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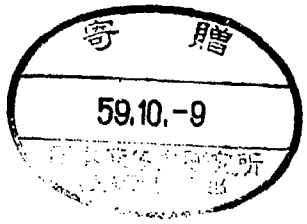

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Quarks and Gluons at Finite Temperature and Density

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Quarks and Gluons at Finite Temperature and Density[+)]

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Abstract

We have run computer simulation in SU(2) lattice gauge theory on a $8^3 \times 2$ lattice including dynamical quark loops. No rapid variation is observed in the value of Polyakov line, while the energy densities of quark and gluon show strong indication of a second order phase transition around $T \simeq 250 MeV$. In order to reduce finite size effects, the results are compared with those of free gas on a lattice of the same size. The quark and gluon energy densities overshoot the free gas values at high temperature. The effects of chemical potential is also studied. The behavior of the energy densities and the number densities are far from a free gas case.

+) Talk given at Warsaw Symposium and Zacopane summer school

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It has been conjectured that systems of quarks and gluons at high temperature and density show completely different behavior from those at zero temperature and normal density [1-3]. At low temperature and low chemical potential, quarks and gluons are confined inside hadrons (Fig.0a). Cabibbo and Parisi have shown that exponentially increasing spectrum does not necessarily mean the limiting temperature a la Hagedorn; rather a system may undergo a second order phase transition. Kislinger and Morley argued through the perturbative calculation that the gluons get mass and the confined mechanism does not work at high temperature (Fig.0b). Collins and Perry conjectured that at very high density hadrons overlap with each other and the quarks move freely inside the overlapped hadrons (Fig.0c).

Monte Carlo (MC) studies of SU(2) Yang-Mills theory in the absence of dynamical quarks by Mclerran and Svetitsky[4] and by Kuti,Polonyi and Szlachanyi[5] have given the first numerical evidence for a second order transition from a confined phase to a deconfined one. Goups at the University of Bielefeld and at the University of Illinois have performed MC simulations of the gluon matter at finite temperature in detail; For SU(3) Yang-Milles theory, they have observed first order phase transition and ideal gas behavior of gluons at high temperature[6].

Such studies of QCD in unusual environment are done not only for a theorist's fun and amusement. We hope that in high energy heavy ion collisions high temperature and density matter might be produced in a controlled experimental environment. To understand the data which might arise from such experiments, we may develop and study models of the quark-gluon system. MC simulation of lattice QCD probably provides the most fundamental information for such an analysis. For the study of hadronic matter, it is important to include quark loops in the calculation since they play a crucial role in screening; The phase transition observed in the pure gauge calculation might be washed out by them[7-8]. In the presence of quark fields, the Polyakov line is no more a good order parameter for the confined and deconfined phases, mathematically because the presence of quark fields breaks the symmetry under the center of the gauge group, or physically because isolated heavy quarks can survive due to the quark pair creation.

We will report here a MC study of the quark gluon system with dynamical quarks. We simulate the finite temperature and baryon number density plasma on a $N_t \times N_s^3$ lattice. The temperature of the system is given by $T = 1/N_t a_t$, where $a_t(g)$ is the lattice distance in the fourth direction. The action is composed of the kinetic term of gauge variables and the fermion part:

$$S = S_G + S_F, \quad S_F = \bar{\psi} \Delta \psi$$

We employ the Wilson form for the action[9]. The matrix Δ has the form,

$$\Delta = I - \sum_{\mu=1}^{4} \{ \kappa_\mu^{(+)} (1-\gamma_\mu) U_{n,\mu} \delta_{n+\hat{\mu},m} + \kappa_\mu^{(-)} (1+\gamma_\mu) U^\dagger_{m,\mu} \delta_{n,m+\hat{\mu}} \} .$$

The chemical potential, μ, is introduced à la Hasenfratz and Karsch[10]:

$$\kappa_4^{(\pm)} = \kappa_t e^{\pm \mu a_t}$$

This change of the hopping parameters results in replacing $p_4 \to p_4 + i\mu$ in the free propagators on a lattice. Following Refs.[11] and [12] we *assume* that the hopping parameters depend upon the lattice distance and the gauge coupling g as

$$\kappa_t = \frac{1}{2} \frac{a_s}{3a_t + a_s} \xi(g)$$

$$\kappa_i^{(+)} = \kappa_i^{(-)} = \kappa_s = \frac{1}{2} \frac{a_t}{3a_t + a_s} \xi(g) . \quad (i = 1,2,3)$$

