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ABSTRACT

Making use of the generating functional of the non-Abelian flavour anomalies of QCD we construct a gauge invariant phenomenological Lagrangian of pseudoscalar and vector mesons, which is equivalent to the extended Wess-Zumino Lagrangian in the low energy approximation. The gauge kinetic term of the hidden local symmetry is necessarily present and the gauge coupling constant is determined by the equivalence.

АННОТАЦИЯ

Построено калибровочно-инвариантный эффективный Лагранжиан псевдоскалярных и векторных мезонов с помощью неабелевой аномалии. Он является эквивалентным с распространенным Лагранжианом Весса и Зумино в низкоэнергическом приближении. Кинетическая энергия калибровочных бозонов скрытой симметрии, появляется автоматически в эффективном Лагранжиане и константа связи эффективной калибровочной теории определена эквивалентностью.

KIVONAT

A nemábeli anomáliák generátorfüggvénye segitségével megkonstruáljuk a pszeudoskalár- és vektormezonokat egyaránt leiró mértékinvariáns fenomenológiai Lagrange-függvényt. Megmutatjuk, hogy ez ekvivalens a kiterjesztett Wess-Zumino féle effektiv hatással az alacsonyenergiás közelitésben. Az ekvivalencia következtében automatikusan megjelenik a Lagrange-függvényben a rejtett lokális szimmetriák mértékbozonjaihoz tartozó kinetikus energia,és az effektiv mértékelmélet csatolási állandója meghatározható.

In a recent paper [1] it was shown that $\mathcal{L}_0$, the usual non-linear σ-model Lagrangian based on the manifold G/H where $G = SU_L(2) \times SU_R(2) \times U(1)$ and $H = U_V(2)$ is equivalent to a Lagrangian L_0 possesing $G_{global} \times H_{local}$ symmetry. The gauge bosons of the hidden local symmetry H_{local} has been successfully identified with the vector mesons ρ , ω . The equivalence holds in the absence of a gauge kinetic term for the vector bosons. The main assumption of [1] was that this term is somehow generated by the underlying dynamics of QCD. After adding this term to L_0 by hand, the vector bosons became dynamical but the exact equivalence between the two theories has been lost. However, their equivalence was still valid as the zeroth order approximation in a low energy expansion.

In the present paper we will show that the emergence of the gauge kinetic terms is the consequence of flavour anomalies of QCD. Following the method of Wess and Zumino [2] but making use of both the usual, Bardeen, and the "spurious", non-topological, anomalies the effective Lagrangian $\mathcal{L}_1$ has been constructed [3,4]. The full Lagrangian $\mathcal{L}_0 + \mathcal{L}_1$ describes the interactions of pseudoscalar mesons in the presence of external electroweak fields up to first order in the low energy approximation [5]. We shall construct the locally gauge invariant Lagrangian L_1 corresponding to $\mathcal{L}_1$. L_1 automatically contains the gauge kinetic terms and is almost uniquely determined by the equivalence.

First we extend the quadratic Lagrangian L_0 of [1] by introducing in addition to the gauge vector field V_μ an axial vector field A_μ as well:

$$L_0 = -\frac{1}{4} f_\pi^2 \, \mathrm{Tr}\{(l_\mu - r_\mu)(l^\mu - r^\mu)\} - \frac{a}{4} f_\pi^2 \, \mathrm{Tr}\{(l_\mu + r_\mu + 2V_\mu)(l^\mu + r^\mu + 2V^\mu)\} - \frac{b}{4} f_\pi^2 \, \mathrm{Tr}\{(l_\mu - r_\mu + 2A_\mu)(l^\mu - r^\mu + 2A^\mu)\}. \qquad (1)$$

Here

$$\ell_\mu = -\xi_L \partial_\mu \xi_L^+ - \xi_L (\mathcal{V}_\mu + \mathcal{A}_\mu)\xi_L^+$$
$$r_\mu = -\xi_R \partial_\mu \xi_R^+ - \xi_R (\mathcal{V}_\mu - \mathcal{A}_\mu)\xi_R^+ \qquad (2)$$

where ξ_L and ξ_R are U(2) valued scalars satisfying the constraint det ξ_L = det ξ_R. $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are matrix valued flavour vector and axial external fields, respectively. Under the $G_{global} \times H_{local}$ symmetry our fields are transformed[+] as follows:

$$\xi'_{L/R}(x) = h(x)\,\xi_{L/R}(x)\,g_{L/R}\,, \qquad h(x) \in H_{local}$$
$$V'_\mu(x) = h(x)\,V_\mu(x)\,h^+(x) + h(x)\partial_\mu h^+(x), \quad (g_L, g_R) \in G_{global} \qquad (3)$$
$$A'_\mu(x) = h(x)\,A_\mu(x)\,h^+(x).$$

