KFKI-1985-24

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CENTRAL
RESEARCH
INSTITUTE FOR
PHYSICS

BUDAPEST

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To appear in Nuovo Cim. 86A /1985/.

HU ISSN 0368 5330
ISBN 963 372 356 6

ABSTRACT

A nonconventional extension of the canonical quantization method for local field theories is presented. This approach lacks some of the difficulties of the conventional one (e.g. there are no ultraviolet divergences in the corresponding S-matrices).

АННОТАЦИЯ

Дается новый метод расширения канонической квантизации для локальных полей. Настоящее приближение не содержит некоторых затруднений общепринятого подхода (например, в S-матрице не появляется ультрафиолетовая дивергенция).

KIVONAT

Lokális térelméletek kanonikus kvantálási módszerének egy nemkonvencionális kiterjesztését alkotjuk meg, amelyben a konvencionális módszerrel együttjáró néhány nehézség nem lép fel (pl. nincs ultraibolya divergencia az S-mátrixban).

One of the most successful theory of physics is quantum (q) mechanics. Its mos powerful method is the canonical quantization which algorithm provides the q version of a wide class of classical (c) mechanical systems. But the conventional extension of this method for quantum field theory still occupies an ambiguous status [1]. On one hand it led, appended by the renormalization prescription, in an experimentally very successful theory, [1,2] while, on the other hand, beside the divergences in the perturbation expansion, further researches revealed some fundamental inconsistencies in this conventional theory [2,3,4] (by Wightman [4] these difficulties in catch words are listed as (1) Haag's theorem, (2) ultraviolet catastrophe and (3) instability of vacuum). Considering this ambiguous status of the conventional procedure, the question arises: Is the conventional procedure the only one for extending the canonical quantization method for field theories or there exists another way? Here we mention an affirmative answer for this question in the cases of classical local field theories (CLFT) given by Lagrangian densities, while the detailed presentation of this approach will be published elswhere [5,6].

Though the present canonical quantization method can be applied for any Lagrangian CLFT, [5] for simplicity's sake, here we presents it for a CLFT of a single real scalar field φ and of Lagrangian

$$L(t,\vec{x}) = [\tfrac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)](t,\vec{x}) \qquad (1)$$

Our method maximally exploits the principle of locality which means that this canonical quantization substitute the identical c mechanical systems described by (1) at each point $\vec{x}$ of the t = constant hyperplane $\mathbb{R}^3$, with their identical q mechanical counterparts [5,6]. In the presentation of this canonical quantization method we closely follow here the introduction and presentation of the q mechanical canonical quantization in standard text books [7]. Thus we start with the discussion of the canonical formalism of CLFT in a completely local fashion. By implementing a Legendre transformation on $L = L(\varphi,\dot{\varphi},\nabla\varphi)$ with the use of the c field equation (in what follows eq.), we obtain the canonical eq.'s

$$\dot{\varphi} = \partial H/\partial\pi, \qquad -\dot{\pi} = \partial H/\partial\varphi - \nabla(\partial H/\partial\nabla\varphi) \qquad (2)$$

where $H = H(\varphi,\pi,\nabla\varphi) = \pi\dot{\varphi} - L$ is the Hamiltonian density. Eq.'s (2) are equivalent to the c field. eq.. Hamiltonian-Jacobi eq. can also be introduced for

the action density $S = S(\vec{x}) = \int dt L(t,\vec{x})$. Along the c mechanical line of thoughts we obtain

$$\partial S/\partial t = -H(\varphi, \partial S/\partial\varphi, \nabla\varphi, t) \tag{3}$$

for S as the function of $\varphi, \nabla\varphi$ and t, where $\partial S/\partial\varphi = \pi$. Poisson brackets of the local physical quantities $F(t,\vec{x}) = F(\varphi,\pi,\nabla\varphi,\nabla\pi)(t,\vec{x})$ can be defined as follows

$$\{F_1,F_2\} = \frac{\delta F_1}{\delta\varphi}\frac{\delta F_2}{\delta\pi} - \frac{\delta F_2}{\delta\varphi}\frac{\delta F_1}{\delta\pi} \tag{4}$$

