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* ^{14}N-TRINUCLEON CLUSTER STATES OF ^{17}F AND ^{17}O

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^{14}N-trinucleon cluster states in ^{17}F and ^{17}O

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Abstract: A cluster model is used to calculate the energies of those states in ^{17}F and ^{17}O which have a ^{14}N-trinucleon cluster-core structure. The non-central terms in the cluster-core potential are deduced phenomenologically and also calculated microscopically. They are found to be intimately related to equivalent terms in the potentials for similar cluster-core decompositions of neighbouring nuclei. The results are compared with the spectrum of states excited in a recent experimental study of three-particle transfer onto ^{14}N.

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1. Introduction

Over the last few years there has been a good deal of theoretical and experimental work on the properties of trinucleon cluster states in light nuclei. These states are best identified by their selective excitation in trinucleon transfer reactions. Their energies typically follow a rotational sequence, with some perturbations due to the presence of non-central forces between cluster and core. Strong electromagnetic E2 transitions are often observed within a given band, and sometimes strong E1 transitions between members of neighbouring bands.

When similar studies were made of alpha cluster states in light nuclei, it was found that several of the higher lying candidate states exhibited very large alpha decay widths. However, in the trinucleon cluster state studies, all the states so far proposed to have trinucleon cluster structure, lie below the core-trinucleon breakup threshold. This means that large triton or helion decay widths are not available for use as an additional indicator of their structure. Nevertheless, the measured energies as well as the static and dynamic electromagnetic properties of the candidate states are exactly in line with the predictions of an assumed core-trinucleon structure, and provide convincing evidence for

this identification.

Trinucleon transfer reactions have recently been used to locate trinucleon cluster states in ^{15}N (Pilt 1983), ^{17}O (Cunsolo et al 1981 and 1983), ^{17}F and ^{17}O (Etchegoyen et al 1984), ^{18}F and ^{18}O (Rae et al 1979) and ^{19}F (Martz et al 1979 and Rae et al 1979). The experimental effort has stimulated a commensurate number of theoretical studies. There have been many applications of the resonating group method to trinucleon cluster states in the nuclei mentioned above (Furutani et al 1980). The simpler, and more intuitively appealing, local potential model of Buck, Dover and Vary (1975) has also been applied to these trinucleon cluster states with equal success (Buck and Pilt 1977 and 1978, Merchant 1983a and 1983b).

In this paper, the Buck, Dover, Vary cluster model will be employed to calculate the properties of trinucleon cluster states based on a ^{14}N core. This will provide a spectrum for comparison with the experimental trinucleon transfer studies of Etchegoyen et al (1984), (who were only able to obtain a rather poor description of their selectively excited states with shell model calculations in a $(0p_{1/2})^{-1}, (0d_{5/2}), (1s_{1/2})$ space) and, at the same time, complete a systematic study of trinucleon clustering among the low (sd)-shell nuclei for the mass range $17 \leq A \leq 19$. A close relationship between the

trinucleon cluster state spectra in all these nuclei will be demonstrated.

2.The Buck, Dover, Vary cluster model

The prescription of the Buck, Dover, Vary model for obtaining the central part of an effective cluster-core potential is to integrate the densities of the two components with an effective nucleon-nucleon interaction. A delta function is usually chosen for this purpose so that the double folding reduces to a single folding, and a local potential of the form;

$$V_o(R) = -\left(\frac{2\pi\hbar^2}{m}\right)\bar{f}\int\rho_A(\underline{R}-\underline{R}')\rho_B(\underline{R}')\,d\underline{R}' \qquad (2.1)$$

results. Here, m is the nucleon mass, ρ_A and ρ_B are the densities of cluster and core respectively, and $\bar{f}$ is an adjustable real strength parameter.

The cluster is then viewed as a single-particle in orbit around the core, and characterized by principal quantum number, N, and orbital angular momentum, L. The major requirements of the Pauli exclusion principle are satisfied, in the present case, by restricting the values of N and L to obey the condition $2N+L \geq 6$. This corresponds to choosing the

quantum numbers of the cluster nucleons so that they are placed in the (sd)-shell or higher, leaving two p-shell orbitals vacant. Then the cluster and core nucleons occupy different major shells, and do not overlap significantly.

The Coulomb interaction is dealt with by treating the cluster as a point charge and the core as a uniformly charged sphere of radius R_c (R_c=3.0 fm has been consistently used in neighbouring nuclei by Merchant 1983a and 1983b). A centrifugal potential is included where appropriate. The single-particle Schrödinger equation may then be solved numerically to obtain the energy and wave function for a state of specified N and L.

When the cluster and/or core has an intrinsic angular momentum, non-central forces may also be present in the interaction. In the earliest applications of this model, these forces were treated phenomenologically as perturbations with great success. The possible forms of the non-central forces were deduced by requiring the potential to remain invariant under space inversion and time reversal. Then the radial parts of their matrix elements were parametrized by constants, chosen to give an optimal fit to the cluster state spectrum.

Subsequently, microscopic calculations have been

performed to evaluate these radial matrix elements. They sum a cluster-nucleon potential over all the nucleons comprising the core, to obtain form factors for the non-central parts of the cluster-core potential. These form factors can then be integrated with the wave functions of relative motion of cluster and core to obtain a strength, for comparison with the appropriate fitted parameter. In general, good agreement between the two is obtained.

It has been shown that very similar moments of inertia and spin-orbit forces can give a good description of many cluster states in neighbouring nuclei in the mass range $16 \leq A \leq 20$. Physically, this is a very satisfactory result, in view of the similarity of the clusters and cores involved in such a restricted mass range. It can be even further reinforced by demonstrating that the parametrizations of the radial matrix elements which Buck and Pilt (1978) used to describe the spectra of ^{18}F and ^{18}O, can be adapted to give an equally good description of trinucleon cluster states in ^{17}F and ^{17}O.