We will analyze the followng quantities:

$$L = \frac{1}{2N_s^3} \sum_x \mathrm{Tr} \prod_{i=1}^{N_t} U_t(i,x)$$

$$\epsilon = \frac{1}{V} \left(-\frac{\partial}{\partial\beta} + \frac{\mu}{\beta}\frac{\partial}{\partial\mu}\right) \log Z - \epsilon_0$$

$$n = \frac{1}{\beta V} \frac{\partial}{\partial\mu} \log Z$$

where

$$\frac{\partial}{\partial\beta} = \frac{1}{N_t} \frac{\partial}{\partial a_t} , \quad V = (N_s a_s)^3 ,$$

$$Z = \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\psi \, e^{-S} .$$

and ϵ_0 stands for the energy density at $T = 0$ and $\mu = 0$.

The energy density, ϵ, may be divided into two parts,

$$\epsilon = \epsilon_G + \epsilon_q = \langle -S_G' \rangle + \langle -S_F' \rangle .$$

where

$$S_\alpha' = \frac{1}{V} \left(-\frac{\partial}{\partial\beta} + \frac{\mu}{\beta}\frac{\partial}{\partial\mu}\right) S_\alpha ,$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\psi \, \mathcal{O} \, e^{-S}$$

We have run the simulation on a 2×8^3 lattice with two flavors using the pseudo-fermion method. The gauge group SU(2) is replaced by its icosahedral subgroup. The reliability of the program and effects of the quark vacuum polarization on ordinary hadronic quantities are discussed in Refs.[13] and [14]. The hopping parameters, κ_t and κ_s, are fixed to be 0.17 which would correspond to a very small quark mass in the quenched approximation. The number of gauge MC iterations is 100 including 20 thermalization except for $4/g^2 = 2.0, 2.2$ and $\mu = 0$ where it is 84. The number of pseudo-fermion iteration is 45 including 15 thermalization for each sweep of gauge fields. For the calculation of ϵ, we need $\partial g^{-2}/\partial a_t$. We employed the formula in Ref.[15] where the quark loops are not included. We used the data on a 4^4 lattice to evaluate ϵ_0 except the pure gauge case for which the data on 8^4 lattice is employed. These procedure introduces some systematic error. However the total contribution from these terms is small.

In Fig.1, we show the expectation value of the Polyakov line as a function of $4/g^2$ for $\mu = 0$. At high temperature (small g^2) the Polyakov line has large expectation value which implies the existence of isolated heavy quarks with only small suppression due to finite temperature effects, while its value is small at low temperature. The characters of the system at high and low temperature are therefore different but heavy quarks are nevertheless deconfined in both phases. Its change is quite smooth, and no indication of a second order transition can be seen.

On the other hand, the gluon and quark densities, which are plotted in Figs.2a and 2b show rapid variation. Moreover their values at high temperature are much larger than that of pure gluon system which agree quite well with the "free gas" behavior at high temperature. Here "free gas" refers to the massless free bose or fermi fields on a lattice of the same size[16]. The energy density of gluons is as large as 3/2 of the "free gas" (the dashed line in the figure). This suggests more degrees

of freedom than those of simple ideal quark gluon gas. The system of quarks and gluons at these temperature regions looks rich in structure.

The rapid change in the energy densities occurs at the small value of $4/g^2$ than in the pure gauge case. This does not mean the transition temperature is lower than that of pure gauge theory because the lattice distance a at the same value of $4/g^2$ is differnt between two theories. In Refs.[13] and [14] it is shown that, once we switch on the quark loops, the gauge configulation are pushed into more ordered state and that the finite size effects become more serious. This may be interpreted as the shrinkage of the lattice distance drived by quark loops.

In order to know the value of temperature in physical unit, we should evaluate the lattice distance a. Note that we cannot use the values in the literature which were obtained without quark loops. We ran a simulation on a $N_t \times N_x \times N_y \times N_z = 8 \times 8 \times 4 \times 4$ lattice at $4/g^2 = 1.6, 1.8$ and 2.0 and measured Wilson loops on t-x plane. The heavy quark potential is estimated by a Stack method[17]. We fit the results to a Martin phenomenological potential[18] with the lattice distance and the constant part of the potential as parameters. The detailed calculation of the potential will be published elsewhere. The obtained temperatures are shown in Fig.2a.