The gauge is fixed by demanding

$$\xi_L = \xi_R^+ = \xi\,, \qquad (4)$$

where ξ can be expressed in terms of the chiral Goldstone bosons:

$$\xi = e^{-\frac{i}{2}P}\,, \quad P = \frac{1}{f_\pi}\pi^a\sigma^a \qquad a = 1,2,3\,, \quad f_\pi = 93\,\text{MeV}. \qquad (5)$$

Using (4),(5) and the identification

$$\mathcal{U} = \xi^{+2}$$

+ of course the presence of $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ breaks G_{global}. However, part of it can be gauged, supplementing (3) by the corresponding transformations on $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$. Though in the applications we shall consider its U(1) subgroup only, corresponding to the interactions of mesons and photons, the general expression (2) proves to be useful in what follows.

the first term in (1) becomes the gauged σ -model Lagrangian:

$$\mathcal{L}_0 = \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\{ D_\mu U^+ D^\mu U \} , \tag{6}$$

where $D_\mu U = \partial_\mu U + \mathcal{L}_\mu U - U \mathcal{R}_\mu$, $\mathcal{L}_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu$, $\mathcal{R}_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu$.

The second and the third terms in (1) are identically zero if we use for V_μ and A_μ the classical equations of motion:

$$\begin{aligned} V_\mu^{cl} &= -\frac{1}{2}(\ell_\mu + r_\mu) \\ A_\mu^{cl} &= -\frac{1}{2}(\ell_\mu - r_\mu) . \end{aligned} \tag{7}$$

Thus $\mathcal{L}_0$ and L_0 are equivalent, though the latter contains two, so far arbitrary, constants a and b . However, when the gauge kinetic term

$$L_{kin} = \frac{1}{2g^2} \mathrm{Tr}\{W_{\mu\nu} W^{\mu\nu}\} , \qquad W_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] \tag{8}$$

is added, the special value $a = 2$ is preferred. As it was shown in [1] this choice yields

(a) $g_{\rho\pi\pi} = g$ $\quad\rho$ universality [6],

(b) $m_\rho^2 = 2 g_{\rho\pi\pi}^2 f_\pi^2$ $\quad$ KSRF relation [7], $\qquad$ (9)

(c) vector meson dominance in $\gamma\pi\pi$ coupling.

Now we turn to the construction of L_1. We recall that the calculation of the effective Lagrangian $\mathcal{L}_1$ proceeds in two steps [3,8,9]. First one looks for local functionals $Z_{WZ}[\mathcal{V}_\mu, \mathcal{A}_\mu]$ and $Z_{NT}[\mathcal{V}^\mu, \mathcal{A}^\mu]$ whose chiral variations are the Bardeen [10] and the non-topological [11] anomalies, respectively. Then the gauged effective Lagrangian is $\mathcal{L}_1 = \mathcal{L}_{WZ} + \mathcal{L}_{NT}$ with

$$\begin{aligned} \mathcal{L}_{WZ}(U, \mathcal{V}_\mu, \mathcal{A}_\mu) &= Z_{WZ}[\tilde{\mathcal{V}}_\mu^{\xi^+}, \tilde{\mathcal{A}}_\mu^{\xi^+}] - Z_{WZ}[\mathcal{V}_\mu, \mathcal{A}_\mu] \\ \mathcal{L}_{NT}(U, \mathcal{V}_\mu, \mathcal{A}_\mu) &= Z_{NT}[\tilde{\mathcal{V}}_\mu^{\xi^+}, \tilde{\mathcal{A}}_\mu^{\xi^+}] - Z_{NT}[\mathcal{V}_\mu, \mathcal{A}_\mu] \end{aligned} \tag{10}$$

where $\tilde{\mathcal{V}}_\mu^{\xi^+}$ and $\tilde{\mathcal{A}}_\mu^{\xi^+}$ are the chiral transforms of $\mathcal{V}^\mu$ and $\mathcal{A}_\mu$ with respect to $\xi^+ = \sqrt{\mathcal{U}}$:

$$\begin{aligned} \tilde{\mathcal{L}}_\mu^{\xi^+} &= \xi \mathcal{L}_\mu \xi^+ + \xi \partial_\mu \xi^+ \\ \tilde{\mathcal{R}}_\mu^{\xi^+} &= \xi^+ \mathcal{R}_\mu \xi + \xi^+ \partial_\mu \xi . \end{aligned} \tag{11}$$