3. Relation between trinucleon cluster states in $^{17}F/^{17}O$ and $^{18}F/^{18}O$

In this section a connection between the lowest lying bands (which have 2N+L=6) of trinucleon cluster states in $^{17}F/^{17}O$ and $^{18}F/^{18}O$ will be examined. In particular, it will be demonstrated that the strength parameters used by Buck and Pilt (1978) to describe the spectra of trinucleon cluster states in $^{18}F/^{18}O$ may be adapted for use in $^{17}F/^{17}O$. When this is done, the predicted energies of the higher spin states are in close agreement with the measured energies of the states excited in three-nucleon transfer onto ^{14}N by Etchegoyen et al (1984). In addition, the energies of the ^{14}N-trinucleon band heads in ^{17}F and ^{17}O may be related to the energy of the head of a band of ^{14}C-^{3}He cluster states in ^{17}O.

3.1 The cluster-core potentials.

Buck and Pilt (1978) were able to obtain a remarkable fit to some 20 proposed trinucleon cluster states in $^{18}F/^{18}O$ (which would have a leading $(p)^{-1}(sd)^{3}$ configuration in a shell-model expansion) as follows. For each cluster-core system they considered a potential containing all the terms allowed by parity and time reversal invariance. They assumed that the core spin-orbit force ($V_{\lambda\sigma}(R)\underline{L}.\underline{S}_1$) could be

ignored, because it had always produced negligible energy shifts in previous calculations. Therefore, their potential for a given cluster-core decomposition (see Fig.1) had the form;

$$V(R) = V_0(R) + V_{SS}(R)\underline{S}_1.\underline{S}_2 + V_{LS}(R)\underline{L}.\underline{S}_2 + V_{TS}(R)\left[(\underline{S}_1.\hat{\underline{R}})(\underline{S}_2.\hat{\underline{R}}) - \tfrac{1}{3}\underline{S}_1.\underline{S}_2\right] \qquad (3.1)$$

It contains central (as described in section 2), spin-spin, cluster spin-orbit and tensor terms. The circumflex accents denote unit vectors. They then considered those states excited in the relevant three-nucleon transfer experiment to be candidate cluster states, and parametrized the non-central forces of equation (3.1) to reproduce their energies as closely as possible.

The candidate states fitted in ^{18}O all had isospin T=1, while those in ^{18}F had T=0. (Analogues of the T=1 states in ^{18}O are of course expected in ^{18}F, but only a few have been located to date). Initially, strengths for the radial matrix elements of the non-central forces in equation (3.1) were fitted to the T=1 candidate states in ^{18}O and the T=0 candidate states in ^{18}F separately. However, the strengths for the two cases were found to bear a striking resemblance to each other. Sometimes they were almost equal in magnitude. Other times, almost equal in magnitude but of opposite sign.

This strongly suggested that some of the non-central forces in the two cases might be almost identical, and others related by simple isospin-dependent terms.

The two separate cluster-core potentials for ^{18}F and ^{18}O can easily be combined into a single potential. This will, in general, contain all the terms of equation (3.1), and similar ones multiplied by $\underline{\tau}_1.\underline{\tau}_2$ (the scalar product of Pauli isospin matrices for cluster and core). In principle, the radial factors multiplying each term will be quite independent of each other. However, the coincidences noted in the last paragraph suggest that some of them may actually be closely related. In any case, the separate sets of strength parameters of Buck and Pilt for ^{18}F and ^{18}O can be combined to give equivalent strengths for each of the various non-central forces in this single, isospin-dependent potential.

It will now be assumed that the radial parts of the non-central forces in the mass-18 trinucleon cluster-core potential are identical to those in the equivalent mass-17 potential. This assumption will be examined in detail in section 4. A phenomenological spectrum of ^{14}N-trinucleon cluster states in ^{17}F and ^{17}O can now be obtained very simply. It is not necessary to calculate the strength parameters for any of the <u>isospin-dependent</u> terms in the cluster-core potential. (Since the ground state ^{14}N

core has isospin T=0, any such terms will not affect the energy splittings of the levels of the spectrum). Strength parameters for the isospin-independent terms may be found using,

$$P(^{17}F \text{ OR } ^{17}O) = \frac{1}{4}\left[P(^{18}F) + 3P(^{18}O)\right] \qquad (3.2)$$

where P is any of the strength parameters of Buck and Pilt (1978), while 1/4 and 3/4 are the appropriate weights to remove the isospin-dependence from these strengths.

The tensor force acting between cluster and core is actually something of a problem for several reasons. The above prescription suggests that a tensor force of the form $\left[(\underline{S}_1 \cdot \hat{\underline{R}})(\underline{S}_2 \cdot \hat{\underline{R}}) - \frac{1}{3}\,\underline{S}_1 \cdot \underline{S}_2\right]$ should be present, but rather small. However, a previous microscopic calculation (Merchant 1983b) suggested that this force should have the same isospin dependence as the spin-spin force. This feature is not apparent in the fits of Buck and Pilt. To complicate matters further, the ^{14}N core has a spin of 1, so that another tensor force of the form $\left[(\underline{S}_1 \cdot \hat{\underline{R}})^2 - \frac{1}{3}S_1^2\right]$ may also be present in the interaction. No information on this type of tensor force can be extracted from the $^{18}F/^{18}O$ cluster state fits because no equivalent force appears there (all clusters and cores have spins of 1/2). In view of these uncertainties, all tensor

forces were ignored in the present phenomenological treatment. (Note that a strength parameter of zero is not very different from the value deduced from the Buck and Pilt fits using equation (3.2)). These tensor forces will, however, be examined more thoroughly in the microscopic treatment to be presented in section 4.

3.2 Comparison of phenomenology with experiment

Using the prescription described in the last subsection, the energies of the states in the 2N+L=6 bands of trinucleon cluster states in ^{17}F and ^{17}O will now be calculated. Initially, a comparison will be made with the energies of the states excited in trinucleon transfer onto ^{14}N by Etchegoyen et al (1984). Then, the full spectrum of trinucleon cluster states in the 2N+L=6 bands, using the 'best' parameter set, will be presented.

The central part of the cluster-core potential generates a series of rotationally spaced centroids having L=0,2,4 and 6 (corresponding to N=3,2,1 and 0 respectively). The spin-spin and spin-orbit forces then split each centroid into a sextuplet. This consists of a doublet when the cluster and core spins (of 1/2 and 1 respectively) couple to a total spin, S, of 1/2 and a quadruplet when they couple to 3/2. The total

angular momentum, J, of a given state is obtained by coupling L with S, i.e. $J=L \circledast S$.