where $\delta F/\delta\varphi = \partial F/\partial\varphi - \nabla(\partial F/\partial\nabla\varphi)$ or $\delta F/\delta\pi = \partial F/\partial\pi - \nabla(\partial F/\partial\nabla\pi)$ is the "functional derivative" of F with respect to φ or π, respectively. The time derivative of a local quantity F can be expressed, with (2) and (4) as $dF/dt = \partial F/\partial t + \{F,H\}$. Poisson's brackets of the basic local variables φ, $\nabla\varphi$, π and $\nabla\pi$ are then as follows

$$\{\varphi,\varphi\} = \{\pi,\pi\} = 0, \qquad \{\pi,\varphi\} = -1 \tag{5a}$$

$$\{\nabla\pi,\pi\} = \{\nabla\varphi,\varphi\} = 0, \qquad \{\nabla\pi,\varphi\} = \{\pi,\nabla\varphi\} = (-\nabla)(-1) \tag{5b}$$

$$\{\nabla\pi,\nabla\pi\} = \{\nabla\varphi,\nabla\varphi\} = 0, \qquad \{\nabla\pi,\nabla\varphi\} = (-\nabla)^2(-1) \tag{5c}$$

Then the canonical quantization of the c theory means that we replace the local quantities $(\varphi,\nabla\varphi,\pi,\nabla\pi)$ with local observables $(\hat{\varphi},\nabla\hat{\varphi},\hat{\pi},\nabla\hat{\pi})$, i.e., with <u>self-adjoint elements</u> of a noncommutative algebra, and Poission's brackets (5a,b,c,) are replaced by commutator brackets

$$[\hat{\varphi},\hat{\varphi}] = [\hat{\pi},\hat{\pi}] = 0, \qquad [\hat{\pi},\hat{\varphi}] = -i\mathbb{I} \tag{6a}$$

$$[\nabla\hat{\varphi},\hat{\varphi}] = [\nabla\hat{\pi},\hat{\pi}] = 0, \qquad [\nabla\hat{\pi},\hat{\varphi}] = [\hat{\pi},\nabla\hat{\varphi}] = (-i\nabla)(-i\mathbb{I}) \tag{6b}$$

$$[\nabla\hat{\pi},\nabla\hat{\pi}] = [\nabla\hat{\varphi},\nabla\hat{\varphi}] = 0, \qquad [\nabla\hat{\pi},\nabla\hat{\pi}] = (-i\nabla)^2(-i\mathbb{I}) \tag{6c}$$

(here $\hbar = c = 1$ are chosen) [6]. A solution of these canonical commutation relations (CCR) with linear self-adjoint operators is

$$\hat{\varphi}\psi = \varphi\cdot\psi(\varphi,\vec{x}), \qquad \hat{\pi}\psi = -i\partial\psi(\varphi,\vec{x})/\partial\varphi \tag{7a}$$

$$\nabla\hat{\varphi}\psi = -i\nabla\varphi\psi(\varphi,\vec{x}), \qquad \nabla\hat{\pi}\psi = -i\nabla(-i\partial\psi(\varphi,\vec{x})/\partial\varphi) \tag{7b}$$

where φ does not depend explicitely on $\vec{x}$, i.e., $[-i\nabla,\varphi] = 0$, and $\psi(\varphi,\vec{x}) \in L^2(\mathbb{R}\times\mathbb{R}^3)$. By von Neumann's theorem [3] the solution (7a) of the CCR (6a) is unique at each point $\vec{x}\in\mathbb{R}^3$, up to unitary equivalence (a more precise formulation of this statement can be found in ref. 5). We note that our method provides the canonical quantization procedure for the approach of CLFT's in ref. 8. For, the q substitute of the trivial phase bundle $\mathbb{R}^2\times\mathbb{R}^3\times(\mathbb{R}^3\times\mathbb{R})$ of the CLFT of Lagrangian (1) is the trivial Hilbert bundle $L^2(\mathbb{R})\times(\mathbb{R}^3\times\mathbb{R})$; the q substitute of the c mechanical system of phase space $\mathbb{R}^2\times\mathbb{R}^3$, described by (1) at $(\vec{x},t)\in\mathbb{R}^3\times\mathbb{R}$ is its q mechanical counterpart of state

space $L^2(\mathbb{R})$. Another note is that in this representation the wave functions $\psi(\varphi,\vec{x})$ and not the field depend explicitely on the points of the 3-space $\mathbb{R}^3$ (as is well-known, [2,3] in the conventional approach the q field $\varphi(t,\vec{x})$ actually does not exist as an operator in a Hilbert space).

