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# MAIN RING VERSION 11

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HIGH ENERGY PHYSICS

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# CHROMATICITY CORRECTION IN THE TRISTAN PHASE I

# MAIN RING VERSION 11

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## Abstract

This report deals with chromaticity correction in the TRISTAN phase I main ring version 11. The program PATRICIA is used to track the trajectories of test particles over 2000 turns. The results show that particles with transverse initial amplitudes of at least 11 $\sigma$ in both planes and with a synchrotron oscillation amplitude of 7 $\sigma_e$ remain stable.

KEYWORDS : storage rings, chromaticity correction

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## §1. Introduction

The low-β insertions that are incorporated into most storage ring designs result in the chromaticity to a higher value,which may seriously modify the performance of the machine and must be corrected by an appropriate arrangement of sextupoles.

From an optical point of view, it is very difficult to compensate the chromaticity, since the very large chromatic aberration, due mainly to the insertions,where the dispersion function $\eta$ is usually zero, has to be compensated in the arcs where $\eta$ is not zero. Therefore, the chromatic perturbations arising from the insertions will propagate into the main lattice and make compensation complicated. It becomes clear that chromaticity correction with sextupoles must be incorporated into the design of some storage rings.

Compared with other similar machines, the natural r.m.s. energy spread of the TRISTAN phase I main ring[1)] is somewhat large and the dispersion function is somewhat smaller, so the chromaticity correction in the TRISTAN phase I main ring is rather difficult.

Chromaticity corrections for the former TRISTAN phase I main ring versions have been described in Ref. (2).

In the TRISTAN phase I main ring version 11,[3)] besides a low-β scheme, a mini-β scheme will be adopted. The uncorrected chromaticity is larger than former versions and a stronger sextupole field is needed. The arrangement and strengths of correcting sextupoles have to be adjusted carefully in order to reduce some undesirable effects of the correcting sextupoles.

This report will deal with the chromaticity correction for the

TRISTAN phase I main ring version 11 including a low-β scheme and a mini-β scheme. The results obtained show that particles with transverse initial amplitudes of at least 11σ in both planes and with a synchrotron oscillation of 7 $\sigma_e$ remain stable over 9 times the damping time. These results demonstrate the validity of the correcting sextupole schemes adopted.

## §2. Correction Scheme

The chromaticity correction method adopted here is based mainly on the W-correction, which attempts to correct the strong first-order chromatic effects arising from the low-β insertion doublet.

According to the definitions in Ref. (4), the chromatic variables in either transverse plane are

$$B = \frac{\Delta\beta}{\beta} = \frac{\beta(\delta) - \beta(0)}{\sqrt{\beta(\delta)\beta(0)}} \tag{1}$$

$$A = \frac{\alpha(\delta)\beta(0) - \alpha(0)\beta(\delta)}{\sqrt{\beta(\delta)\beta(0)}} \quad , \tag{2}$$

where δ is a momentum deviation $\frac{\Delta p}{p}$.

The equations of motion for A, B are

$$\frac{dB}{ds} = -2A\frac{d\phi}{ds} \tag{3}$$

$$\frac{dA}{ds} = \beta\,\Delta K + 2B\frac{d\phi}{ds} \quad , \tag{4}$$

where $\Delta K = K(\delta) - K(0)$, $\phi = \frac{1}{2}\left[\phi(\delta) + \phi(0)\right]$, K is the gradient parameter. Other parameters have their usual meanings.

In an achromatic region, where $\Delta K = 0$, it follows that

$$\frac{d^2A}{d\phi^2} + 4A = 0 \tag{5}$$

$$\frac{d^2B}{d\phi^2} + 4B = 0 \tag{6}$$

and W is invariant, if

$$W = \frac{1}{2}\left[B_v^2 + B_h^2 + A_v^2 + A_h^2\right]^{1/2} \quad , \tag{7}$$

where v and h represent the vertical and horizontal plane, respectively.

If the chromaticity from the normal cell is rather small, which is corrected in the usual way with sextupoles located close to each quadrupole, and the chromaticity from the RF section is also small, we can see from the eqs. (5) and (6) that the very large chromatic perturbations arising from the low-ß doublet oscillate at twice the betatron frequency and propagate into the main lattice through the RF section without change, except in phase. The perturbations A and B at any given point in the main lattice are just functions of the phase advance between the insertion quadrupoles to that point.