We can expect the rapid change of the quark gluon energy density between $T \simeq 200$ and 350 MeV, which may be more drastic in case of SU(3). This picture is consistent with the recent analyses by Celik et al.[19], by Fucito and Solomon[20], and by Gavai et al.[21].

Next we shall study the effects of the chemical potential. We plot the MC data in Figs.3, 4 and 5 as a function of the chemical potential for $4/g^2 = 1.4$. As the chemical potential increases, the value of Polyakov line increases very slowly and monotonously, while the thermodynamic quantities show peculiar behavior. At large chemical potential, isolated heavy quarks can survived longer than at the zero chemical potential. The gluon energy density shown in Fig.4a increases quickly when we increase the chemical potential, *i.e.*, the gluons are not independent of quark matter density and exibit behavior far from that of an "free gas". However it falls suddenly at large chamical potential. The quark energy density in Fig.4b increases like a "free gas" but the value is much higher. At these chemical potential regions, the free quark-gluon picture is not correct. There might be other degrees of freedom. The number density, n, shown in Fig.5 also overshoots the "free gas" values at large chemical potential in a similar manner to quark energy density. To obtain the system with large chemical potential, higher density is required than that estimated from ideal gas equation.

This calculation was done at CERN and Frascati. I am grateful to the theory divisions there for their hospitality and to N.Oshima for his advise in the numerical computation. I am indebted to the participants and organizers of Warsaw symposium and of Zacopane summer school, 1984 for constructive criticism, especially for A.Bialas and L.D.Mclerran for valuable discussions and careful reading of the manuscript. I thank T.Minamoto for permitting me to use his characters in the figures.

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Figure Captions

Fig.1 Expectation value of the Polyakov line as a function of $4/g^2$ for zero chemical potential. The circles are the data obtained in the presence of quark loops. The crosses correspond to the pure gluon matter.

Fig.2a Gluon energy density, ϵ_G/T^4, as a function of $4/g^2$ for zero chemical potential. The dashed line gives the free field limit on the same lattice.

Fig.2b Quark energy density, ϵ_q/T^4, as a function of $4/g^2$ for zero chemical potential. (Two flavor.) The dashed line gives the free field limit on the same lattice.

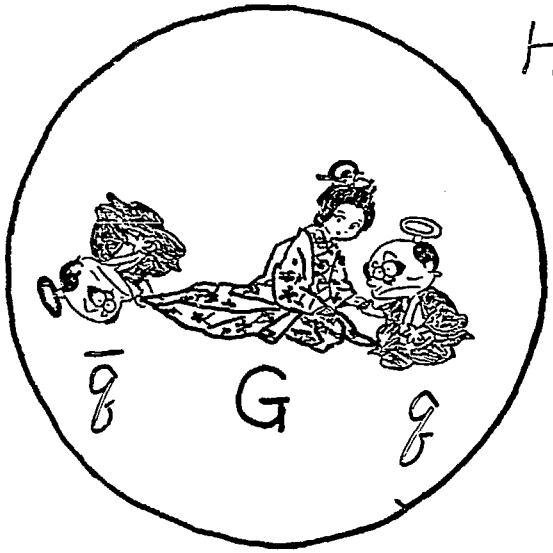
Fig.3 Expectaion value of the Polyakov line as a function of chemical potential for fixed temperature.

Fig.4a Gluon energy density, ϵ_G/T^4, as a function of chemical potential for fixed temperature. The dashed line gives the free field limit on the same lattice.

Fig.4b Quark energy density, ϵ_q/T^4, as a function of chemical potential for fixed temperature. (Two flavor.) The dashed line gives the free field limit on the same lattice.

Fig.5 Quark baryon number density, n/T^3, as a function of chemical potential for fixed temperature. (Two flavor.) The dashed line gives the free field limit on the same lattice.