Evaluation of the angular momentum algebra for the non-central forces allows us to write down a simple, phenomenological expression for the energy of each cluster state,

$$E(J,L,S) = C + A\,L(L+1) + \alpha\left[2S(S+1) - \frac{11}{4}\right] + (-1)^{S+1/2}\left(\frac{\xi}{3}\right)\left[J(J+1) - L(L+1) - S(S+1)\right] \quad (3.3)$$

Here, A is the rotational parameter (determined by the central potential), α the spin-spin force strength and ξ the spin-orbit force strength. They are all obtained from Buck and Pilt (1978) using equation (3.2). C is the energy of the band head. This is chosen to reproduce the energy (8.43 MeV) of the lowest $7/2^+$ state populated in the experiment of Etchegoyen et al (1984). In the model, this is formed by coupling $L=2 \circledast S=3/2$. At this stage, mixing of states with equal J but different S by the $\underline{L}.\underline{S}_2$ force is being ignored (as are any tensor forces).

A comparison of the energies predicted by equation (3.3), using all of the parameter sets A-E of Buck and Pilt (1978), with the five levels in $^{17}F/^{17}O$ strongly populated by Etchegoyen et al (1984) is presented in table 1. The

agreement is quite good for all the parameter sets, and best of all for set B, (which is markedly better than the shell-model energies of $(p)^{-2}(sd)^{3}$ states quoted by Etchegoyen et al (1984)). The full spectrum of cluster states in the 2N+L=6 band has been calculated using parameter set B. The energies are listed in table 2 (which gives the values of L and S that couple to J) and displayed in Figure 2 in order of ascending excitation energy. These results will be discussed further in section 5, after the microscopic calculation has been presented.

3.3 The energies of the bandheads

The bandhead energies, C, used in conjunction with the various parameter sets A-E, range from 8.2 MeV to 8.6 MeV. However, in principle, C can be related to the energy of the head of a band of ^{14}C-^{3}He cluster states in ^{17}O. This latter band has been investigated experimentally by Cunsolo et al (1981 and 1983) and theoretically by Merchant (1983a). There would then by no free parameters in equation (3.3).

Merchant used the common parametrization of equation (2.1),

$$V_o(R) = -V_o\left[\frac{1 + \cosh(R_o/a_o)}{\cosh(R/a_o) + \cosh(R_o/a_o)}\right] \qquad (3.4)$$

with R_o=2.0 fm, a_o=1.3 fm to describe the ^{14}C-^{3}He cluster states in ^{17}O. Since the ground state ^{14}C core has isospin T=1, two bands of helion cluster states (one having T=1/2, the other T=3/2) are expected in ^{17}O. Experimentally, the energies of their respective bandheads are found to lie at 6.8 MeV (reproduced by the potential of equation (3.4) when V_o=123.6 MeV) and at 13.1 MeV (requiring V_o=105.1 MeV). Actually, the energy difference between these two bandheads can be attributed to an explicit isospin dependence in the central potential of the form $V_I(R)\,\underline{\tau}_1\cdot\underline{\tau}_2$. The situation can thus be described by a single bandhead at C=11.0 MeV (produced by V_o=111.7 MeV) which is split into two by the isospin dependent force.

The energies of the heads of the ^{14}N-^{3}H band in ^{17}O and the ^{14}N-^{3}He band in ^{17}F can now be calculated by solving the single-particle Schrödinger equation for the cluster-core relative motion. The same nuclear potential is used as for the ^{14}C-^{3}He cluster states (V_o=111.7 MeV, R_o=2.0 fm and a_o=1.3 fm in equation (3.4)), and only the Coulomb potential

(described in section 2) is varied by changing the charge product $Z_1 Z_2$ to the appropriate value for each cluster-core system. For example, setting $Z_1 Z_2 = 7$, the head of the $2N+L=6$ band of ^{14}N-^{3}H cluster states in ^{17}O is predicted to lie at 9.76 MeV below the ^{14}N-^{3}H breakup threshold. This corresponds to an excitation energy of 8.86 MeV in ^{17}O. Since ^{17}F and ^{17}O are mirror nuclei, the equivalent bandhead in ^{17}F is expected at a very similar excitation energy. Indeed, with $Z_1 Z_2 = 14$, the $2N+L=6$ ^{14}N-^{3}He cluster state bandhead in ^{17}F is predicted to lie 6.98 MeV below the ^{14}N-^{3}He breakup threshold. This corresponds to an excitation energy of exactly 8.86 MeV in ^{17}F. Therefore the bandhead energies to be used in equation (3.3) for the $2N+L=6$ bands of trinucleon cluster states in ^{17}F and ^{17}O are calculated to lie at equal excitation energies and to be close to the range of fitted values.

3.4 Comparison with neighbouring nuclei

Taking together all the calculations on bands of trinucleon cluster states in nuclei in the mass range $17 \leq A \leq 19$ we find the following similarities. In ^{17}F (decomposed into ^{14}N-^{3}He), ^{17}O (decomposed into ^{14}C-^{3}He and ^{14}N-^{3}H), ^{18}F (decomposed into ^{15}N-^{3}He and ^{15}O-^{3}H), ^{18}O (decomposed into ^{15}N-^{3}H) and ^{19}F (decomposed into ^{16}O-^{3}H) all the $2N+L=6$ bands can be

accurately generated from central potentials having exactly the same geometries (i.e. R_0 2.0 fm and a_0=1.3 fm in equation (3.4)) and closely similar depths. Of course, this is precisely what would be expected from folding the densities of a trinucleon and such similar cores, and in itself lends credence to the proposal that all these states have a basically similar structure.

All these cluster state spectra exhibit a spin-orbit splitting which may be attributed to a force of the form,

$$V_{so}(R) = -V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{R}\left|\frac{df(R)}{dR}\right| 2\underline{L}\cdot\underline{S}_2 \qquad (3.5)$$

where the function f(R) describes the geometry of the central force, and V_{so} = -2.5 MeV for <u>all</u> the cluster-core systems mentioned. This again emphasizes the similarity of all these states. This close interrelationship suggests very strongly that there is a real physical significance to the picture of these states as having a core-trinucleon cluster structure.