The value of W defined by eq. (7) is evidently a measure of the overall chromatic error in both planes. According to Ref. (4), if at the first sextupole in the regular lattice, $B_h = 0$, $B_v = 0$ and $A_h A_v < 0$, the sextupole will minimize the first-order chromatic error, with minimum higher-order effects. As a result, the perturbations produced by the low-ß doublet are reduced towards the main lattice.

The above conditions strongly depend on the phase advance between the perturbation and the sextupole, so a flexible tunable insertion, in which the phase advance can be adjusted over a wide range, is very important for chromaticity correction.

When the above conditions are satisfied, the first-order chromatic error will be decreased to its lowest value, but not zero. In order to maintain a higher luminosity and reduce the higher-order chromatic effects, we impose the condition that the $W = 0$ at both the interaction point and the centre of the arc.[5] Therefore, 4 families of sextupoles are required. Considering that 2 other families of sextupoles are needed for local correction of the main lattice, then a total of 6 independent families of sextupoles are necessary for correcting the first-order chromatic perturbations. Calculation of the strengths of the correcting sextupoles required for the above conditions have been written into the program MAGIC at KEK.[6]

Besides correcting the first-order chromatic perturbations, sometimes non-linear variations of the β functions and tunes must be further investigated.

As is well known, the general equation of motion for the β function is:

$$\frac{d^2}{ds}(\beta^{1/2}) + K\beta^{1/2} - \beta^{-3/2} = 0 \quad . \qquad (8)$$

Putting $K \rightarrow K + \Delta K$ and $\beta \rightarrow \beta + \Delta\beta$, and keeping terms up to second order in $\Delta\beta$ and $\Delta K$, we have:

$$\frac{d^2}{d\psi^2}\left(\frac{\Delta\beta}{\beta}\right) + 4\nu^2\frac{\Delta\beta}{\beta} = -2\nu^2\beta^2\Delta K - \nu^2\Delta K\beta^2\frac{\Delta\beta}{\beta} + 3\nu^2\left(\frac{\Delta\beta}{\beta}\right)^2 , \qquad (9)$$

where $\psi = \phi/\nu$, $\nu$ being the betatron tune.

If we only keep the linear terms on the right hand side of eq.

(9), it follows

$$\frac{d^2}{d\psi^2}\left(\frac{\Delta\beta}{\beta}\right) + 4\nu^2 \frac{\Delta\beta}{\beta} = -2\nu^2 \frac{\Delta\beta}{\beta}\Delta K \quad , \qquad (10)$$

which is the linear Courant-Snyder equation for $\beta$. The solution of eq. (9) can be written as

$$\frac{\Delta\beta}{\beta} = \left(\frac{\Delta\beta}{\beta}\right)^{(1)} + \left(\frac{\Delta\beta}{\beta}\right)^{(2)} \quad ,$$

$\left(\frac{\Delta\beta}{\beta}\right)^{(1)}$ is the linear part of $\frac{\Delta\beta}{\beta}$, which corresponds to the solution of eq. (10). The problem of using linear theory to minimize $\frac{\Delta\beta}{\beta}$ has been discussed. $\left(\frac{\Delta\beta}{\beta}\right)^{(2)}$ is the term which is quadratic in $\frac{\Delta p}{p}$. When the working point is close to a half-integer value, the maximum variation of $\beta$ with $\delta$ can be written as[7)]

$$\left(\frac{\Delta\beta}{\beta}\right)_{max} = \left(\frac{\Delta\beta}{\beta}\right)^{(1)}_{max} + \left(\frac{\Delta\beta}{\beta}\right)^{(2)}_{max} \qquad (11)$$

$$\left(\frac{\Delta\beta}{\beta}\right)^{(1)}_{max} = \frac{2}{|n - 2\nu|}|a_n|\,\delta \qquad (12)$$

$$\left(\frac{\Delta\beta}{\beta}\right)^{(2)}_{max} = \left[\frac{3}{(n - 2\nu)^2}C\,|a_n| + \frac{1}{(n - 2\nu)^2}|a_n|^2\right]\delta^2 \quad , \qquad (13)$$

where $C = -\frac{1}{4\pi}\int\beta(K - K'\eta)ds$, n is the integer closest to $2\nu$ and $a_n$ is the half-width of the half-integer stop band at $\nu = \frac{n}{2}$,

$$a_n = -\frac{1}{4\pi}\int ds\,\beta\,(K - K'\eta)e^{-in\psi} \quad ., \qquad (14)$$

where $\eta$ is the dipersion function, K' is the strength of sextupole.

