|<0.03$	Hard pions
Ali & Hussain[h]		Similar to g
Pratrap & Smith[i]	0.002	Fermion loop
Raval & Ramachandran[j]	0.012	Hard pions
Dicus et al.[k]		Similar to i
Efimov & Ivanov[l]	0.042	Non-local quark model (Russian bag)
Ivanov & Shekhter[m]		Similar to VDM calculations
Bergstrom & Snellman[n]	0.5	$q\bar{q}$ with QCD potentials
Bramon & Masso[o]		"Q^2 duality" (similar to VDM)

[a]S.D. Drell, Nuovo Cimento 11, 692 (1959).
[b]S. Berman and D. Geffen, Nuovo Cimento 18, 1192 (1960).
[c]M. Gell-mann and Zachariasen, Phys. Rev. 121, 275 (1961).
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[e]G. Barton and B. Smith, Nuovo Cimento 36, 436 (1965).
[f]A. Mukherjee, Jour. Phys. G4, 797 (1978).
[g]B.L. Young, Phys. Rev. D2, 606 (1970).
[h]A. Ali and F. Hussain, Phys. Rev. D3, 1207 (1971)
[i]M. Pratrap and J. Smith, Phys. Rev. D5, 2020 (1972).
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[k]D.A. Dicus et al., Phys. Rev. D15, 1286 (1977).
[l]G.V. Efimov and M.A. Ivanov, JETP Lett. 32, 55 (1980).
[m]A.I. Ivanov and V.M. Shekhter, Sov. J. Nucl. Phys. 32, 410 (1980).
[n]L. Bergstrom and H. Snellman, Zeit. Phys. C8, 363 (1981).
[o]A. Bramon and E. Masso, Phys. Lett. 104B, 311 (1981).

$$a = \frac{m_\pi^2}{12\ m_q^2} + O\left(\frac{m_\pi}{m_q}\right)^4 .$$

In the old vector dominance approach, one gets an expression for the form factor at small momentum transfers of the form

$$F(k_1^2,0) = 1 + \left(\frac{m_\pi^2}{m_V^2}\right) k_1^2 ; \qquad a = \frac{m_{\pi^\circ}^2}{m_V^2} .$$

The vector dominance prediction is reproduced for quark masses around 225 MeV.

The experimental situation is still confusing. Table II presents a compilation of experimental results on both π° and η° decays.

Table II Experimental limit for the π° and η form factor slope

Source	Technique	a
Samios[a]	Bubble chamber	-0.24±0.16
Kobrak[b]	Bubble chamber	-0.15±0.10
Devons[c]	NaI & spark chambers	0.01±0.11
Burger[d]	Magnetic spectrometer	0.016±0.10
Fischer[e]	π°'s from K decays (mag. spect.)	0.10±0.03
Jane[f]	η decays	-0.22±0.45
Schardt[g]	Branching ratio measurement	0.18±0.38
Dzhelyadin[h]	$\eta\to\mu^+\mu^-\gamma$	0.57±0.12

[a]N. Samios et al., Phys. Rev. 121, 275 (1961).
[b]H. Kobran et al., Nuovo Cimento 20, 115 (1961).
[c]S. Devons et al., Phys. Rev. 184, 1356 (1969).
[d]J. Burger, NEVIS-190, unpublished thesis (1972).
[e]J. Fischer et al., Phys. Lett. 73B, 359 (1978).
[f]M.R. Jane et al., Phys. Lett. 59B, 103 (1975).
[g]M.A. Schardt et al., Phys. Rev. D23, 639 (1981).
[h]R.I. Dzhelyadin et al., Phys. Lett. 94B, 548 (1980).

Although a recent Russian experiment[9] with good statistics on $\eta\to\mu^+\mu^-\gamma$ is in excellent agreement with the vector dominance prediction and implies that constituent quark mass of order 250 MeV be used in quark loop models, the recent π° form factor results from Fischer et al.[10] would imply quark masses in the 120–150 MeV range and are higher than the vector dominance prediction by a factor of 3.