The Z functionals are determined only up to chiral invariant terms, but $\mathcal{L}_{WZ}$ and $\mathcal{L}_{NT}$ do not depend on this ambiguity. We note that Z_{NT} and Z_{WZ} are vector gauge invariant. Z_{NT} is given by [3,11]:

$$\begin{aligned} Z_{NT} = \frac{N_c}{12\pi^2} \mathrm{Tr} \Big\{ & \frac{1}{4} \mathcal{W}_{\mu\nu} \mathcal{W}^{\mu\nu} - \mathcal{A}_\mu \mathcal{A}_\nu \mathcal{W}^{\mu\nu} - \frac{1}{2} (\mathcal{D}_\mu \mathcal{A}^\mu)^2 \\ & - \frac{1}{2} \mathcal{A}_\mu \mathcal{A}_\nu \mathcal{A}^\mu \mathcal{A}^\nu + \frac{3}{2} (\mathcal{A}_\mu \mathcal{A}^\mu)^2 \Big\} . \end{aligned} \tag{12}$$

Z_{WZ} is local only in five dimensions. In the language of differential forms it is given by [9] :

$$Z_{WZ}^{(5)} = \frac{-iN_c}{12\pi^2} \mathrm{Tr} \Big\{ 3\mathcal{W}^2 \mathcal{A} + 2\mathcal{W}\mathcal{A}^3 + (\mathcal{D}\mathcal{A})^2 \mathcal{A} + \frac{3}{5} \mathcal{A}^5 \Big\} . \tag{13}$$

We look for $G_{global} \times H_{local}$ invariant Lagrangians L_{NT} and L_{WZ} which are equivalent to $\mathcal{L}_{NT}$ and $\mathcal{L}_{WZ}$ up to first order in the low energy approximation. That is we have to require

$$\begin{aligned} \mathcal{L}_{NT}(\mathcal{U}, \mathcal{V}_\mu, \mathcal{A}_\mu) &= L_{NT}(\ell_\mu, r_\mu, V_\mu^{cl}, A_\mu^{cl}) \\ \mathcal{L}_{WZ}(\mathcal{U}, \mathcal{V}_\mu, \mathcal{A}_\mu) &= L_{WZ}(\ell_\mu, r_\mu, V_\mu^{cl}, A_\mu^{cl}) \end{aligned} \tag{14}$$

Apart from uninteresting selfcouplings of the external fields (14) is satisfied if we choose

$$\begin{aligned} L_{NT} &= Z_{NT}[V_\mu, A_\mu] + Z_{INV}^{(4)}[V_\mu, A_\mu] \\ L_{WZ} &= Z_{WZ}^{(5)}[V_\mu, A_\mu] + Z_{INV}^{(5)}[V_\mu, A_\mu] , \end{aligned} \tag{15}$$

since from (7) and (11) we have

$$V_\mu^{cl} = \tilde{\mathcal{V}}_\mu^{\xi^+} , \qquad A_\mu^{cl} = \tilde{\mathcal{A}}_\mu^{\xi^+} . \tag{16}$$

$Z^{(4)}_{INV}$ and $Z^{(5)}_{INV}$ denote chiral invariant contributions to (12) and (13), respectively. They do not contribute to $\mathcal{L}_1$ but they are present in L_1 in general. The most general choice for them is

$$Z^{(4)}_{INV} = \frac{\varkappa}{32\pi^2} \mathrm{Tr}\left\{ L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \right\}, \quad Z^{(5)}_{INV} \equiv 0 \quad , \tag{17}$$

where $L_{\mu\nu}$ and $R_{\mu\nu}$ are the field strength tensors of $L_\mu = V_\mu + A_\mu$ and $R_\mu = V_\mu - A_\mu$.

Thus our complete Lagrangian

$$L = L_0 + L_{NT} + L_{WZ} \tag{18}$$

contains the arbitrary constants a, b and $\varkappa$. We choose $a = 2$ as in [1] to maintain the property (9c). Moreover, we choose $\varkappa = 1$ in order to cancel the strong momentum dependence in the $\rho\pi\pi$ coupling thus maintaining properties (9a) and (9b) as well. The parameter b can be determined from the axial meson mass.