From eqs. (12) and (13), we can see that: (1) the second-order variation of $\beta$ is propotional to C. It shows that a higher chromaticity will result in larger non-linear chromatic effects. (2) the second-order term varies inversely as the square of $(2\nu - n)^2$. This shows that the tunes should be chosen to be far from the half integer values. (3) the maximum variation in the $\beta$ function with $\delta$ strongly depends on the $|a_n|$, so we must adjust the strengths and arrangement to make $|a_n|$ minimum; sometimes this is very effective in reducing the nonlinear chromatic effects.

## §3. Chromaticity Correction in TRISTAN Phase I Main Ring Version 11

As stated above, in this version the mini-$\beta$ scheme will be adopted. The first insertion quadrupole magnet will be replaced by a superconducting quadrupole magnet and the distance between the quadrupole and the interaction point is as short as 2.7 m. The beta-function at the interaction point is reduced to $\beta^*_y$ = 0.05 m vertically and $\beta^*_x$ = 0.8 m horizontally and the luminosity is about 9.3 x $10^{31}$ $cm^{-2}s^{-1}$.

Due to the small beta-functions at the interaction point, the $\beta_{max}$ near the interaction point is about 180 m and 160 m in vertical and horizontal planes respectively; the uncorrected chromaticities for this version are $\zeta_x \gtrsim -61$ and $\zeta_y \gtrsim -95$ in the two planes for the mini-$\beta$ scheme, and $\zeta_x \gtrsim -67$, $\zeta_y \gtrsim -95$ for the low-$\beta$ scheme. These values make the chromaticity correction more difficult than former versions.

The procedures for the chromaticity correction are as follows:

(1) For the requirement of the chromaticity correction, the low-$\beta$ insertion and the dispersion suppressor have been rematched.

In the version 11, 21 families of quadrupoles are used in the whole ring, in which 7 families are for the dispersion suppressor, 6 families for the low-$\beta$ insertion and 2 families for the RF section. When matching the suppressor into the low-$\beta$ insertion, the strict periodicity of the $\beta$ function is kept in the RF section. There are 2 degrees of freedom in parameters to be used for adjusting the phase advance when the dispersion suppressor is unchanged. In order to give more freedom in the parameters for changing the phase advance, we use a somewhat different matching procedure to rematch the low-$\beta$ insertion and the dispersion suppressor.

i) In the dispersion suppressor, the number of families of quadrupoles is reduced from 7 to 5 under this circumtances of keeping the necessary flexibility.

Calculations[8)] using the program MAGIC show that we can match the normal cell of 54°, 60° and 90° in phase advance into the low-$\beta$ insertion without additional aperture, and the beta functions in both planes are quite smooth.

ii) The condition which keeps the strict periodicity of the $\beta$ functions in the RF section is given up; 8 families of quadrupoles are used to match the dispersion suppressor into the interaction point. Besides matching the Twiss parameters, there are still 4 degrees of freedom in parameters for changing the phase advance. This enables the working point to be adjusted over a wide range, while keeping the rest of the optics unchanged. The results show that $\nu_x$ can be adjusted from 31 to 33, $\nu_y$ from 37 to 39 and the beta functions in the RF section are almost periodic. The ratio of horizontal and vertical $\beta$ function at the interaction point can be held constant while the $\beta$ functions at the interaction point are increased by a factor of 3.

According to such a matching procedure, 19 families of quadrupoles are used in the whole ring.

Some parameters of this version are given in Table 1.

(2) In the chromaticity correction for this version, 6 independent families of sextupoles are adopted. As stated above, we require that the W-value be zero at both the interaction point and the symmetry point of the arc. According to these conditions, the strengths of the sextupoles can be calculated. A total 240 sextupoles are used in the whole ring.

At first, the phase advance between the insertion quadrupole doublet and the first sextupole is adjusted over quite a large range in order to find a favourable region where

i) particles with larger transverse initial amplitudes remain stable over at least one damping time.

ii) the perturbations produced by the low-$\beta$ doublet are reduced towards the main lattice.

iii) the necessary strengths of the correcting sextupoles are as low as possible and the non-linear effects of the sextupoles are tolerable.