An experiment is in progress at TRIUMF[11] to remeasure the π° electromagnetic form factor using large sodium iodide detectors in a configuration similar to that used by Nemethy and Devons[12] in the late '60s. We expect to get a with a precision of 0.02 and clearly establish if VDM has to be ruled out for π°. The experimental arrangement is shown in Fig. 1.

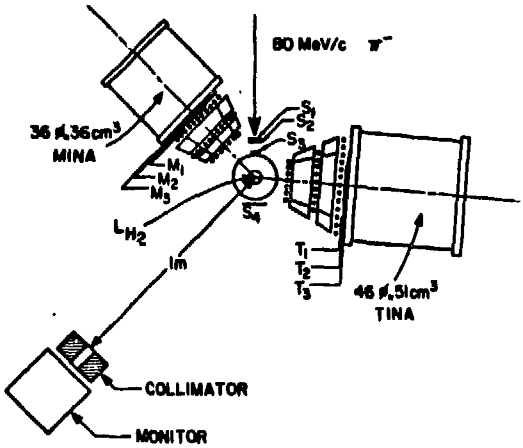

Fig. 1. Experimental set-up for exp. 217 at TRIUMF. M_1, M_2, M_3 and T_1, T_2, T_3 are 3 scintillator arrays 1 mm thick each. ••• are multiwire proportional chambers; S_1, S_2, S_3, S_4 are beam telescope scintillators.

If we now allow both photons to be off mass shell the calculation of the $\pi^\circ \to e^+e^-$ rate becomes more involved. Recently Ametller et al,[8] and Pitch and Bernabeu[7] have carried out such a calculation using modern computation techniques. The soft pion limit approximation is still in good agreement with the results of dispersion calculation and the conclusion is that the Q^2 duality is very successful.

Identical predictions are obtained for the branching ratio $\Gamma(\pi^\circ \to e^+e^- / \pi^\circ \to \gamma\gamma)$ from models based on VDM (bound quark states) or triangle model (relativistic quark loops) if one identifies the form factor cut-off m_V (vector meson mass) to $2M_q$ (twice quark mass). The rate for the decay $\pi^\circ \to e^+e^-$ is given by

$$R_{e^+e^-} = R^{Im}_{e^+e^-} + R^{Re}_{e^+e^-} .$$

The imaginary part (absorptive part) is known as the unitarity prediction (quark mass independent)

$$R^{Im}_{e^+e^-} = 4.75\ 10^{-8} .$$

The real part represents the diffractive contribution and is quark mass dependent. The current experimental limit[13] is $1.8 \pm 0.6\ 10^{-7}$, about four times the unitarity limit.

For quark masses of order 100-150 MeV (necessary to explain the slope parameters) the total rate is predicted to be $7.1 - 6.6\ 10^{-8}$. In general almost all model-independent predictions are obtained for $\pi \to e^+e^-$ and $\eta \to \mu^+\mu^-$, which are very close to the unitarity predictions. So here again π° is in conflict with the prediction whereas η° decays agree experimentally.

Other contributions have been considered that could enhance the branching ratio. Weak neutral current effects have been studied by Herczeg and Michel[14] and appear to be very small ($\sim 10^{-9}$). Possible exotic coupling between quarks and leptons (lepto-quark) or Higgs exchanges have also been considered.

Tupper et al.[15] have also questionned the validity of the 2 photon approximation claiming that the argument leading to the neglect of radiative correction to the $\pi^\circ \to e^+e^-$ based on the fact that this branch is suppressed due to CP conservation, does not apply if photons are also emitted from the leptons.

The rate for $\pi^\circ \to e^+e^-\gamma$ where γ is a soft bremsstrahlung photon could be non-negligible and should be included in the analysis of the $\pi^\circ \to e^+e^-$ data. We then would measure a combination of the two rates $\pi^\circ \to e^+e^-$ and $\pi^\circ \to e^+e^-\gamma$ which could be significantly above the unitarity bound (this contribution would be much smaller in the case of $\eta \to \mu^+\mu^-\gamma$ explaining the relatively good agreement in that case).

It is up to the experimenters to clarify this situation as more statistics are needed before a firm conclusion can be drawn. High intensity K beams could provide the necessary tagged π° beams to answer the above questions.

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