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S. KOX, A. GAMP, R. CHERKAOUI, A.J. COLE, N. LONGEQUEUE, J. MENET, C. PERRIN and J.B. VIANO

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DIRECT MEASUREMENTS OF HEAVY ION TOTAL REACTION CROSS-SECTIONS AT 30 AND 83 MeV/nucleon

S. Kox, A. Gamp[(a)], R. Cherkaoui[(b)], A.J. Cole,
N. Longequeue, J. Menet, C. Perrin, J.B. Viano

Institut des Sciences Nucléaires (IN2P3-USMG)
53 Avenue des Martyrs - F 38026 Grenoble Cédex FRANCE

ABSTRACT Measurements of σ_R have been performed at 30 and 83 MeV/nucleon incident energy for 15 heavy ion systems using the attenuation method. The data are compared with predictions of microscopic calculations and a semi empirical formula is given for σ_R.

Nuclear reactions ^{12}C + ^{12}C, ^{27}Al, ^{40}Ca, $^{54,56,57}Fe$, $^{64,66,68}Zn$ ^{89}Y at 83 MeV/nucl. ^{12}C + ^{12}C ; ^{20}Ne + ^{12}C, ^{27}Al + ^{56}Fe, ^{64}Zn at 30 MeV/nucl. measured total reaction cross-sections.

(a) also Hahn-Meitner-Institut für Kernforschung Berlin, Germany

(b) Present address : University of Rabat, Rabat, Morocco

1. Introduction

The total reaction cross-section σ_R is one of the most fundamental quantities characterizing nuclear reactions. We recently reported a study of the variation of σ_R with energy for the $^{12}C + ^{12}C$ system[1], which showed clearly the deviation of σ_R from the geometrical limit. This effect is well known for nucleon-nucleus collisions for which systematic measurements have been carried out over a wide range of energy and target [2-5]. More recently DeVries et al. reported the same observation for the $\alpha + ^{12}C$ system [6]. For these light systems DeVries et al. [6,7,8] succeded in reproducing the trend of $\sigma_R(E)$ with microscopic calculations in which the energy dependance of σ_R was linked with that of the average nucleon-nucleon total cross-section. It is thus of interest to know if such deviations are also systematically present for heavier systems and to what extent nuclear reactions for heavy ion collisions may be explained in terms of individual nucleon-nucleon interactions. For heavy ion systems extensive data for σ_R exist only below 20 MeV/nucl. [9-12] and a few results are available at higher energy [13-14]. Values of σ_R are generally extracted from elastic scattering data and are thus to some extent model dependant. It was therefore considered worthwhile to obtain direct measurements using the attenuation method [15] in the energy region 20-200 MeV/nucl. In this paper we present measurements of σ_R for various systems at two bombarding energies (30 and 83 MeV/nucl).

Section 2 will briefly resume the attenuation method and experimental set up. The results are reported in section 3. In section 4 we present the microscopic calculations developped by Karol [16] which are compared and discussed with data in section 5. In section 6 we propose a new empirical parameterization for σ_R.

2. Attenuation method and experimental set up

The total reaction cross-section σ_R is proportional to the difference between the number of incoming beam particles and the corresponding number of outgoing particles after the target not having undergone a reaction, i.e. elastically scattered particles and residual beam. In the attenuation method [15] the direct and simultaneous measurement of these two numbers is thus sufficient to obtain σ_R (given the target thickness). A detailed description of the set up has been reported elsewhere [1,17] and we therefore present only a brief description of the main characteristics of our experiment. The beam is defined and counted before the target by two detectors (counter 1 and active collimator 2). After the target the outgoing particles are detected by a mosaïc (wheel) of 19 counters. The function of the mosaïc is to identify and to count the residual beam and the elastically scattered particles. This "wheel" is made up of a central detector placed in the beam surrounded by two concentric rings of 6 and 12 counters respectively. All the 21 detectors are formed by plastic scintillators optically coupled to fast phototubes. The atomic charge of scattered particles detected after the target is identified by measuring the time of flight between counter 1 (start) and any counter of the mosaic (stop) and the corresponding light output produced by energy loss (ΔE) in the stop detectors. In spite of the fact that the beam intensity is limited to $10^4 - 10^5\ s^{-1}$ (by the count rate of the detectors placed in the beam) a measurement can be achieved in a few hours for a given system. The limitation of solid angles covered by our mosaic (less than 4π) and of identification of scattered particles give rise to corrections which are detailed in ref. 1. It should be emphasized that for measurements presented herein the sum of these corrections never exceeded 10 % of the final value of σ_R.