A useful guide in choosing the linear optics is that[9] the horizontal and vertical phases across half of the insertion differ by $\pi/2$. In some situations, this condition can be used to minimize the strengths of the correcting sextupoles.

After a preliminary choice, there are two regions of the working point which are favourable for chromaticity correction, one of them is $\nu_x \approx 33.7$ and $\nu_y \approx 39.7$, where larger betatron and synchrotron amplitudes are allowed and the strengths of the correcting sextupoles are acceptable.

(3) In order to improve the non-linear effects of the correcting sextupoles, a further adjustment will be necessary which includes:

i) adjusting the working point carefully.

ii) adjusting the strengths and arrangement of the first two families of sextupoles while keeping the natural chromaticity zero.

iii) changing the distribution of the β functions and the arrangement of the correcting sextupoles in the wiggler section.

Because the program PATRICIA can compute the half-width of the half-integer stop band $a_n$ at $\nu = n/2$, and indicate which sextupole makes the largest contribution to $a_n$, all the above adjustments must make $a_n$ minimum.

(4) Since the beam-beam interactions apparently cause the particles to have very large transverse amplitudes but the momentum distribution remains Gaussian and falls off very rapidlly,[10] the stability of the particles with as large as possible transverse amplitude and with a synchrotron oscillation only within bucket height (corresponding to 7 $\sigma_e$) is given more attention.

The stability of the betatron oscillation in the corrected machine has been investigated by means of the program PATICIA, which tracks the trajectories of test particles over 9 times the damping time.

(5) The chromaticity corrections for several schemes in version II have been studied and the results are as follows:

i) mini-β (I) $\beta^*_x = 0.8$ m $\beta^*_y = 0.05$ m

$\xi_x = -60.79$ $\xi_y = -94.98$

the stability limits: 11 $\sigma_x$, 11 $\sigma_y$, 7 $\sigma_e$

ii) mini-β (II) $\beta^*_x = 0.9$ m $\beta^*_y = 0.055$ m

$\xi_x = -58.7$ $\xi_y = -88.93$

the stability limits: 12 $\sigma_x$, 12 $\sigma_y$, 7 $\sigma_e$ .

iii) low-β $\beta^*_x = 1.6$ m $\beta^*_y = 0.1$ m

$\xi_x = -66.52$ $\xi_y = -94.87$

the stability limits: 11 $\sigma_x$, 11 $\sigma_y$, 7 $\sigma_e$ .

The stability regions found by tracking the particles for various schemes are shown in Figs. 1, 2 and 3. Figure 4 shows the phase-space diagrams for up to 800 turns ($\sim$ 4 times of the damping time), in both planes, for various schemes. The variations of the β functions and dispersion function $\eta$ at the interaction point with momentum deviation $\Delta p/p$ are shown in Figs. 5, 6 and 7.

The typical arrangement and the strengths of the sextupoles are given in Tables 2, 3 and 4.

## §4. Conclusion

A flexible insertion which can be tuned over a wide range of phase advance is very important for chromaticity correction. Some strategy which includes changes in the strengths and arrangement of the first two sextupole families is very useful for chromaticity correction when the W-correction method is adopted.

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Table 1 General parameter of TRISTAN main ring V 11