4. The Semi-microscopic model

The non-central forces in the ^{14}N-trinucleon potential may be calculated microscopically by combining the results of two earlier studies. They are the calculations of alpha cluster states in ^{18}F (Buck et al 1979) and of trinucleon cluster states in ^{18}F and ^{18}O (Merchant 1983b). The former work will be followed in describing the ^{14}N core as two p-shell holes in an inert, doubly closed shell ^{16}O nucleus coupled to a total spin of 1, and isospin 0. There are three possible combinations of p-shell orbitals which can achieve such a coupling. They are labelled by amplitudes c_1 (both holes in $p_{1/2}$ orbitals), c_2 (one hole in a $p_{1/2}$ orbital and the other in a $p_{3/2}$ orbital) and c_3 (both holes in $p_{3/2}$ orbitals).

An effective cluster-core potential can be obtained by summing a nucleon-trinucleon potential over all the core nucleons. This is equivalent to the practically simpler procedure of taking an ^{16}O-trinucleon potential and adding to it the interaction between the trinucleon and the two coupled holes. Two detailed examples of the method are given by Buck et al (1979) and Merchant(1983b), so only an outline will be sketched here, before giving the results.

4.1 The nucleon-trinucleon potential

An effective cluster-core potential is obtained by making a tensor decomposition of a nucleon-trinucleon potential. The various moments can then be integrated with a wave function describing the two p-shell holes to produce radial form factors for each of the non-central terms in the cluster-core potential.

The first problem is therefore to decide how to obtain a nucleon-trinucleon potential. It may, in principle, contain an abundance of spin and isospin dependent terms. Unfortunately, there is not yet enough information available from nucleon-trinucleon scattering experiments (using polarized beams and targets) to uniquely specify all of those possible terms from a phase shift analysis. Although some progress has recently been made in microscopically producing a momentum space p-^{3}He potential with full spin-dependence (Paez and Landau 1984), this is not applicable to the present calculation which requires a coordinate space nucleon-trinucleon potential.

Considerations of invariance under space inversion and time reversal restrict the terms contained in a nucleon-trinucleon potential, $V_{nt}(x)$, to the following ten

possibilities

$$V_{nt}(x) = \bar{U}_0(x) + \bar{U}_{\tau\tau}(x)\,\underline{\tau}_n\cdot\underline{\tau}_t + \bar{U}_{\sigma\sigma}(x)\,\underline{\sigma}_n\cdot\underline{\sigma}_t + \bar{U}_{\sigma\tau}(x)(\underline{\sigma}_n\cdot\underline{\sigma}_t)(\underline{\tau}_n\cdot\underline{\tau}_t)$$
$$+\bar{U}_{LS_t}(x)\,\underline{l}_{rel}\cdot\underline{\sigma}_t + \bar{U}_{LS_t\tau}(x)(\underline{\tau}_n\cdot\underline{\tau}_t)\,\underline{l}_{rel}\cdot\underline{\sigma}_t + \bar{U}_{LS_n}(x)\,\underline{l}_{rel}\cdot\underline{\sigma}_n$$
$$+\bar{U}_{LS_n\tau}(x)(\underline{\tau}_n\cdot\underline{\tau}_t)\,\underline{l}_{rel}\cdot\underline{\sigma}_n + \bar{U}_T(x)\left[(\underline{\sigma}_n\cdot\hat{\underline{x}})(\underline{\sigma}_t\cdot\hat{\underline{x}}) - \tfrac{1}{3}\underline{\sigma}_n\cdot\underline{\sigma}_t\right]$$
$$+\bar{U}_{T\tau}(x)(\underline{\tau}_n\cdot\underline{\tau}_t)\left[(\underline{\sigma}_n\cdot\hat{\underline{x}})(\underline{\sigma}_t\cdot\hat{\underline{x}}) - \tfrac{1}{3}\underline{\sigma}_n\cdot\underline{\sigma}_t\right] \qquad (4.1)$$

where $\underline{\sigma}_n$, $\underline{\sigma}_t$, $\underline{\tau}_n$ and $\underline{\tau}_t$ are the Pauli spin and isospin matrices for the nucleon and trinucleon, x is the separation distance of the nucleon and the centre of mass of the trinucleon, and $\underline{l}_{rel}$ is their relative orbital angular momentum.

In the present calculation it is not necessary to retain any of the five isospin dependent terms in equation (4.1). These forces inevitably produce similar isospin dependent forces in the derived cluster-core potential, and since the ^{14}N core is an isosinglet, (T=0), they will have no effect on the cluster state spectrum. Of the remaining five isospin independent terms, the nucleon spin-orbit and tensor terms may also be ignored. The former is known from earlier calculations (Merchant 1983b) to have a negligible influence on the cluster state spectrum, while there is no conclusive experimental evidence, so far, for the existence of the

latter. Therefore, only three terms out of the original ten in equation (4.1) need to be specified.

The real part of the optical potential of DeVries et al (1972) will be used for the central part of the potential, $\bar{U}_o(x)$. The two remaining terms, namely the spin-spin and trinucleon spin-orbit potentials, will be taken from Merchant (1983b). In that work, the various non-central terms in the nucleon-trinucleon potential were assigned Gaussian geometries of equal width. Their strengths were then adjusted until they could correctly reproduce the energies of the seven p-wave resonances in ^{4}He, as calculated by de Shalit and Walecka (1966). This procedure was repeated for a large number of different widths. The trinucleon cluster state spectra in ^{18}F and ^{18}O were then calculated with the potential appropriate to each width and compared with experiment. The experimental spectra were most closely reproduced (see tables 8 and 9 of Merchant 1983b) using a width parameter, κ, of 0.2 fm^{-2} in conjunction with harmonic oscillator nucleon-trinucleon relative motion wave functions of energy constant $\hbar\omega$ = 23 MeV. Therefore, the three non-negligible terms (for the present calculation) of equation (4.1) were explicitly written as

$$V_{nt}(x) = \frac{\bar{U}_o}{1 + e^{(x - x_o A^{1/3})/a}} + (\bar{U}_{ss}\underline{\sigma}_n \cdot \underline{\sigma}_t + \bar{U}_{LS_t}\underline{l}_{rel} \cdot \underline{\sigma}_t) e^{-\kappa x^2} \qquad (4.2)$$

where $\bar{U}_o$ = -37.4 MeV, x_o = 1.49 fm (amu)$^{-1/3}$, a = 0.144 fm, $\bar{U}_{SS}$ = 5.32 MeV, $\bar{U}_{LS_t}$ = -2.18 MeV and χ = 0.2 fm^{-2}.