The experiments reported in this work were performed with the ^{12}C beam of the synchrocyclotron at CERN (83 MeV/nucl) and with ^{12}C and ^{20}Ne beams delivered by the SARA facility of the ISN, Grenoble (30 MeV/nucl).

3. Experimental results

The systems studied are as follow :

^{12}C + ^{12}C, ^{27}Al, ^{40}Ca, $^{54,56,57}Fe$, $^{64,66,68}Zn$, ^{89}Y at 83 MeV/nucl.

^{12}C + ^{12}C ; ^{20}Ne + ^{12}C, ^{27}Al, ^{56}Fe, ^{64}Zn at 30 MeV/nucl.

The resulting values of σ_R are plotted with their corresponding error bars in figure 1 and numerical values are presented in table 1. The figure shows clearl a change in the dependance of σ_R with the mass numbers of the colliding nuclei when the beam energy increases from 30 to 83 MeV/nucl. Each set of values can be reasonably described by a linear dependance on $(A_P^{1/3} + A_T^{1/3})^2$ but the slope changes significantly. This change (corresponding to a decrease of σ_R with energy) can be interpreted as an energy dependant transparency in nuclear reactions for heavy ion collisions. This effect was already observed for the $^{12}C + ^{12}C$ system [1]. For this system microscopic calculations [8] reproduced the variation of σ_R [1] and the increasing transparency was related to the energy dependance of the nucleon-nucleon total cross-section Figure 1 shows that a transparency is observed systematically over a wide range of mass of the colliding nuclei.

4. Microscopic calculations

In light systems, the observed transparency has been explained via microscopic calculations as resulting from the decrease with increasing energy of the nucleon-nucleon total cross-section. For the systems studied herein the corresponding calculations were not available. However the basic ingredients of such calculations appear in a model used successfully by Karol at much higher energies [16]. The basic assumption made in this model is that nuclear reactions are produced by individual nucleon-nucleon interactions occuring in the volume of overlap of the colliding nuclei. Semi-classicaly σ_R can be written as a function of the impact parameter b :

$$\sigma_R = \int_0^\infty 2\pi b\ (1-T(b))\ db \qquad (1)$$

Karol obtained the expression (2) for the transparancy function T(b) by using the concept of mean free path of a nucleon in nuclear matter and adapting it for heavy ion collisions involving straight line trajectories.

$$T(b) = \exp(-\overline{\sigma_T^{nn}} \int_{-\infty}^{+\infty} \rho_t(b,z)\ \rho_p(b,z)\ dz) \qquad (2)$$

where z is a coordinate in the beam direction which specifies with the impact parameter b, the separation $r = \sqrt{b^2 + z^2}$ of the mass centres of projectile and target and $\overline{\sigma_T^{nn}}$ is the isospin averaged nucleon-nucleon total cross-section. The integrand in eq.(2) represents the overlap of the density distributions of the nuclei.

It was shown in ref. 16 that only the tail of the nuclear densities (i.e. large impact parameters) gives rise to values of T(b) different from 0. Thus an adequate description of T(b) was obtained by fitting the tails of realistic nuclear density distributions by gaussian functions

$$\rho(r) = \rho(0)\ \exp(-r^2/a^2) \qquad (3)$$

where $\rho(0)$ and a were calculated using semi empirical formulae (see ref. 16 for more details).