The dynamics can be specified in this local approach by the unitary map $t \to \exp\{-i\hat{H}t\}$; $\psi(\varphi,\vec{x},t) = \exp\{-i\hat{H}t\}\psi(\varphi,\vec{x},0)$ where $\hat{H} = H(\hat{\varphi}, \hat{\pi}, \nabla\hat{\varphi}) = \frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\nabla\hat{\varphi})^2 + V(\hat{\varphi})$ is the local Hamiltonian of the q system. This prescription provides Schrödinger's eq.

$$i\,\frac{\partial\psi}{\partial t} = [-\frac{1}{2}\,\frac{\partial^2}{\partial\varphi^2} - \frac{1}{2}\,\varphi^2\Delta + V(\varphi)]\psi \tag{8}$$

This specification of the dynamics is confirmed by the following facts. (1) Eq. (8) implies a continuity eq. $\partial|\psi|^2/\partial t + \mathrm{Div}\, J = 0$ with the current $J = [(i/2)(\psi\partial\psi^*/\partial\varphi - \psi^*\partial\psi/\partial\varphi), (i/2)\varphi^2(\psi\nabla\psi^* - \psi^*\nabla\psi)]$. Thus $|\psi|^2$ can be interpreted as a probability density for a measurement that at time t the field $\hat{\varphi}$ at the point $\vec{x}$ gives the value φ when the measurement of the 3-position $\hat{\vec{x}}$ gives the value $\vec{x}$. (2) Eq. (8) implies an Ehrenfest theorem. Namely, the c field eq. holds true in expectation value

$$\ddot{\bar{\varphi}} - \overline{\Delta\varphi} = -\overline{\partial V/\partial\varphi} \tag{9}$$

where, e.g., $\bar{\varphi} = \langle\psi|\hat{\varphi}|\psi\rangle = \int d\varphi d^3\vec{x}\ \psi^*(\varphi,\vec{x},t)\hat{\varphi}\ \psi(\varphi,\vec{x},t)$ etc.. This results can be derived directly from (8) or in an easier way in Heisenberg's picture. Namely, the local observables $\hat{F} = F(\hat{\varphi},\hat{\pi},\nabla\hat{\varphi},\nabla\hat{\pi})$ evolve in time as follows $\hat{F}(t) = \exp\{-i\hat{H}t\}\cdot\hat{F}(0)\cdot\exp\{i\hat{H}t\}$ or in differential from $d\hat{F}/dt = \partial\hat{F}/\partial t + i[\hat{H},\hat{F}]$. Then, by applying (6a,b,c) and (7a,b), we get for $\hat{\varphi}$ and $\hat{\pi}$

$$\dot{\hat{\varphi}} = i[\hat{H},\hat{\varphi}] = \partial\hat{H}/\partial\hat{\pi}, \qquad -\dot{\hat{\pi}} = i[\hat{\pi},\hat{H}] = \partial\hat{H}/\partial\hat{\varphi} - i\nabla(\partial\hat{H}/\partial\nabla\hat{\varphi}) \tag{10}$$

taking into account that $\nabla\hat{\varphi} = -i\nabla\hat{\varphi} = \hat{\varphi}(-i\nabla)$ (10) as operator eq.'s are equivalent in form with the c canonical eq.'s (2). Using the specific form of $\hat{H}$ we have $\dot{\hat{\varphi}} = \hat{\pi}$ and $\dot{\hat{\pi}} = i\nabla(\nabla\hat{\varphi}) - \partial\hat{V}/\partial\hat{\varphi}$ or $\ddot{\hat{\varphi}} - \Delta\hat{\varphi} = -\partial\hat{V}/\partial\hat{\varphi}$, i.e., the c field eq. recovers in the quantized theory as an operator eq.. Taking the expectation value of this eq. in any state, we obtain (9). (3) The c limit of (8) can be performed in the same way as in q mechanics. For this purpose let $\psi(\varphi,\vec{x},t) = A(\varphi,\vec{x},t)\exp\{iS(\varphi,\vec{x},t)\}$. Then (8) implies the continuity eq. for $A^2 = |\psi|^2$ and the following eq. for S:

$$\frac{\partial S}{\partial t} + \frac{1}{2}\,\frac{\partial^2 S}{\partial\varphi^2} + \frac{1}{2}(\varphi\nabla S)^2 + V(\varphi) - \frac{1}{2A}(\frac{\partial^2}{\partial\varphi^2} + \varphi^2\Delta)A = 0 \tag{11}$$