With this choice we have

$$\begin{aligned} L_{NT}(\varkappa=1) = \frac{1}{4\pi^2} \mathrm{Tr} \Big\{ & \frac{1}{2} W_{\mu\nu} W^{\mu\nu} + \frac{1}{4} (D_\mu A_\nu - D_\nu A_\mu)(D^\mu A^\nu - D^\nu A^\mu) \\ & - \frac{1}{2}(D_\mu A^\mu)^2 + (A_\mu A^\mu)^2 \Big\} \end{aligned} \tag{19}$$

$$L^{(5)}_{WZ} = \frac{-i}{4\pi^2} \mathrm{Tr} \left\{ 3W^2 A + 2WA^3 + (DA)^2 A + \frac{3}{5} A^5 \right\}.$$

The physical vector and axial fields are ρ^a_μ, $\tilde{A}^a_\mu$, where

$$\begin{aligned} V_\mu &= -\frac{ig}{2} \rho^a_\mu \sigma^a \\ \tilde{A}_\mu &= -\frac{ig_A}{2} \tilde{A}^a_\mu \sigma^a . \end{aligned} \qquad a = 0,1,2,3. \tag{20}$$

Here

$$\tilde{A}_\mu = A_\mu - \frac{i}{2} \partial_\mu P \quad . \tag{21}$$

These fields together with π^a diagonalize the quadratic part of L.

From (19) we see that the vector and axial vector kinetic terms are necessarily present in L_1 as a consequence of the equivalence between L_1 and the extended chiral Lagrangian $\mathcal{L}_1$. Moreover, the values of the vector and axial coupling constants are also fixed by the equivalence. They can be read off from (19) :

$$g = 2\pi \qquad g_A = 2\sqrt{2}\,\pi \quad . \tag{22}$$

From (1) and (22) we can calculate the ρ mass:

$$m_\rho = 2\sqrt{2}\,\pi f_\pi = 826\ \mathrm{MeV} \quad . \tag{23}$$

We have also calculated some decay width using our effective Lagrangian L. Table I. contains the numerical results of these simple calculations together with the corresponding experimental values [12].

Our results are only slightly different from those of earlier calculations [13,14,15]. The difference is mainly due to our use of (22) while in other calculations $g = g_{\rho\pi\pi}$ was calculated from the $\rho \to \pi\pi$ width.

The rather good agreement between our results and experiment supports the hypothesis that the vector mesons ρ, ω are indeed the effective gauge bosons of the hidden local symmetry of QCD. So far it is not known whether this is a general phenomenon, i. e. in all strongly interacting theories if some global chiral symmetry G breaks down to a smaller one, H with the emergence of composite G/H massless Goldstone bosons then among the other composite states there are always present massive vector bosons corresponding to a hidden and spontaneously broken H_{local} gauge symmetry.

Finally we would like to comment on an attempt [14] to directly identify the ρ and A_1 meson fields with the vector and axial fields present in the gauged Wess-

Zumino Lagrangian $\mathcal{L} = \mathcal{L}(\mathcal{U}, \mathcal{V}_\mu, \mathcal{A}_\mu)$. The authors of ref. [14] concentrated on the Lagrangian $\mathcal{L}_{WZ}$ and introduced the interactions between vector, axial and pseudoscalar fields by using, instead of (15), the Lagrangian

$$\bar{L}^{(5)}_{WZ} = Z^{(5)}_{WZ}\left[V_\mu^{\xi^+}, A_\mu^{\xi^+}\right] - Z^{(5)}_{WZ}\left[V_\mu, A_\mu\right] \quad . \tag{24}$$

While it is evident that (24) and (15) are entirely different functionals, there can be effective vertices which turn out to be the same. This happens to be the case with the VVP vertex which is the most important one in the phenomenological applications. It is given by

$$L^{(5)}_{VVP} = \frac{3}{8\pi^2} \mathrm{Tr}\left\{(dV)^2 dP\right\} = \frac{3}{8\pi^2}\, d\, \mathrm{Tr}\left\{(dV)^2 P\right\} \tag{25}$$

in both cases.

However, it has been noted [15] that (24) is inconsistent in that in general it fails to reproduce the low energy theorems of current algebra incorporated in $\mathcal{L}_{WZ}$. On the other hand, we constructed our Lagrangian (15) so as to reproduce $\mathcal{L}_{WZ}$ (and $\mathcal{L}_{NT}$) when (7) and (16), the equation of motion of the vector and axial fields in the low energy approximation, are used.

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Decay	Calculated width from L	Experiment
$\Gamma(\rho \to \pi\pi)$	163 MeV	154 ± 5 MeV
$\Gamma(A_1 \to \rho\pi)$	420 MeV	315 ± 45 MeV
$\Gamma(\omega \to \pi^0\gamma)$	877 keV	861 ± 60 keV
$\Gamma(\rho \to \pi\gamma)$	92 keV	71 ± 9 keV
$\Gamma(\omega \to 3\pi)$	7.25 MeV	8.90 ± 0.27 MeV

Table I. The calculated decay widths of vector and axial mesons and the corresponding experimental values [12].

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