4.2 The form factors for the non-central cluster-core forces

In this subsection, expressions for the radial form factors in the derived ^{14}N-trinucleon potential will be presented. The radial wave functions of the two p-shell holes which describe the ^{14}N ground state are taken as harmonic oscillator radial wave functions, with width parameter b=1.77 fm (Donnelly and Walker 1969). This choice allows those form factors which are derived from the Gaussian terms in equation (4.2) to be evaluated in closed form. The tensor form factor derived from the Wood-Saxon central potential, $\bar{U}_o(x)$, can only be specified numerically.

A ^{14}N-trinucleon effective potential can now be derived. It takes the form

$$V(R) = V_o(R) + 4V_{SS}(R)\underline{S}_1\cdot\underline{S}_2 + 2V_{LS}(R)\underline{L}\cdot\underline{S}_2$$
$$+4V_{TS}(R)\left[(\underline{S}_1\cdot\hat{\underline{R}})(\underline{S}_2\cdot\hat{\underline{R}}) - \tfrac{1}{3}\underline{S}_1\cdot\underline{S}_2\right] + 4V_{TR}(R)\left[(\underline{S}_1\cdot\hat{\underline{R}})^2 - \tfrac{1}{3}S_1^2\right] \qquad (4.3)$$

The central term, $V_o(R)$, is the cosh potential of equation (3.4). The non-central force form factors are,

$$V_{SS}(R) = \sqrt{\tfrac{3}{2}}C_{101}\left(F_{SS}(R)\right)_0 + \frac{2m_t}{m+m_t}C_{110}\left[\left(F_{LS_t}(R)\right)_0 - \left(F^{+}_{LS_t}(R)\right)_1\right] \qquad (4.4)$$

$$V_{LS}(R) = -V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{R}\left|\frac{df}{dR}\right| + \frac{m}{m+m_t}\left[\left(F^{-}_{LS_t}(R)\right)_1 - \left(F_{LS_t}(R)\right)_0\right] \quad (4.5)$$

$$V_{TS}(R) = 4\sqrt{\frac{3}{2}}C_{121}\left(F_{SS}(R)\right)_2 - \frac{4m_t}{m+m_t}C_{110}\left[\left(F_{LS_t}(R)\right)_2 - \left(F^{+}_{LS_t}(R)\right)_1\right] \quad (4.6)$$

$$V_{TR}(R) = \frac{3}{2}C_{220}\left(F_0(R)\right)_2 \quad (4.7)$$

where m is the nucleon mass and m_t is the trinucleon mass. The radial factors, $(F_{SS}(R))_2$ etc, are integrals of the radial parts of the different terms in the nucleon-trinucleon potential multiplied by the p-shell harmonic oscillator functions (see the definitions given in equations (4.21) and (4.22) of Merchant 1983b). The 'C-factors' are geometric coefficients involving sums over Wigner 9-j symbols. They show how the various derived forces depend on the precise p-shell orbitals occupied by the two holes which comprise the ^{14}N ground state core. The factors C_{101}, C_{121} and C_{220} are given in equation (4.24) of Buck et al (1979), and C_{110} may be evaluated from equation (4.22) of that reference. It is,

$$C_{110} = \frac{2}{3}\left(C_1^2 + \frac{1}{2}C_1C_2 + \frac{9}{4}C_2^2 - \frac{1}{2}\sqrt{\frac{5}{2}}C_2C_3 + \frac{1}{2}C_3^2\right) \quad (4.8)$$

The explicit values of c_1, c_2 and c_3 to describe the ground state ^{14}N core are given in equation (3.6) of Buck et al (1979).

4.3 Similarities between the $^{17}F/^{17}O$ and $^{18}F/^{18}O$ calculations

By inspection of the form factors given in the last subsection it is possible to understand why the phenomenological calculation of section 3 was so successful. The central potentials used to describe the trinucleon cluster states in $^{17}F/^{17}O$ and $^{18}F/^{18}O$ are very similar and produce almost identical rotationally spaced centroids in all of these nuclei. These are then split principally by spin-spin and cluster spin-orbit interactions. The form factors associated with these latter two forces in $^{17}F/^{17}O$ (equations 4.4 and 4.5) have an identical structure to the equivalent form factors in $^{18}F/^{18}O$ (equations 4.37 and 4.42 of Merchant 1983b).

The similarity of the trinucleon cluster state spectra in $^{17}F/^{17}O$ and $^{18}F/^{18}O$ becomes even clearer if we consider the limiting case in which the ^{14}N core is described simply as two $p_{1/2}$ holes with their spins aligned so as to couple to total spin 1. (The ^{15}N or ^{15}O cores for $^{18}F/^{18}O$ are of course described as a single $p_{1/2}$ hole.) This can be achieved by setting $c_1=1$ and $c_2=c_3=0$. Then $C_{101} = -\frac{1}{3}\sqrt{\frac{2}{3}}$ and $C_{110} = \frac{2}{3}$ so the form factors $V_{SS}(R)$ and $V_{LS}(R)$ for $^{17}F/^{17}O$ become <u>identical</u> to the equivalent form factors in $^{18}F/^{18}O$. The only difference between the forces is that in $^{18}F/^{18}O$ the

trinucleon interacts with a single $p_{1/2}$ hole ($S_1 = 1/2$) and in $^{17}F/^{17}O$ with two aligned $p_{1/2}$ holes ($S_1 = 1$). This limiting description of the ^{14}N core is not very different from that actually used, where $c_1 = 0.9495$. Therefore the form factors for $^{17}F/^{17}O$ are only slightly different from those in $^{18}F/^{18}O$. We can thus understand why the strengths of Buck and Pilt, which parametrize radial integrals of these form factors in $^{18}F/^{18}O$, can also be used successfully in $^{17}F/^{17}O$.