With these densities an analytical form is obtained for T(b) :

$$T(b) = \exp - [\ \pi^2\, \overline{\sigma_T^{nn}}\, \frac{\rho_t(0)\rho_p(0)a_p^3 a_t^3}{(a_t^2 + a_p^2)}\ \exp\ (-\frac{b^2}{a_t^2 + a_p^2})\,] \qquad (4)$$

This expression developped by Karol to reproduce data at relativistic energies [16] can also be obtained with the optical limit of Glauber theory [18]. As our results have been obtained at medium energy we decided to take into account the Coulomb forces by substituting $T(b')$ for $T(b)$ in formula 1 with $b'(b)$ being the classical distance of closest approach. Thus σ_R is given by :

$$\sigma_R = \int_0^\infty 2\pi b \; (1 - T(b'))db \tag{5}$$

The root mean square radii used in the semi empirical formulae of ref. 16 have been taken from the litterature [19]. $\overline{\sigma_T^{nn}}$ was obtained as the isospin averaged cross-section of the free nucleon-nucleon total cross-section [20].

5. Results and discussion of microscopic calculations

The predictions of the calculations are compared with experimental data in figure 1. The broken curve is drawn to show the general trend of the predictions. An overall agreement is clearly observed over the whole range of mass for the two energies. Thus simple microscopic calculations describe the trend of σ_R for heavy ion collisions even at low energies where the optical limit of Glauber theory is probably inaccurate. The small differences observed between predictions and data could be due to the fact that we have not included the Pauli blocking effect in the calculations. The effects of Fermi motion, Pauli blocking and the nuc[illegible] potential have been included for nucleon-nucleus system by Di Giacomo and al. [7] who observed an improvment of the description of the data. More refined calculations including these effects for heavy ion collisions are under consideration. The overall agreement suggests strongly that the transparency observed in this study for heavy ion systems over a wide range of mass can be related to the behaviour of $\overline{\sigma_T^{nn}}$ as for light projectiles. It should be noted that the predictions of the calculations are only sensitive to geometrical properties and to the parameters (nuclear densities and nucleon-nucleon total cross-section) at the surface of the colliding nuclei. Thus the total reaction cross-section σ_R is a quantity which yields information essentially for large impact parameters so that questions concerning the relevance of the simple model used herein at small impact parameters cannot be answered.

As mentioned above the starting point in such calculations [8,1] resides in the definition of the mean free path for nucleons in nuclear matter.

Microscopically the mean free path Λ of a nucleon in nuclear matter of constant density ρ is defined as

$$\Lambda = (\rho \, \overline{\sigma_T^{nn}})^{-1} \qquad (6)$$

Here $\overline{\sigma_T^{nn}}$ is the isospin averaged nucleon-nucleon total cross-section.

This simple definition of mean free path does not apply to the case of the collision of nuclei having a realistic (i.e. not a constant sharp edged) density. For this case a local mean free path can be defined in the overlap region of the nucleon densities at each step of the collision. This quantity $\Lambda(b,z)$ is implicitely contained in the microscopic calculations developped by Karol [16].

$$\Lambda(b,z) = \left(\overline{\sigma_T^{nn}} \int_{-\infty}^{+\infty} 2\pi d\eta \int_0^{\infty} \rho_t(b,z,\eta,r)\rho_t(b,z,\eta,r)\ r dr\right)^{-1}$$

$$= \left(\overline{\sigma_T^{nn}}\ \rho(b,z)\right)^{-1} \tag{7}$$

where ρ_t, ρ_p are the nuclear densities of the colliding nuclei, η is the distance between the two mass centres and r is the projected distance of any point onto the η axis. The coordonate z is taken in the beam direction as defined in figure 4 together with the impact parameter b. The success of the model in predicting σ_R values indicates that equation (7) is at least correct for large impact parameters for which $(1-T(b)) \ll 1$. As an exemple we have calculated values of $\Lambda(b,z)$ for the system $^{12}C + {}^{12}C$ at 83 MeV/nucl. choosing the impact parameter $b_{1/e}$ such that

$$T(b_{1/e}) = 1/e \tag{8}$$

The value of $b_{1/e}$ is 5.4 fm and the dependance of $\Lambda(b,z)$ on z is shown in figure 2. For the highest overlap densities (z = 0), the local mean free path is of the order of 2 fm and increases for larger z tending to infinity as $\rho(b,z)$ tends to zero.