| | |
|---|---|
| Beam energy | $E_0$ = 30 GeV |
| Circumference | c = 3018 m |
| Average radius of curved section | R = 480.34 m |
| Number of coll. point | N = 4 |
| Revolution frequency | $f_{rev}$ = 99.33 KHz |
| Synchrotron oscillation frequency | $f_s$ = 10.058 KHz |
| Damping time | $\tau_x$ = 2.08 ms |
| | $\tau_y$ = 2.08 ms |
| | $\tau_E$ = 1.04 ms |
| Partion number | $J_x$ = 1.00005 |
| | $J_E$ = 1.99995 |
| Natural energy spread | $\sigma_E/E_0 = 0.164 \times 10^{-2}$ |
| Natural horizontal emittance | $\varepsilon_{xo} = 0.1798 \times 10^{-6}$ m rad |
| Bucket height | $\Delta E/E = 1.089 \times 10^{-2}$ |
| Betatron oscillation tune | |
| mini-ß | $\nu_x/\nu_y$ = 33.79/39.72 |
| low-ß | $\nu_x/\nu_y$= 33.7500/39.7496 |
| Phase advance per normal cell | $\mu_x = \mu_y = 60°$ |
| Natural chromaticity | |
| mini-ß | $\xi_x$ = - 60.79 |
| | $\xi_y$ = - 94.98 |
| low-ß | $\xi_x$ = - 66.52 |
| | $\xi_y$ = - 94.87 |
| Luminosity | |
| mini-ß | $L = 9.315 \times 10^{31}$ $cm^{-2}s^{-1}$ |
| low-ß | $L = 4.657 \times 10^{31}$ $cm^{-2}s^{-1}$ |
| Beta function at coll. paint | |
| mini-ß | $\beta^*_x/\beta^*_y$ = 0.8 m/0.05 m |
| low-ß | $\beta^*_x/\beta^*_y$ = 1.6 m/0.1 m |

## Table 2 Lattice Layout (Mini-beta)

| Name | Length (m) | | | | | | |
|---|---|---|---|---|---|---|---|
| Mini-ß insertion | 97.4728 | LC0 | QCSH | LC1 | QC1H | LC2 | QC3H |
| and RF section | | LC4 | QC4H | LRF | QC5H | LRF | QC6H |
| | | LRF | | | | | |
| | | 3 | [QRDH | LRF | QRFH | LRF] | QRDH |
| | | LRF | QRFH | LBF | | | |
| Dispersion suppressor | 61.63 | BXW | L.55 | 2 | [QS3H | LBF | BX |
| | | L.55 | QS4H | LBF | BX | L.55] | QS5H |
| | | L.55 | SD1 | L.55 | BX | LBF | QS6H |
| | | L425 | SF1 | L425 | BX | L.55 | QS7H |
| | | L.55 | SD3 | L.55 | BX | LBF | |
| Normal cell | 193.44 | 4[cell (SF2, SD3) cell (SF3, SD1), cell (SF1, SD2)] | | | | | |
| Wiggler cell | 24.7171 | QW4H | L425 | SF2 | L425 | BX | L.55 |
| | | QW3H | L.55 | SD3 | L.55 | BX | LBF |
| | | QW2H | L425 | SF3 | L425 | LW2 | QW1H |
| | | LW1 | | | | | |
| Cell (SF, SD) | | QFH | L425 | SF | L425 | BX | L.55 |
| | | ODH | L.55 | SD | L.55 | BX | LBF |

Drift lengths

| Name | length (m) | Name | length (m) |
|---|---|---|---|
| LC0 | 2.70 | LC1 | 0.8 |
| LC2 | 11.4366136 | LC4 | 5.0 |
| LRF | 6.0736219 | L.55 | 0.55 |
| L425 | 0.425 | LBF | 0.3 |
| LW2 | 1.647043 | LW1 | 4.5 |

Table 3 Lattice Layout (Low-beta)

| Name | Length (m) | | | | | | |
|---|---|---|---|---|---|---|---|
| Low-ß insertion | 97.4728 | LCO | QC1H | LC2 | QC2H | LC3 | QC3H |
| and RF section | | LC4 | QC4H | LRF | QC5H | LRF | QC6H |
| | | LRF | | | | | |
| | | 3 | [QRDH | LRF | QRFH | LRF] | QRDH |
| | | LRF | QRFH | LBF | | | |
| Dispersion suppressor | 61.63 | BXW | L.55 | 2 | [QS3H | LBF | BX |
| | | L.55 | QS4H | LBF | BX | L.55] | QS5H |
| | | L.55 | SD1 | L.55 | BX | LBF | QS6H |
| | | L425 | SF1 | L425 | BX | L.55 | QS7H |
| | | L.55 | SD3 | L.55 | BX | LBF | |
| Normal cell | 193.44 | 4[cell (SF2, SD3) cell (SF3, SD1), cell (SF1, SD2)] | | | | | |
| Wiggler cell | 24.7171 | QW4H | L425 | SF2 | L425 | BX | L.55 |
| | | QW3H | L.55 | SD2 | L.55 | BX | LBF |
| | | QW2H | L425 | SF3 | L425 | LW2 | QW1H |
| | | LW1 | | | | | |
| Cell (SF, SD) | | QFH | L425 | SF | L425 | BX | L.55 |
| | | ODH | L.55 | SD | L.55 | BX | LBF |