If $S(\varphi,\vec{x},t)$ is of the form $S = S_1(\varphi,t) + s(\vec{x})$ and s is of the form $s = \ln\varphi$ then $(\varphi\nabla s)^2 = (\nabla\varphi)^2$ and so, neglecting the last term proportional to $\hbar^2$ in the usual units in (11), we have got back Hamilton-Jacobi's eq. (3) of the c theory [9].

Up to now we formulated the canonical quantization of CLFT considered and the local description of the dynamics of the quantized theory. We now turn to the global description of the quantized system. As we saw the quantization substitute the "connected trivial fibre bundle" of c mechanical systems described by (1) with a "connected trivial fibre bundle" of q mechanical systems [5,8] (the "connection" is described by the term $(\nabla\varphi)^2$ of (1) in the c theory and by the term $\varphi^2\Delta$ of (8) in the q theory). We can treat this infinite set of connected q mechanical systems globally by the tools of q statistical mechanics [5]. For this purpose we use the following representation space of the CCR's (6a,b,c). The wave function can be generated as $\psi(\varphi,\vec{x},t)= \Phi(\varphi,\hat{\vec{p}},\hat{\vec{x}},t)\chi(\vec{x})$, where $\hat{\vec{p}} = -i\nabla$. Then the set of the (operator-valued) functions $\Phi(\varphi,\hat{\vec{p}},\hat{\vec{x}},t)$ equipped with the $\hat{A}$-valued scalar product $\langle\Phi_1|\Phi_2\rangle_{\hat{A}} = \int d\varphi\Phi^*_1(\varphi,\hat{\vec{p}},\hat{\vec{x}},t)\Phi_2(\varphi,\hat{\vec{p}},\hat{\vec{x}},t)$ constitutes a Hilbert space $H_{\hat{A}}$ over $\hat{A}$ where $\hat{A}$ is a *-algebra of linear operators in $L^2(\mathbb{R}^3)$ [5,10]. One may consider this representation space $H_{\hat{A}}$ as the local state space of the q system; the elements Φ with the property $\langle\Phi|\Phi\rangle_{\hat{A}} = 1$ (1 is the unit in $\hat{A}$) may be called local states; the local observables are represented by self-adjoint operators ($\hat{A}$-module homomorphisms) in $H_{\hat{A}}$; the expectation value of a local observable $\hat{F}$ in the local state Φ is $\hat{F} = \langle\Phi|\hat{F}|\Phi\rangle_{\hat{A}}\in\hat{A}$; $H_{\hat{A}}$ carries all the informations one gets by local measurements [5,11].

The global state space of the q system is then obtained by averaging over the local state space $H_{\hat{A}}$. The physically interesting measures for this averaging are provided by von Neumann's density operators, $0<\hat{\rho}<1$, in $\hat{A}$. Then the global state spaces are the Hilbert spaces $H^\rho = Tr\hat{\rho}\cdot H_{\hat{A}} := \{\Phi|\Phi\in H_{\hat{A}},\ Tr\hat{\rho}\langle\Phi\ \Phi\rangle_{\hat{A}} < \infty\}$ equipped with the complex scalar products $\langle\Phi_1|\Phi_2\rangle_\rho = Tr\hat{\rho}\langle\Phi_1|\Phi_2\rangle_{\hat{A}}$. The global states are the rays of H^ρ, the global observables are represented by self-adjoint operators in H^ρ (we note that a global observable corresponds to every local observable in $H_{\hat{A}}$) and the expectation value of a global observable $\hat{f}$ corresponding to the local one, $\hat{F}$, in the global state Φ is $\bar{f} = Tr\rho\langle\Phi|\hat{F}|\Phi\rangle_{\hat{A}}$. H^ρ carries all the information obtained from the infinite set of q mechanical systems in the q statistical state $\hat{\rho}$ of the measuring apparatus [5,6].