Finally we remark that the model may be extended to calculate other observables. Thus Chauvin et al. [21] have fitted elastic scattering data with the same parameters we used to calculate values of σ_R. They have obtained a nice agreement with $^{12}C + {}^{12}C$ data at 30 and 83 MeV/nucl.

6. Empirical parameterization for σ_R

As they reproduce the trend of σ_R, microscopic calculations can be used to study the origins of the variation of σ_R with energy and to establish a parametrization of σ_R as a function of the masses of the colliding nuclei. For this study we also considered other data from the litterature [2,6,13]. In all 47 distinct cases taken into account a good overall agreement between microscopic calculations and the data has been obtained as shown in fig. 4d. Figure 3 shows the calculated value of (1-T(b)) at two energies (30 and 83 MeV/nucl) for two sets of 3 systems each having approximately the same value of $(A_p^{1/3} + A_t^{1/3})^2$. For each system the dashed area represents the region of impact parameter becoming transparent when the beam energy increases from 30 to 83 MeV/nucl. This region of transparency is clearly located at the higher impact parameters and the form and the thickness of this region depend weakly on the system studied. A second observation is that σ_R for a given value of $(A_p^{1/3} + A_t^{1/3})^2$ depends on the mass assymetry of the system. This can be explained by a geometrical argument. For given values of impact parameter b and $(A_p^{1/3} + A_t^{1/3})^2$ the volume of overlap depends on the mass assymetry of the system. As this volume increases with the symetry of the system, reactions can occur at higher impact parameters for nearly symetric systems such as $^{20}Ne + {}^{12}C$ than for more assymetric ones such as $^{1}H + {}^{64}Cu$ and $^{4}He + {}^{40}Ca$. These observations can be used to improve the usual parametrization of σ_R

$$\sigma_R = \pi r_0^2 \,(A_p^{1/3} + A_t^{1/3} - x)^2 \,(1 - \frac{B_C}{E_{CM}}) \qquad (9)$$

This formula is a modification of the black disk model [22] and is used without the Coulomb term at relativistic energies [13].

As remarked by Karol [16] the overlap term x must be varied for each system to reproduce the data.

We propose to replace the overlap term by two distinct terms in order to reproduce the data at a given energy with an unique set of parameters. These two terms will respectively describe the transparency effect and the mass assymetry effects. The proposed parametrization of σ_R is

$$\sigma_R = \pi r_0^2 \left(A_p^{1/3} + A_t^{1/3} + \frac{a\, A_p^{1/3} \cdot A_t^{1/3}}{A_t^{1/3} + A_p^{1/3}} - c\right)^2 \left(1 - \frac{B_C}{E_{CM}}\right) \qquad (10)$$

The assymetry term has been parametrized by observing that the volume overlap increases nearly as $A_p^{1/3}\, A_t^{1/3}$ for a given value of $(A_p^{1/3} + A_t^{1/3})$.

The term c is energy dependant and is related to the thickness of the region of impact parameter becoming transparent for nuclear reactions as the energy increases.

The improvment due to the assymetry term can be observed in *figure 4. Figure 4a represents the ratio of experimental values of σ_R with that* calculated using formula 9 with r_0 = 1.25 fm and x = 0. Figure 4b shows the influence of changing x with energy to improve the fit with formula 9. It is clear that an unique set of values of r_0 and x in formula 9 at a given energy cannot describe correctly data obtained with different types of projectile. Finally the figure 4c shows the same ratio but with σ_R calculated with formula (10) using the parameters listed in table 2. An overall agreement is observed for this great variety of systems. The most important discrepancies are for systems for which the Coulomb correction is very important ($(1 - B_C/E_{CM}) < 0.7$). For these systems, (mainly p + ^{120}Sn, ^{208}Pb, at 30 MeV/nucl) the effect of the nuclear potential should be taken into account. The effect of the nuclear potential has been taken into account for p + ^{208}Pb and p + ^{120}Sn at 30 MeV/nucl. in the microscopic calculations using the parametrization of the nuclear potential given by Menet [4]. Apart from this problem formula (10) seems to be able to reasonably reproduce σ_R values over a wide range of mass and energy.