So far the tensor force has been ignored. However, the tensor form factor, $V_{TS}(R)$, in $^{17}F/^{17}O$ is also simply related to that in $^{18}F/^{18}O$. In the same limit of $c_1 = 1$ and $c_2 = c_3 = 0$ the $V_{TS}(R)$ for $^{17}F/^{17}O$ and $^{18}F/^{18}O$ are proportional to each other, with the constant of proportionality being 4/3. This tensor force is rather small and only produces minor changes in the cluster state spectra.

However, there is another tensor force present in the ^{14}N-trinucleon potential which is absent from the $^{15}N/^{15}O$-trinucleon system. This force is essentially due to the interaction of the trinucleon with the quadrupole deformation of the ^{14}N core. Its form factor, $V_{TR}(R)$, is derived from the central part of the nucleon-trinucleon force, and is therefore quite large. This tensor interaction, $V_{TR}(R)\left[(\underline{S}_1 \cdot \hat{\underline{R}})^2 - \frac{1}{3}S_1^2\right]$, should be comparable with the exactly

analogous one calculated in the $^{14}N-\alpha$ system (Buck et al 1979). In this case, it will be much larger than the other tensor interaction, $V_{TS}(R)\left[(\underline{S}_1 \cdot \hat{\underline{R}})(\underline{S}_2 \cdot \hat{\underline{R}}) - \frac{1}{3}\underline{S}_1 \cdot \underline{S}_2\right]$. Unfortunately, the limited experimental data available at present are not sufficient to demonstrate its effects.

5. Conclusions

5.1 Comparison of the microscopic calculation with phenomenology

The form factors for the non-central forces of equations (4.4) to (4.7) were calculated using the microscopic prescription of the last section. They were then integrated with cluster-core relative motion wave functions for L = 0,2,4,6 (generated by solving the single-particle Schrödinger equation as described in section 2). The resulting strengths are compared with the values deduced from Buck and Pilt's parameter set B in table 3.

The spin-spin parameter, α, is in excellent agreement. It decreases somewhat in magnitude as L increases, and the range of values -0.336 to -0.250 straddles the fitted value of -0.300. The spin-orbit parameter, ξ, shows hardly any

L-dependence and is fairly close to the fitted values, but a little overestimated. The tensor parameter, β_s, increases with increasing L, and is of comparable magnitude to the fitted value, but opposite sign.

This result was also encountered in the $^{18}F/^{18}O$ calculation. It is inescapable since the spin-spin and tensor forces are derived from the same terms in the nucleon-trinucleon potential. The only way out of the impasse would seem to be that there might be additional contributions to the tensor force which were not included in these calculations, such as a tensor force in the nucleon-trinucleon potential. In any case this force is of minor importance here, since $V_{TR}(R)$ is found to be much bigger.

The tensor force $V_{TR}(R)\left[(\underline{S}_1 \cdot \hat{\underline{R}})^2 - \frac{1}{3}S_1^2\right]$ is generated by the interaction of the cluster with the quadrupole deformation of the ^{14}N core. As such, it is closely comparable with a similar interaction in the ^{14}N-α system (the equivalent parameter is listed in table 2 of Buck et al 1979). The calculated magnitude of the strength parameter associated with $V_{TR}(R)$ in $^{17}F/^{17}O$ increases with increasing L, and its range of values from -0.69 to -1.15 is indeed very similar to the range for the ^{14}N-α system of -0.62 to -1.02

We conclude that the microscopic calculation is able to

justify the use of Buck and Pilt's $^{18}F/^{18}O$ strength parameters to evaluate ^{14}N-trinucleon cluster spectra in $^{17}F/^{17}O$. It gives good predictions of the effects of the spin-spin and spin-orbit forces, but suggests that a large tensor force varying like $\left[(\underline{S}_1 \cdot \hat{\underline{R}}) - \frac{1}{3}S_1^2\right]$ should also be present in the cluster-core interaction. These calculated forces are very similar to equivalent forces in cluster-core decompositions of neighbouring nuclei.

5.2 Comparison with experiment

The full spectrum of ^{14}N-trinucleon cluster states in the $2N+L=6$ bands in ^{17}F and ^{17}O has been evaluated microscopically and phenomenologically. The results are presented in table 2 and Figure 2. Higher lying bands having $2N+L=7,8,\ldots$ etc. are also possible but were not calculated since there is no experimental data to compare them with. The dominant values of L and S associated with each state are also given in table 2.

The microscopic calculations use the parameters listed in table 3 for the various values of L in conjunction with the central potential of equation (3.4). This has $V_0 = 113.7$ MeV so that the lowest $7/2^+$ state lies at the same energy as its experimental counterpart to facilitate comparison with the

phenomenological and experimental spectra. Mixing of states with equal J but different S by the $\underline{L}.\underline{S}_2$ force, and of states with equal J but different L by the tensor forces was included in the microscopic calculation. However, it was very small, and it still makes sense to describe a state of given J as being formed predominantly by coupling one value of L with one value of S.

The microscopic results give a slightly inferior agreement with the energies of the five levels located experimentally by Etchegoyen et al (1984) than the phenomenology did. This discrepancy can be attributed mainly to an overestimate of the rotational and spin-orbit parameters, A and ξ. This could be reduced somewhat, if the requirement of using the same central potential geometry (and associated spin-orbit force) as for other trinucleon cluster states in neighbouring nuclei was relaxed. This was not done because there is so little experimental data for comparison at the moment that it is not worth obscuring the results by introducing more than a minimum of free parameters (i.e. just the band head, and even that is close to expectations from the $^{14}C-^{3}He$ cluster states in ^{17}O).

The experiment of Etchegoyen et al (1984) was designed so that only states having fairly high spins (L- matching to

states with $J \geq 7/2$ was achieved in practice) were heavily populated. However, many trinucleon cluster states with low spins are also predicted at similar excitation energies n ^{17}F and ^{17}O. At the moment there is not really enough experimental data on states in ^{17}F and ^{17}O above 8 MeV to confidently assign any of these predicted low spin states to experimental counterparts. Nevertheless, by searching through the compilations of Ajzenberg-Selove (1982) for mirror states with spin-parity values and excitation energies compatible with those calculated, we might speculate that the $3/2^+$ at 8.76 MeV in ^{17}F and 8.897 MeV in ^{17}O could correspond to one of our predicted cluster states. It is included in table 2 and Figure 2 on this basis. There is simply too little experimental data at present to do better.