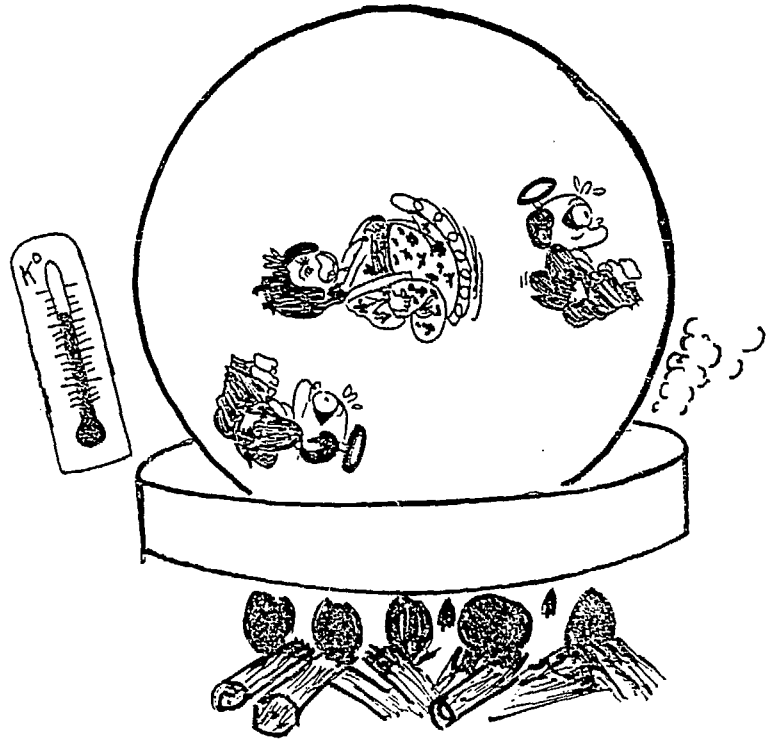
Fig.0a



Hadron
at
$T=0$
$\mu=0$

Confined Phase

Fig.0b



$T \gg 0$

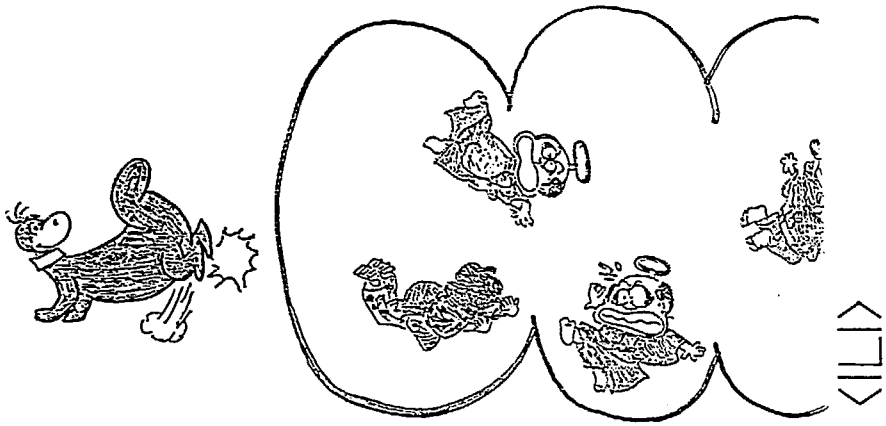


Fig.0c