Summary and conclusions

We have measured σ_R for heavy ion collisions at two bombarding energies (30 and 83 MeV/nucl). A transparency for nuclear reactions has been clearly established over a wide range of mass of the colliding systems. A comparison between data and predictions of a microscopic calculation developped by Karol has been performed. The overall agreement suggests strongly that as for lighter projectiles the transparency of σ_R can be linked with the behaviour of the total cross-section for nucleon-nucleon collisions σ_T^{nn}. The predictions of the microscopic calculations have shown that the region of impact parameter becoming transparent with increasing energy is located at the surface of the colliding nuclei and that the thickness of this region is nearly independant of the system studied. A mean free path Λ for $^{12}C + ^{12}C$ at 83 MeV/nucl. in the transparent surface region of typically 2 fm was found. A dependance of σ_R with the assymetry of the colliding system is also observed and has been explained with pure geometrical arguments. All these observations were employed to establish a new parametrization of σ_R for all masses and energies in which only one parameter is energy dependant.

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Table captions

Table 1

Numerical values of total reaction cross-sections σ_R measured using the attenuation method.

Table 2

Parameters of formula (10) obtained with a χ^2 minimization fitting procedure. The data used in addition to ours for this study are extracted from the references indicated opposite each incident energy.

Table 1

System	σ_R (mb)
E_{lab}/ nucleon = 30 MeV/nucl	
$^{12}C + ^{12}C$	1315 ± 40
$^{20}Ne + ^{12}C$	1550 ± 100
$^{20}Ne + ^{27}Al$	2130 ± 120
$^{20}Ne + ^{56}Fe$	2770 ± 200
$^{20}Ne + ^{64}Zn$	2850 ± 180

System	σ_R (mb)
E_{lab}/ nucleon = 83 MeV/nucl	
$^{12}C + ^{12}C$	960 ± 30
$^{12}C + ^{27}Al$	1400 ± 40
$^{12}C + ^{40}Ca$	1550 ± 60
$^{12}C + ^{54}Fe$	1815 ± 100
$^{12}C + ^{56}Fe$	1810 ± 100
$^{12}C + ^{57}Fe$	1820 ± 80
$^{12}C + ^{64}Zn$	1900 ± 140
$^{12}C + ^{66}Zn$	2010 ± 140
$^{12}C + ^{68}Zn$	2120 ± 140
$^{12}C + ^{89}Y$	2020 ± 160

Table 2

$E_{lab.}$ (MeV/nucl)	r_0(fm)	a	c
30 [4,6,7]	1.05	1.9	0.6
83 [6,7]	1.05	1.9	1.4
600-900 [5,13]	1.05	1.9	1.9

Figure captions

Figure 1

Plot of the experimental data with corresponding error bars and of the predictions of the microscopic calculations based on the formalism developped by Karol[16]. The lines are drawn to guide the eye and to illustrate the general trend of data and predictions.

Figure 2

Local density $\rho(b,z)$ and local mean free path $\Lambda(b,z)$ of nucleonsin the region of volume overlap of the system $^{12}C + {}^{12}C$ at 83 MeV/nucl as a function of the coordinate z. The impact parameter has been fixed to $b_{1/e} = 5.4$ fm where $T(b_{1/e}) = 1/e$.

Figure 3

Variation of the reaction probability $(1-T(b))$ as a function of the impact parameter b for E_{lab}/nucl. = 30 and 83 MeV/nucl. Each set of three systems has approximately the same value of $(A_t^{1/3} + A_p^{1/3})^2$. The dashed area represents the transparent region and the vertical line is drawn to show the variation of the mean position of this region with the mass assymetry of the system.

Figure 4

Ratio of the experimental to the calculated value of σ_R. The calculated values of σ_R are respectively

3a - $\pi r_0^2 (A_t^{1/3} + A_p^{1/3})^2 (1 - B_C/E_{CM})$ $r_0 = 1.25$ fm

3b - overlap model

$\pi r_0^2 (A_t^{1/3} + A_p^{1/3} - x)^2 (1 - B_C/E_{CM})$ $r_0 = 1.4$ fm $x = 0.1$ 30 MeV/nucl.

$x = 0.7$ 83 MeV/nucl.

