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RESISTIVE MODE IN ROTATING PLASMA COLUMNS INCLUDING THE HALL CURRENT

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ABSTRACT

A new resistive mode is shown to exist in rotating plasma columns. The mode is localized in the neighbourhood of the radius where the angular velocity of the bulk plasma is equal to minus half the local angular velocity of the ions. This singular point is caused by the Hall term in the generalized Ohm law. The growth rate of the mode scales with $\eta^{1/2}$, where η is the plasma resistivity.

RESUMO

É demonstrado que um novo modo resistivo é instável em colunas de plasma com rotação. O modo é localizado na vizinhança do raio onde a velocidade angular do plasma é igual a metade da velocidade angular local dos ions e em sentido contrário. Este ponto singular é causado pelo termo de Hall na lei de Ohm generalizada. A razão de crescimento do modo é proporcional a $\eta^{1/2}$, onde η é a resistividade do plasma.

1. INTRODUCTION

Resistive modes are known to occur in magnetic confinement configurations in which the magnetic field reverses direction inside the plasma [1,2,3]. However, the experimental results agree only qualitatively with the theoretical predictions. In rotating plasma columns,the reversed--field configuration tends to stay stable for many growth periods before becoming unstable [2]. It has been suggested that the rotation of the plasma column [2], kinetic effects [3], the Hall effect [4,5], or a component of the magnetic field normal to the neutral sheet [6] may have stabilizing effects on resistive modes. In this paper, I show that the rotation of the plasma column 'rgether with the Hall effect are destabilizing effects giving rise ' : , ew resistive mode.

Th : 'w portance of the Hall term in the generalized Ohm law was long ago rect ;ized by Ware [7]. The effect of the Hall current is to tie the motion ⸱ the field lines mainly to the electrons allowing a slippage of the lin ⸱ with respect to the main fluid even when the plasma resistivity is neglected. From a more formal point of view, the Hall term, although sm 1 in comparison with the inductive term in the generalized Ohm law, introduces a singular perturbation because it increases the order of the magnetohydrodynamic equations with respect to the equations without it. In the absence of the Hall term, resistive modes develop in the neighbourhood of the point where the confining magnetic field vanishes [8,9]. In the presence of the Hall term, new singular points appear in the ideal magnetohydrodynamic equations[10]. Resistive modes can now develop in the neighbourhood of these points with growth rates that differ considerably from the growth rate of the tearing mode. This point has so far been overlooked in the studies of the influence of the Hall effect on the tearing mode[4,5].

2. BASIC EQUATIONS AND EQUILIBRIUM

I use the resistive magnetohydrodynamic equations including the contribution of the Hall current in the equation for the evolution of the magnetic field $\vec{B}$, i.e.,

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho(\vec{V}\cdot V)\vec{V} = \frac{1}{4\pi} (\nabla \times \vec{B}) \times \vec{B} - \nabla p, \tag{1}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) - \nabla \times (\frac{\eta c^2}{4\pi} \nabla \times \vec{B}) - \frac{c\epsilon_H}{4\pi} \nabla \times [(\nabla \times \vec{B}) \times \frac{\vec{B}}{\rho}] . \tag{2}$$

$$\frac{\partial p}{\partial t} + \Gamma p \nabla \cdot \vec{V} + \vec{V} \cdot \nabla p = 0 , \tag{3}$$

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \vec{V} + \vec{V} \cdot \nabla \rho = 0 , \tag{4}$$

and $\nabla \cdot \vec{B} = 0$. Here the Hall parameter ϵ_H is given by the mass to charge ratio of the ions, η is the plasma resistivity, Γ is the ratio of specific heats, ρ and p are respectively the density and pressure of the plasma, and $\vec{V}$ is the fluid velocity. The equilibrium configuration is an infinite cylindrical plasma column. with azimuthal symmetry and confined by a longitudinal magnetic field $\vec{B}_0 = B_0(r)\vec{z}$. Then, neglecting the plasma resistivity, the equilibrium relationship is given by

$$\frac{d}{dr} (p + \frac{B_\theta^2}{8\pi}) = \rho \Omega^2 r, \tag{5}$$

where r is the radial coordinate of a cylindrical coordinate system (r,θ,z). The Hall term has not been neglected in the derivation of Eq. (5); it simply does not appear in the equilibrium equation. This is quite different from the special equilibrium studied by Kadish [11].