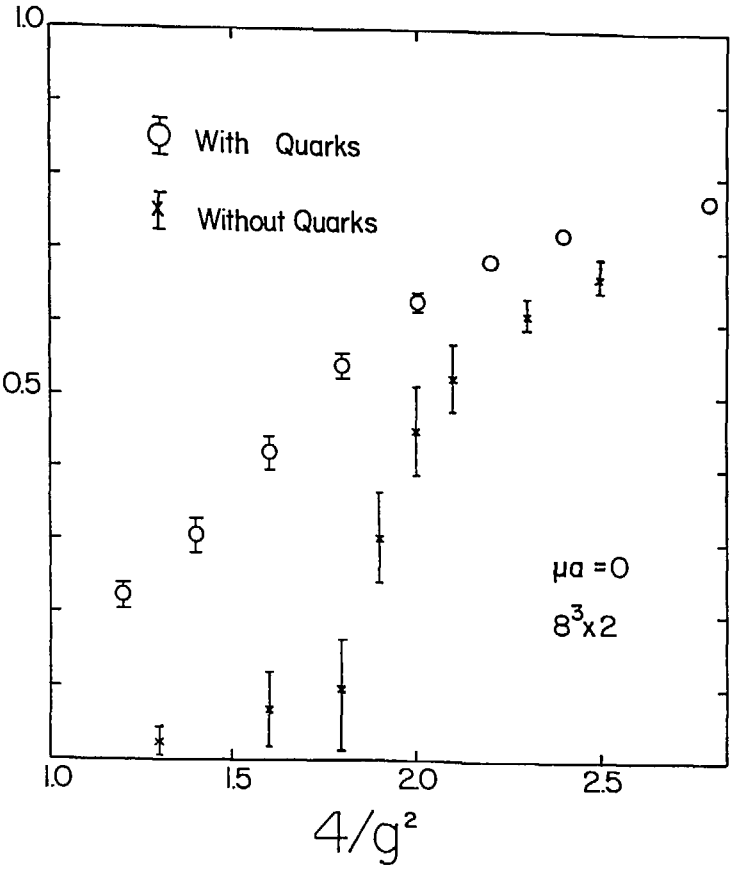


Fig.1

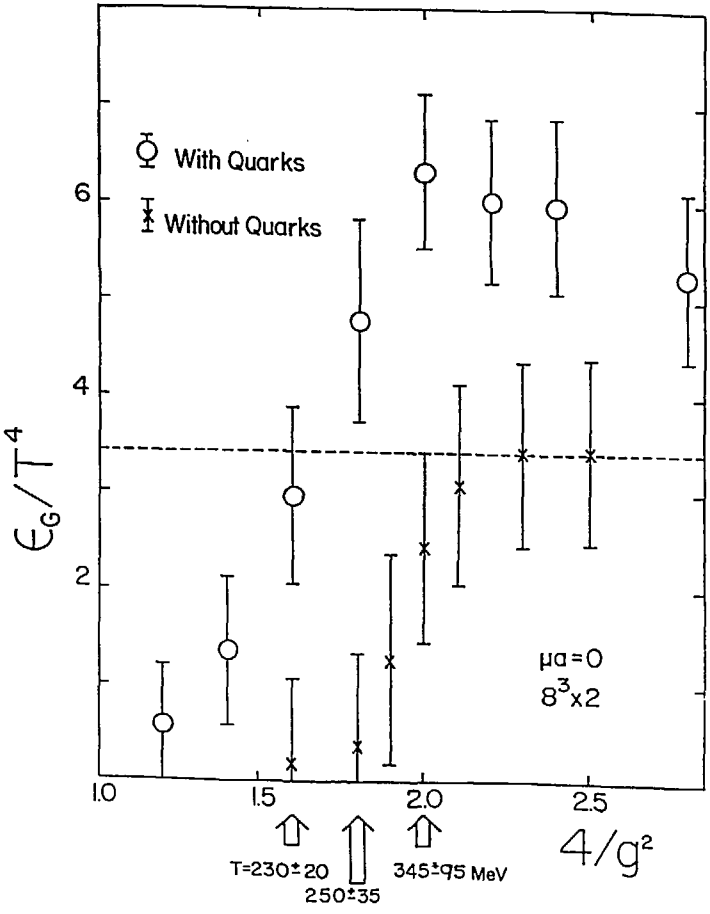

With Quarks
Without Quarks
ϵ_G/T^4
6
4
2
μa=0
8^3x2
1.0
1.5
2.0
2.5
$4/g^2$
T=230±20
250±35
345±95 MeV