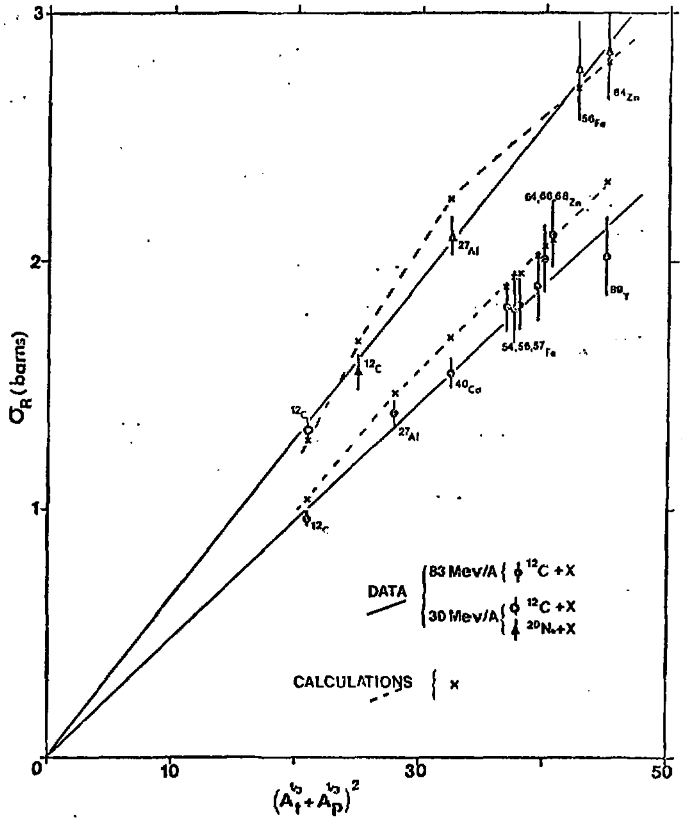
$x = 1.3$ 600-900 MeV/nucl.

3c - formula (7) using the parameters listed table 2

3d - microscopic calculations based on the formalism of Karol [16].

The parameters used for the figure 3b and 3c have been obtained using a X^2 minimization fitting procedure.

Fig 1



σR (barns)
3
2
1
0
10
30
50
(At^1/3 + Ap^1/3)^2
DATA
83 Mev/A { 12C + X
30 Mev/A { 12C + X
20Ne + X
CALCULATIONS
12C
27Al
40Ca
56Fe
64Zn
64,66,68Zn
54,56,57Fe
89Y