3. PERTURBED EQUATIONS

I consider only the m=0 mode, i.e., perturbations of the form $\vec{V}_1(r,z,t) = \vec{V}_1(r)\exp(-i\omega t + ikz)$, where ω is the frequency of the mode and k is the longitudinal mode number. Then, introducing the variable $V_{1r} = -i\omega\xi_r$ and neglecting the azimuthal component of the perturbed velocity (this component is irrelevant for the m=0 mode), the following set of equations is derived from Eqs. (1)-(4).

$$\frac{d}{dr}\left\{p\left[\frac{\rho\omega^2}{\rho\omega^2-k^2\Gamma p}\frac{1}{r}\frac{d}{dr}(r\xi_r)\right] + \frac{\rho\omega^2}{p\omega^2-k^2\Gamma p}\frac{dp}{dr}\xi_r\right\} + \rho\omega^2\xi_r -$$

$$- \rho\Omega^2 r\left[\frac{\rho\omega^2}{\rho\omega^2-k^2\Gamma p}\frac{1}{r}\frac{d}{dr}(r\xi_r) + \frac{k^2 dp/dr}{\rho\omega^2-k^2\Gamma p}\xi_r\right] - \frac{d\rho}{dr}\Omega^2 r\xi_r =$$

$$= \frac{d}{dr}\left[\Gamma p\frac{ikdB_0/dr}{4\pi(\rho\omega^2-k^2 Tp)}B_r + i\frac{B_0}{4\pi k}\frac{1}{r}\frac{d}{dr}(rB_r)\right] -$$

$$- \rho\Omega^2 r\frac{(ikdB_0/dr)B_r}{4\pi(\rho\omega^2-k^2\Gamma p)} - \frac{ikB_0}{4\pi}B_r \tag{6}$$

and

$$B_r = ik(B_0 + 2\Omega c \epsilon_H)\xi_r + i \frac{\eta c^2}{4\pi\omega} (\frac{1}{r} \frac{d}{dr} r \frac{dB_r}{dr} - \frac{1 + k^2 r^2}{r^2} B_r),$$

(7)

where B_r is the r-component of the perturbed magnetic field and η and Ω have been assumed constant.

Equations (6) and (7) are solved using a boundary layer approach as usual in the analysis of resistive modes [8]. The ideal magnetohydrodynamic equation,

$$\frac{d}{dr} \left\{ [\frac{\Gamma p \rho \omega^2}{\rho\omega^2 - k^2 \Gamma p} + \frac{B_0(B_0 + 2\Omega c \epsilon_H)}{4\pi}] \frac{1}{r} \frac{d}{dr} (r\xi_r) \right\} +$$

$$+ \frac{d}{dr} [\frac{\Gamma p k^2 2\Omega c \epsilon_H dB_0/dr}{4\pi(\rho\omega^2 - k^2 \Gamma p)} \xi_r] + \left\{ \rho\omega^2 r \frac{d}{dr} (\frac{\rho^2 \omega^2 \Omega^2}{\rho\omega^2 - k^2 \Gamma p}) - \right.$$

$$- \frac{k^2 \rho^2 \Omega^4 r^2}{\rho\omega^2 - k^2 \Gamma p} - r\Omega^2 \frac{d\rho}{dr} - \rho\Omega^2 r \frac{2\Omega c \epsilon_H k^2}{4\pi(\rho\omega^2 - k^2 \Gamma p)} \frac{dB_0}{dr} -$$

$$\left. - \frac{k^2 B_0(B_0 + 2\Omega c \epsilon_H)}{4\pi} \xi_r \right) = 0 ,$$

(8)

is obtained by taking $\eta = 0$, substituting Eq. (7) into Eq. (6), and using Eq. (5). In the marginal stability limit , $\omega \to 0$, and when the Hall effect is neglected, $\epsilon_H = 0$, Eq. (8) has a singular point at the radial position where B_0 vanishes. In this case, a resistive instability develops in the neighbourhood of this point with the growth rate of the mode proportional to $\eta^{1/3}$. This mode is just the rotational version of the gravitational resistive mode with the gravity replaced by the centrifugal acceleration[12]. When the Hall effect is not neglected, $\epsilon_H \neq 0$, the singular point is

shifted to the point $r = r_0$ such that $B_0 + 2\Omega c\epsilon_H = 0$, that is, to the radius where the plasma rotates with half the local angular velocity of the ions and in the opposite sense. In the neighbourhood of this point, Eq. (8) has a singular solution which is given asymptotically by

$$\xi_r \sim \frac{r}{r - r_0} + a_0 \ell n \left(\frac{r - r_0}{r_0} \right), \tag{9}$$

where

$$a_0 = \frac{\dfrac{\rho^2 \Omega^4 r_0^2}{\Gamma p} - \Omega^2 (r \dfrac{d\rho}{dr})_0}{\dfrac{2\Omega c\epsilon_H}{4\pi r_0^2} (r \dfrac{dB_0}{dr})_0} + \frac{\rho \Omega^2 r^2}{\Gamma p}. \tag{10}$$