To see some advantages of the present approach we consider the perturbation theory using the interaction picture. Let $V(\varphi) = \frac{1}{2}m^2\varphi^2 + V_I(\varphi)$. Then in the interaction picture $\hat{\varphi}(t)$ satisfies free field eq. while the (local) states are governed by the interaction local Hamiltonian $\hat{H}_I = V_I(\hat{\varphi})$ according to Tomonaga-Schwinger's eq. $i\partial\Phi/\partial t = \hat{H}_I\Phi$. The free field eq. from (10) is $\Box\hat{\varphi} + m^2\hat{\varphi} = 0$. The solution is $\hat{\varphi}(t) = \hat{a}\hat{c}(t) + \hat{a}^+\hat{c}^*(t)$, where $[\hat{a},\hat{a}^+] = 1$ and $\hat{c}(t) = \int dP(\vec{k})(1/\sqrt{2k_o})\exp(-ik_o t)$, $k_o = \sqrt{\vec{k}^2 + m^2}$, and $P(\vec{k})$ is the spectral decomposition of $\hat{\vec{p}}$. Then the free local Hamiltonian takes the form $\hat{H}_o = (\hat{N} + \frac{1}{2})\hat{p}_o$, $\hat{N} = \hat{a}^+\hat{a}$ and $\hat{p}_o = (\hat{\vec{p}}^2 + m^2)^{1/2}$. In Fock's representation H in $H_{\hat{A}} = H\otimes\hat{A}$ [10] is spanned by Fock's basis $\Phi(n) = (1/\sqrt{n!})(\hat{a}^+)^n\Phi(0)$. The local observable $\hat{\vec{P}} = (\hat{N} + \frac{1}{2})\hat{\vec{p}}$ can be called the local 3-momentum observable of the q system [5]. Generelly, we can say that the free field operator eq. describes a "trivial fibre bundle" of harmonic oscillators, or, another way of inter-

pretation is that $\hat{a}^+$ creates a Klein-Gordon particle of 4-momentum observable $(\hat{p}_o,\hat{\vec{p}})$ from the local vacuum $\Phi(0)$. Thus in the local state $\Phi(n)$ there are n Klein-Gordon particles of 4-momentum observable $(\hat{p}_o,\hat{\vec{p}})$. So the free field description in the present approach is physically equivalent with the conventional one. Some formal eq.'s of the conventional free field theory now recover as mathematically well-defined eq.'s [5]. Thus, e.g., we have $[\hat{\varphi}(t_1),\hat{\varphi}(t_2)] = -i\hat{\Delta}(t_1-t_2)$ for the non-equal time commutators of $\hat{\varphi}$, where $\hat{\Delta}(t_1-t_2) = \int dP(\vec{k}) \oint_C (2\pi)^{-1} \cdot (m^2-k_o^2+\vec{k}^2)^{-1} \exp[ik_o(t_1-t_2)]dk_o$, or for the contraction of $\hat{\varphi}(t_1)$ and $\hat{\varphi}(t_2)$ we have $\hat{\varphi}(t_1)\hat{\varphi}(t_2)=\langle 0|T(\hat{\varphi}(t_1)\hat{\varphi}(t_2))|0\rangle_{\hat{A}}=i\hat{\Delta}_F(t_1 - t_2)$ where $\hat{\Delta}_F$ is Feynman's propagator, $\hat{\Delta}_F(t_1-t_2) = \int dP(\vec{k}).(2\pi)^{-1}\int dk_o \cdot .(m^2-k_o^2+\vec{k}^2-i0)^{-1}\exp[ik_o(t_1-t_2)]$ [5].

For describing scattering problems in the interaction picture the local S-matrix is obtained along the usual line of thought and is given by the expression $\hat{S} = T \exp[-i\int dt \hat{H}_I(t)]$. The terms of $\hat{S}$ can be expanded by applying Wick's first theorem and the corresponding terms can be represented by Feynman's graph techniques [2,5]. These terms do not contain infinite factors when their matrix elements are taken between any two local (or global) states from their domain [5]. This statement follows from the fact that in this local approach only the local interaction Hamiltonian $\hat{H}_I$ appears in $\hat{S}$ and not the total one, "$\int d^3\vec{x}\hat{H}_I(\vec{x})$". Mathematically this implies that there are no 3-momentum integrals for the internal lines of Feynman's graphs. The remaining energy integrals have the same structure as in q mechanics, so they do not produce ultraviolet divergences. Physically, the local Hamiltonian does not contain contribution of far away fluctuations.