Fig 2

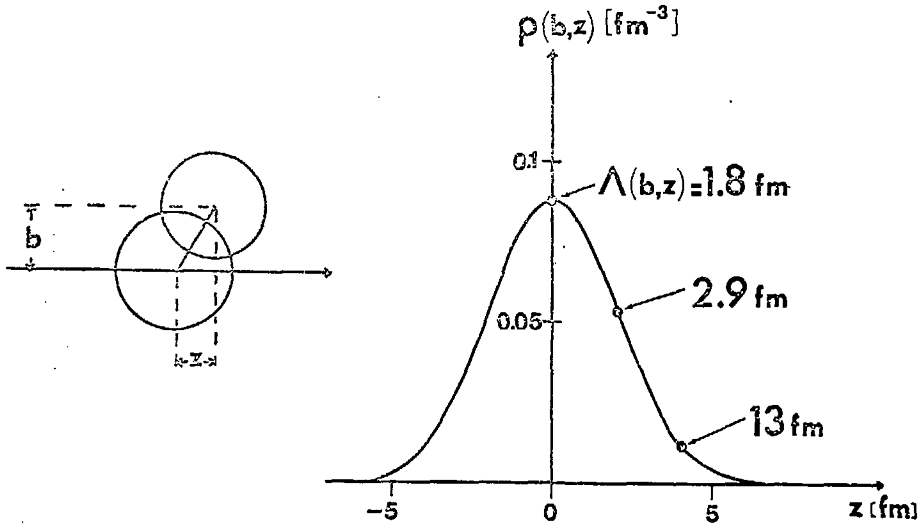


Fig 3

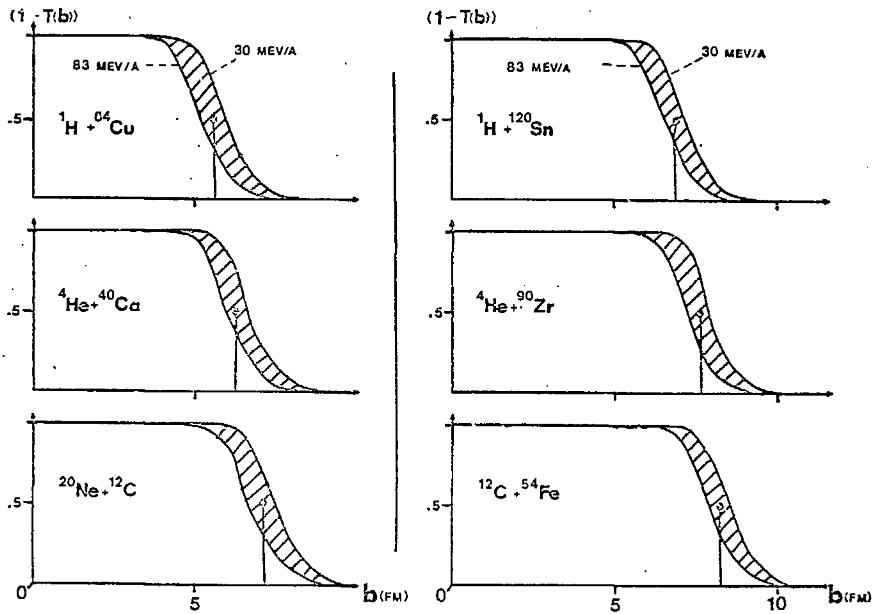
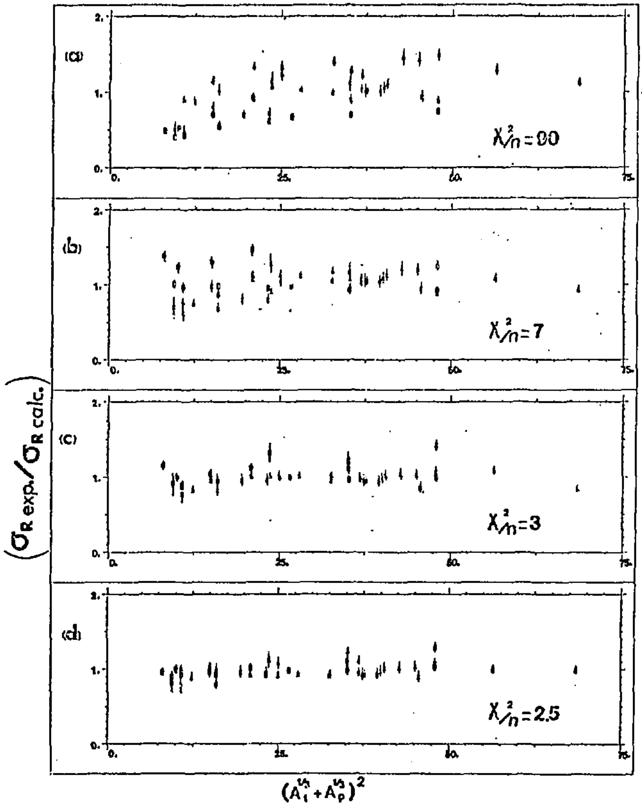

(1 - T(b))
83 MEV/A
30 MEV/A
$^{1}H + ^{64}Cu$
$^{4}He + ^{40}Ca$
$^{20}Ne + ^{12}C$
(1 - T(b))
83 MEV/A
30 MEV/A
$^{1}H + ^{120}Sn$
$^{4}He + ^{90}Zr$
$^{12}C + ^{54}Fe$
.5
0
5
10
b (FM)

Fig 4



(a)
(b)
(c)
(d)
$\chi^2/n = 7$
$\chi^2/n = 3$
$\chi^2/n = 2.5$
0.
1.
2.
25.
50.
75.
$(\sigma_{R\,exp.}/\sigma_{R\,calc.})$
$(A_t^{1/3} + A_p^{1/3})^2$