Fig.2a

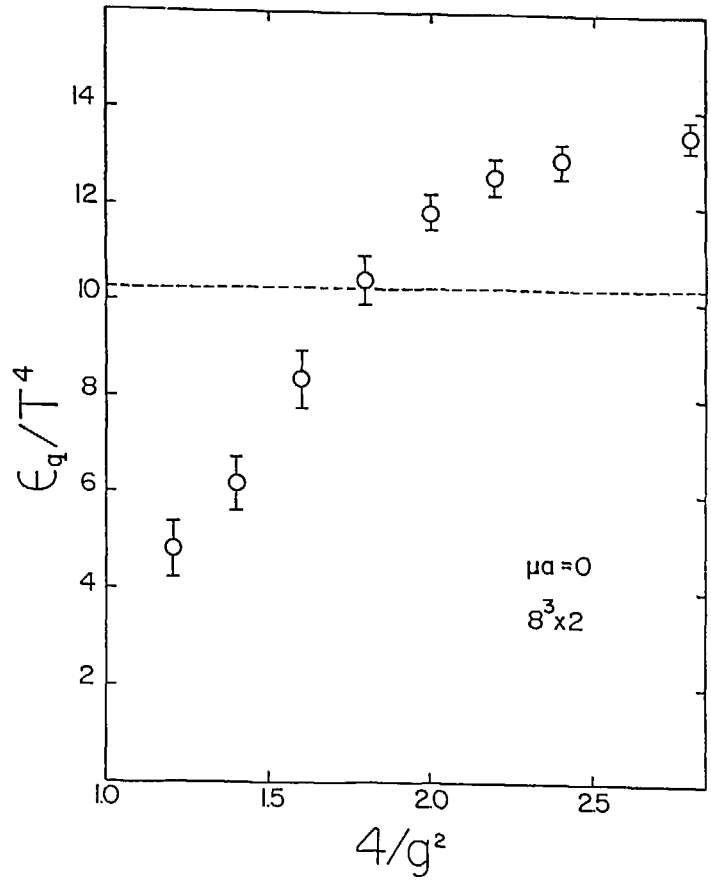

ϵ_q/T^4
14
12
10
8
6
4
2
μa=0
8^3x2
1.0
1.5
2.0
2.5
$4/g^2$

Fig.2b

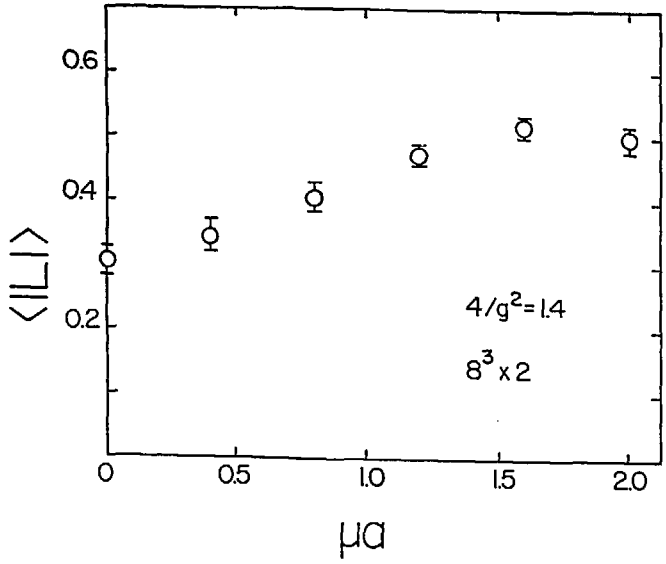

0.6
0.4
0.2
⟨|L|⟩
0
0.5
1.0
1.5
2.0
μa
4/g²=1.4
8³x2

Fig.3

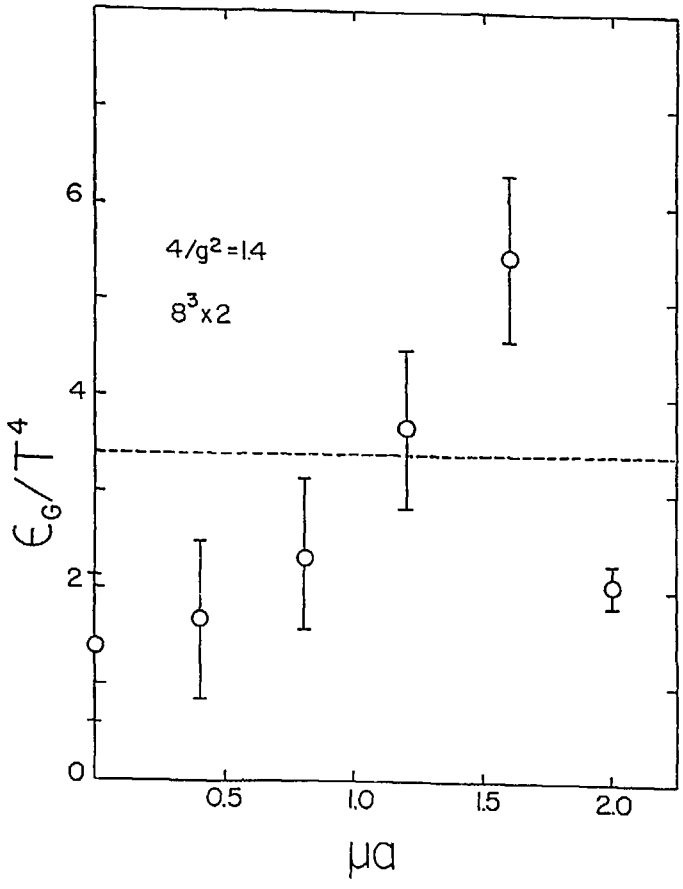