The main difference between the microscopic and phenomenological spectra is due to the large tensor force in the former. Unfortunately, the cluster states located experimentally to date are the very ones which are least sensitive to this force. Therefore a firm confirmation of the presence of such a force must await further experimental studies to locate more ^{14}N-trinucleon cluster states in ^{17}F and ^{17}O. However, we can conclude that very similar trinucleon cluster states have now been seen (and

satisfactorily described with a consistent set of parameters within the framework of the Buck, Dover, Vary cluster model) in all the flourine and oxygen isotopes with masses 17 and 18, as well as in ^{19}F.

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Table 1

A comparison of the phenomenologically predicted energies of trinucleon cluster states in ^{17}F and ^{17}O (using the parameter sets A-E of Buck and Pilt (1978)) with those found in three-nucleon transfer onto ^{14}N by Etchegoyen et al (1984).

$2J^{\pi}$	Energy (MeV)						
	Parameter Set					Experiment	
	A	B	C	D	E	^{17}F	^{17}O
7^+	8.43	8.43	8.43	8.43	8.43	8.43	8.48
11^+	10.48	10.64	10.57	10.56	10.55	10.7	10.7
7^+	11.83	11.79	12.54	11.68	11.79	11.9	12.0
9^+	13.53	13.36	14.32	13.11	13.11	13.51	13.53
15^+	13.92	14.31	14.27	14.09	14.09	14.84	14.88

Table 2

The predicted trinucleon cluster state spectra in ^{17}F and ^{17}O obtained microscopically and with parameter set B.

J^{π}	L	S	Energy (MeV)			
			Theory		Experiment	
			Micro	Set B	^{17}F	^{17}O
$3/2^+$	0	3/2	7.53	7.68		
$1/2^+$	0	1/2	9.54	9.48		
$7/2^+$	2	3/2	8.44	8.43	8.43	8.48
$5/2^+$	2	3/2	8.07	8.83		
$3/2^+$	2	3/2	9.21	9.12	(8.76)	(8.90)
$1/2^+$	2	3/2	10.09	9.29		
$5/2^+$	2	1/2	11.16	10.69		
$3/2^+$	2	1/2	10.46	10.40		
$11/2^+$	4	3/2	10.98	10.64	10.7	10.7
$9/2^+$	4	3/2	10.42	11.27		
$7/2^+$	4	3/2	11.99	11.79	11.9	12.0
$5/2^+$	4	3/2	13.38	12.19		
$9/2^+$	4	1/2	14.43	13.36	13.51	13.53
$7/2^+$	4	1/2	13.12	12.84		
$15/2^+$	6	3/2	15.50	14.31	14.84	14.88
$13/2^+$	6	3/2	14.56	15.17		
$11/2^+$	6	3/2	16.55	15.92		
$9/2^+$	6	3/2	18.73	16.55		
$13/2^+$	6	1/2	19.57	17.49		
$11/2^+$	6	1/2	17.81	16.74		

Table 3

A comparison of the microscopically calculated parameters, α, Σ, β_S and β_R with the values de uced from fit B of Buck and Pilt (1978).

Parameter	Fit B	Microscopic Calculation			
		L=0	L=2	L=4	L=6
α	-0.300	-0.336	-0.324	-0.296	-0.250
Σ	-0.172	-0.244	-0.242	-0.238	-0.231
β_S	-0.147	0.114	0.121	0.137	0.166
β_R	------	-0.688	-0.743	-0.884	-1.148

Figure Captions

Figure 1

A cluster of mass A_2, charge Z_2 and spin S_2 orbits a core of mass A_1, charge Z_1 and spin S_1. The separation of their centres of mass is R.

Figure 2

A comparison of the microscopically, (a), and phenomenologically, (b), predicted energies of the ^{14}N-^{3}He cluster states in ^{17}F with the states excited by Etchegoyen et al (1984). 2J is twice the spin value of a given level. One speculative experimental cluster state candidate is also included in the column labelled 'Expt' and drawn with dot-dash lines. It is the $3/2^+$ at 8.76 MeV (Ajzenberg-Selove 1982). A very similar diagram would also be appropriate for the triton cluster state spectrum in ^{17}O.