Now I consider a resistive layer around $r = r_0$ such that $\eta \neq 0$ in the layer. The basic small parameter inside the layer is given by $\epsilon = \tau_A / \tau_r$, where $\tau_A = r_0 \sqrt{4\pi\rho} / (rdB_0/dr)_0$ is the Alfvén transit time and $\tau_r = 4\pi r_0^2 / \eta c^2$ is the resistive diffusion time. The different variables are scaled in terms of the small parameter ϵ by defining the following dimensionless quantities of order one

$$\xi = \frac{\xi_r}{r} \ ; \ \Lambda = \frac{-i\omega\tau_A}{\epsilon^a} \ ; \ x = \frac{r - r_0}{r_0 \epsilon^b} \ ; \ \psi = \frac{iB_r}{kr_0 (rdB_0/dr)_0 \epsilon^c} \ .$$

The constants a, b, and c are determined by requiring that the term proportional to η in Eq. (7) becomes of the same order as the relevant terms in Eqs. (5) and (7) and that the resulting set of equations has an asymptotic solution that matches the ideal magnetohydrodynamic solution (9). This can be accomplished by choosing $a = b = c = 1/2$ and $\psi = \psi_0 + \epsilon^{1/2} \psi_1, \psi_0 = $ const. Then Eqs. (6) and (7) reduce to

$$\frac{d^2\xi}{dx^2} + (A_1 - A_2 x) \xi = A_2 \psi_0 \ , \tag{11}$$

where

$$A_1 = \frac{\Omega^2 \tau^2 k^2 r^2_0}{\Lambda^2} \left[\frac{(rdp/dr)_0}{\Gamma p} - \left(\frac{r}{\rho} \frac{d\rho}{dr} \right)_c \right]$$

and

$$A_2 = \frac{2\Omega c \epsilon_H}{(rdB_0/dr)_0} \frac{k^2 r_0^2}{\Lambda}$$

An integral representation of the proper solution of Eq. (11) can be found using the technique discussed by Johnston[13], viz.,

$$\xi = \psi_0 A_2 \int_{-\infty}^0 \exp\left(- A_2 tx + \frac{A_2^2}{3} t^3 + A_1 t\right) dt. \tag{12}$$

To match the inside and outside solutions, I require that $\Delta'|_{in} = \Delta'|_{out}$, where Δ' is the jump in the logarithmic derivative of ψ across the singular layer[8]. Writing $\Delta|_{in}$ in terms of the normalized variables, the eigenvalue equation can be written as

$$\Lambda \int_{-1}^1 dx \left(1 + x \frac{\xi}{\psi_0}\right) = r_0 \Delta'|_{out}. \tag{13}$$

Substituting Eq. (12) into Eq. (13) and carrying out the integration, I obtain $\Lambda = r_0 \Delta'|_{out}/2$. Then, the growth rate of the mode, $\gamma = - i\omega$, is given by

$$\gamma \approx \frac{r_0 \Delta'|_{out}}{2} \left(\frac{\tau_A}{\tau_r} \right)^{1/2} \tau_A^{-1}. \tag{14}$$

This equation shows that γ scales with $\eta^{1/2}$, which is quite different from the scaling of the growth rate of the tearing mode or the resistive gravitational mode.

4. DISCUSSION

Previous work by Kappraff et al indicate that the effect of the Hall current on resistive modes in rotating plasma columns is stabilizing [5]. In this paper, I have shown that this is not a general result. The combined effects of rotation and Hall current leads to a new resistive mode. The mode develops around the singular point where the plasma column rotates with half the local angular velocity of the ions and in the opposite sense. The growth rate of the mode depends on the angular velocity Ω through the parameter $a_.$, which comes in $\Delta'|_{out}$. In closer agreement with experimental results [1-3], and for typical profiles of the confining magnetic field, the mode grows slower than the resistive modes usually considered to explain the instabilities observed in reversed-field configurations [5,9,17]. The effect of viscosity has been neglected in this paper. However, it is possible that viscosity play an important role further decreasing the growth rate of the mode [14]. Work on the effect of viscosity is currently in progress and the results will soon be reported.

Acknowledgements

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