6
4
2
0
ϵ_G/T^4
0.5
1.0
1.5
2.0
μa
4/g²=1.4
8³x2

Fig.4a

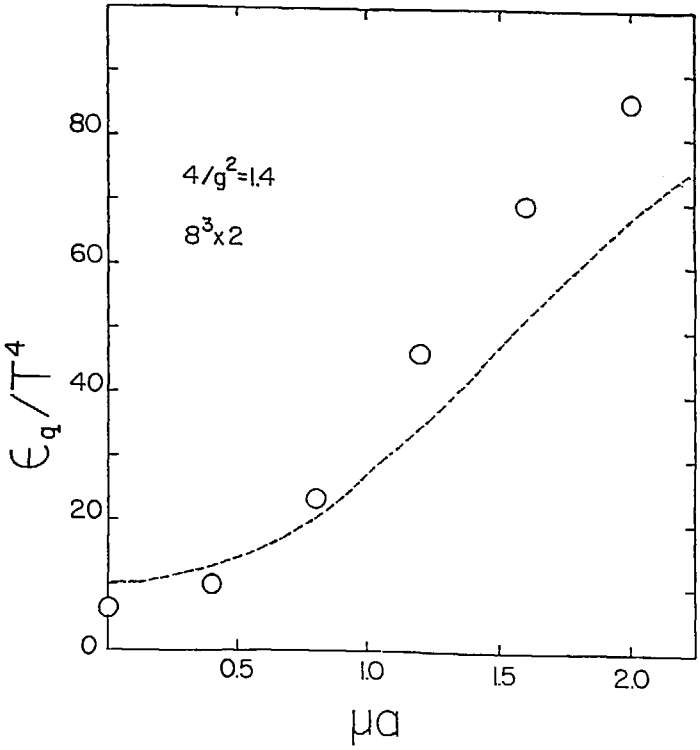

$4/g^2=1.4$
8^3x2
ϵ_q/T^4
μa
0
20
40
60
80
0.5
1.0
1.5
2.0

Fig.4b

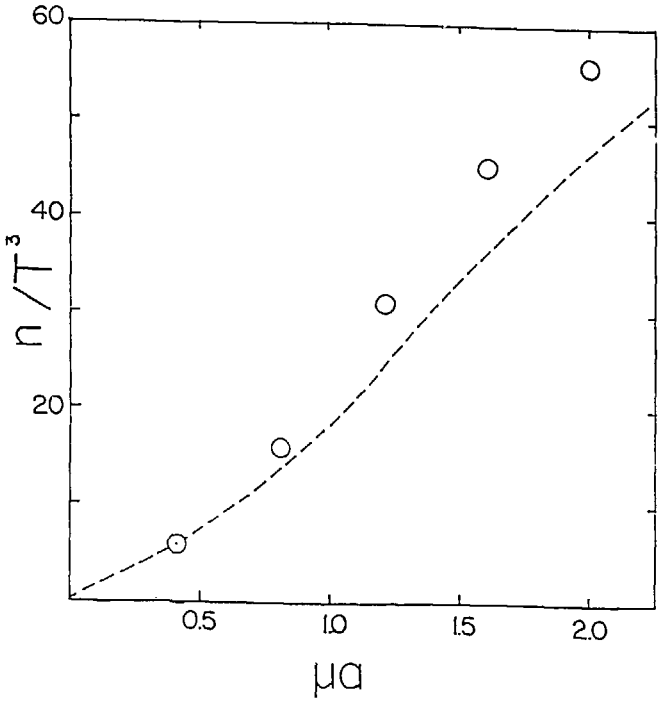

n/T^3
μa
20
40
60
0.5
1.0
1.5
2.0

Fig.5