Drift lengths

| Name | length (m) | Name | length (m) |
|---|---|---|---|
| LCO | 4.5 | LC2 | 1.0 |
| LC2 | 7.9366136 | LC4 | 5.0 |
| LRF | 6.0736219 | L.55 | 0.55 |
| L425 | 0.425 | LBF | 0.3 |
| LW2 | 1.647043 | LW1 | 4.5 |

Table 4 Sextupole parameters

mini-β (I)

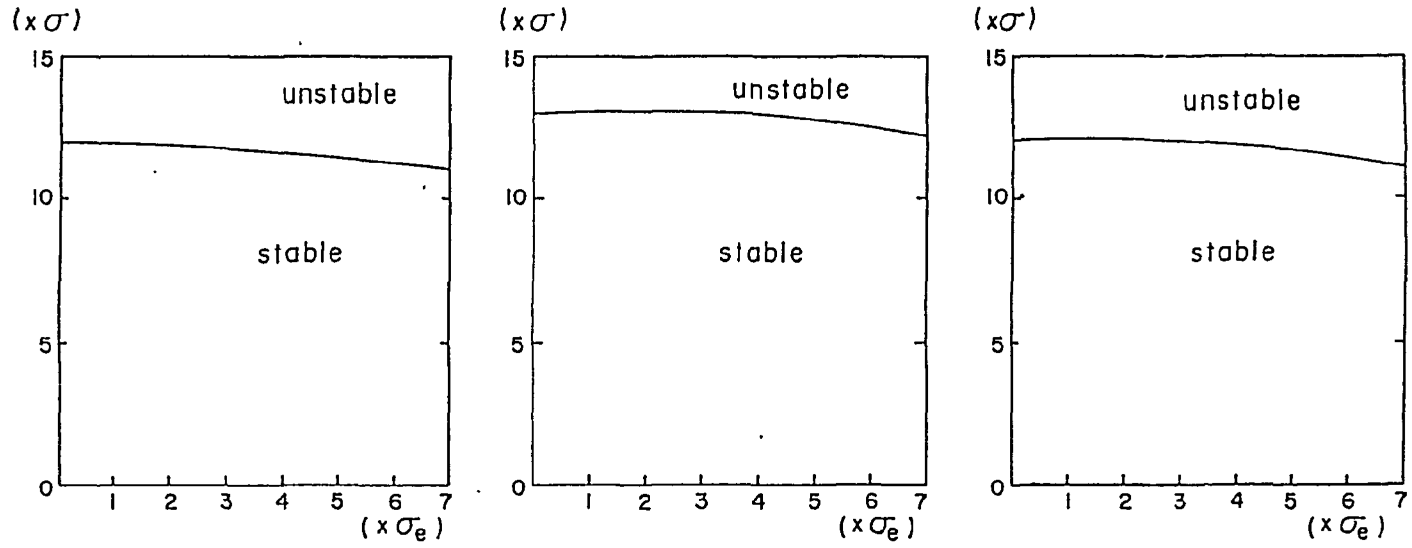
| Name | No. | Strength | ($\frac{1}{B\rho}$ B"ℓ, $M^{-2}$) |
|---|---|---|---|
| SF1 | 40 | - 0.33194 | |
| SF2 | 40 | - 0.56680 | |
| SF3 | 40 | - 0.67137 | |
| SD1 | 40 | 0.23539 | |
| SD2 | 32 | 1.28292 | |
| SD3 | 48 | 1.40219 | |

mini-β (II)

| Name | No. | Strength | ($\frac{1}{B\rho}$ B"ℓ, $M^{-2}$) |
|---|---|---|---|
| SF1 | 40 | - 0.32508 | |
| SF2 | 40 | - 0.52978 | |
| SF3 | 40 | - 0.64015 | |
| SD1 | 40 | 0.27415 | |
| SD2 | 40 | 1.17849 | |
| SD3 | 40 | 1.32184 | |

low-β

| Name | No. | Strength | ($\frac{1}{B\rho}$ B"ℓ, $M^{-2}$) |
|---|---|---|---|
| SF1 | 40 | - 0.35250 | |
| SF2 | 40 | - 0.53649 | |
| SF3 | 40 | - 0.75326 | |
| SD1 | 40 | 0.29849 | |
| SD2 | 48 | 1.25178 | |
| SD3 | 32 | 1.50000 | |




(Mini-Beta I) (Mini-Beta II) (Low-Beta)

Fig. 1 Stability limit vs. synchrotron oscillation amplitude.