As to the Lorentz invariance of the theory we note that only non-unique (infinitely many different) representations of the Lorentz group can be given in the local state space $H_{\hat{A}}$. This and Erenfest's theorem implies that a unique Lorentz invariance holds only in mean value in this theory [12]. Nevertheless the global scattering amplitudes $\langle\Phi_1|\hat{S}\Phi_2\rangle_\rho = Tr\hat{\rho} \cdot \langle\Phi_1|\hat{S}\Phi_2\rangle_{\hat{A}}$ of local states are Lorentz scalars and thus the global scattering amplitudes of wave-packets (describing physical particles), too. Furthermore in the cases of free fields we can select out uniquely local (and the corresponding global) states which describe relativistic many particle states. Then these states describe the in and out states of (adiabatically) interacting relativistic particles, however Lorentz invariance in the process of interaction is fulfilled only in mean value in this approach [6].

We conclude here that a nonconventional extension of the canonical quantization method for CLFT's exists, which lacks the difficulties of the conventional one formulated in Haag's theorem and in the ultraviole catastrophe [2,3]. In this approach the interaction picture exists for nontrivial scattering problems, too. The corresponding S-matrices have the same structure as the conventional ones and are free of ultraviolet divergences. This formalism is an explicite example that the q dynamics of a system can be locally implemented, according to the physical measuring situation.

REFERENCES

[1] For an overview of the field quantization problems see: I. Segal, Physica Scripta 24, 827-831 (1981).

[2] P. Roman: Introduction to Quantum Field Theory, John Wiley and Sons, Inc., New York 1969.

[3] N.N. Bogolubov, A.A. Lugonov and I.T. Todorov: Introduction to Axiomatic Quantum Field Theory. W.A. Benjamin, London-Amsterdam 1975.

[4] A.S. Wightman: in High Energy Electromagnetic Interactions and Field Theory, ed. M. Lévy, Gordon and Breach, New York 1967.

[5] M. Banai: Central Research Institute for Physics preprint, KFKI-1983-51; and Int. J. Theor. Phys. 23, 1043-1063 (1984): and cf. M. Banai, Hadronic J. 5, 1812-1841 (1982)

[6] M. Banai and B. Lukács: in preparation.

[7] See, e.g., L.D. Landau and E.M. Lifshitz: Quantum Mechanics, Pergamon, London-Paris 1958.

[8] J. Kijowski and W.M. Tulczyjew: A Symplectic Framework for Field Theories, Spinger, Berlin-New York 1979.

[9] Remind that in q mechanics the coordinates q in the wavefunctions $\psi(q,t)$ do not depend explicitely on the time t while in Hamilton-Jacobi's eq. they do. Now the situation is the same. In q theory φ does not depend explicitely on $(\vec{x},t)$ while in c theory it does.

[10] G.G. Kasparov: J. Operator Theory 4, 133-150 (1980). ($H_{\hat{A}}$ equals the tensor product $L^2(\mathbb{R})\otimes\hat{A}$. A mathematically more precise discussion of $H_{\hat{A}}$ can be found in ref. 5.)

[11] M. Banai: Int. J. Theor Phys. 20, 147-169(1981). For a connection with a q relativistic spacetime model see: M. Banai and B. Lukács, Lett. Nuovo Cim. 36, 533-538 (1983); M. Banai, Quantization of Spacetime and the corresponding Quantum Mechanics, Found. Phys. 15 (1985) N°7-9 (to appear); cf. ref. 5.

[12] E. Prugovečki has shown that a consistent unification of c relativity and q theory implies stochastic Poincaré group in a stochastic spacetime. Cf. E. Prugovečki, Stochasitc Quantum Mechanics and Quantum Spacetime, D. Reidel, Dordrecht-Boston 1984.

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Szakmai lektor: Margaritisz Tamás
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Készült a KFKI sokszorosító üzemében
Felelős vezető: Töreki Béláné
Budapest, 1985. február hó