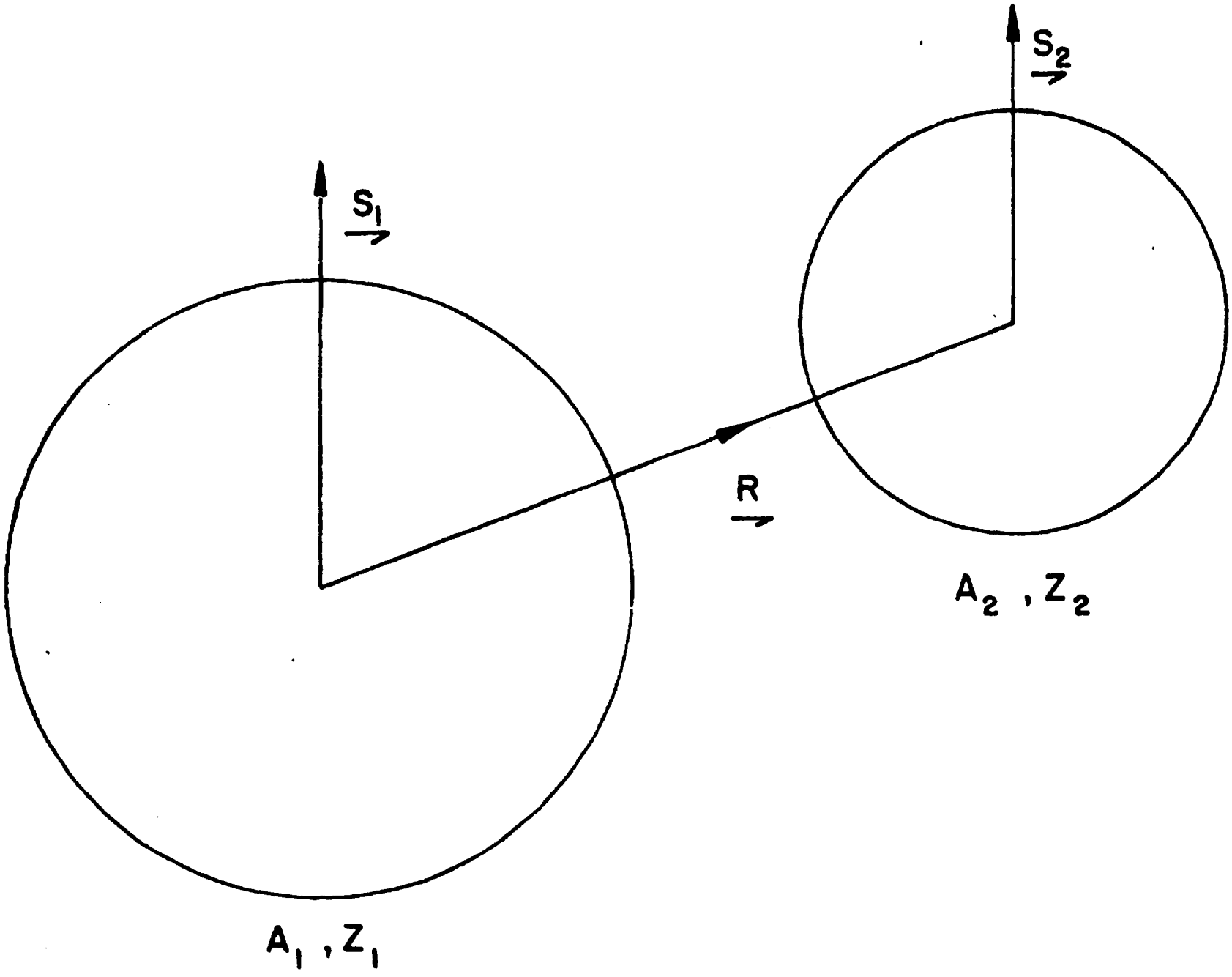
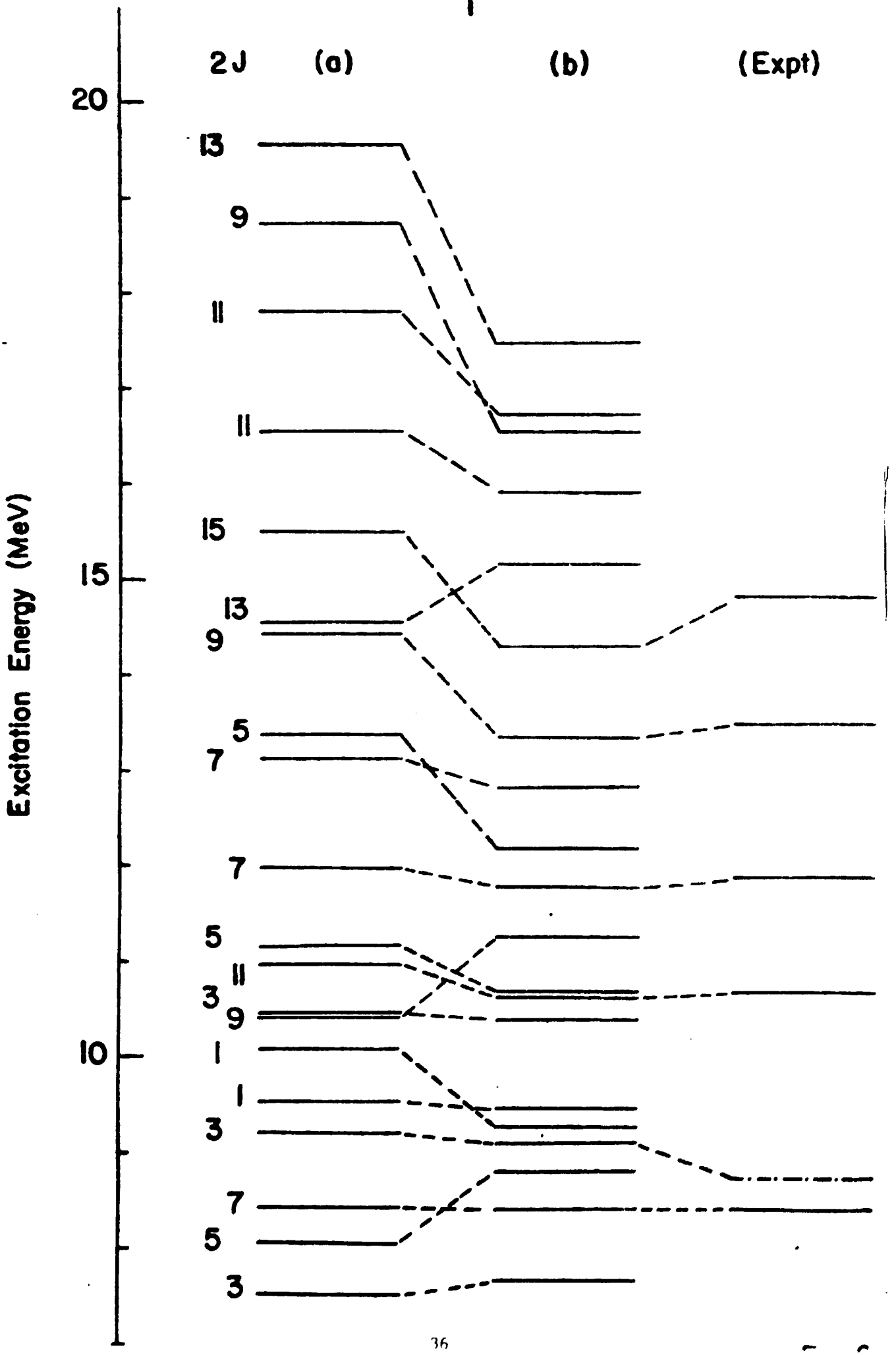


Fig. I


2J
(a)
(b)
(Expt)
Excitation Energy (MeV)
20
15
10
13
9
11
11
15
13
9
5
7
7
5
11
3
9
1
1
3
7
5
3