$\sigma$ : number of standard deviations of transverse amplitude.

$\sigma_e$: number of standard deviations of momentum deviation.

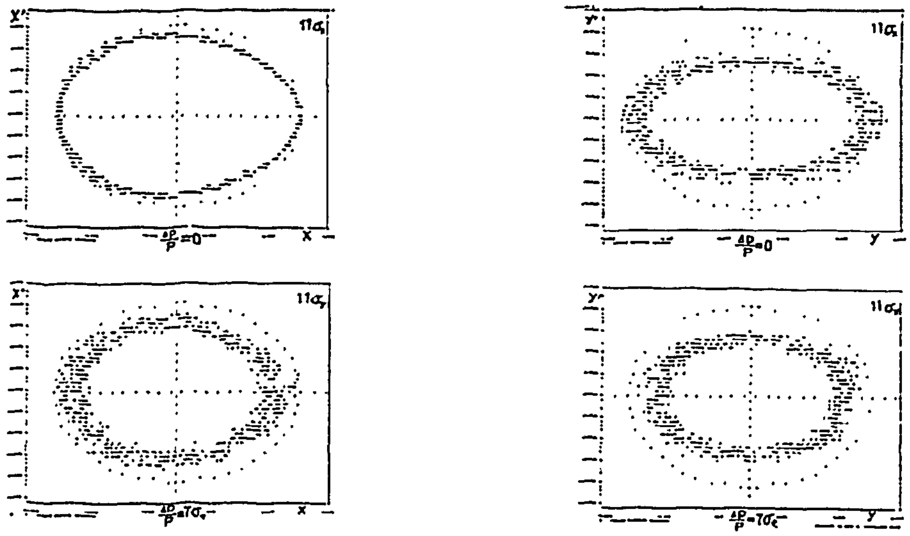


Fig. 2 Phase space diagram when tracking with the program PATRICIA. Particles are shown with 11 standard deviations of transverse amplitude and 0 and 7 momentum deviation. (Mini-Beta I)

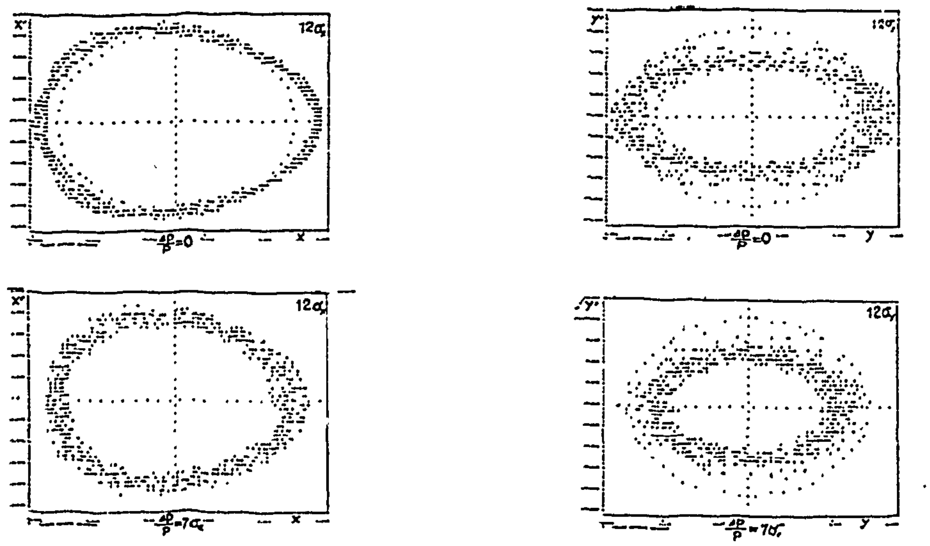


Fig. 3 Phase space diagram when tracking with the program PATRICIA. Particles are shown with 12 standard deviations of transverse amplitude and 0 and 7 momentum diviation. (Mini-Beta II)

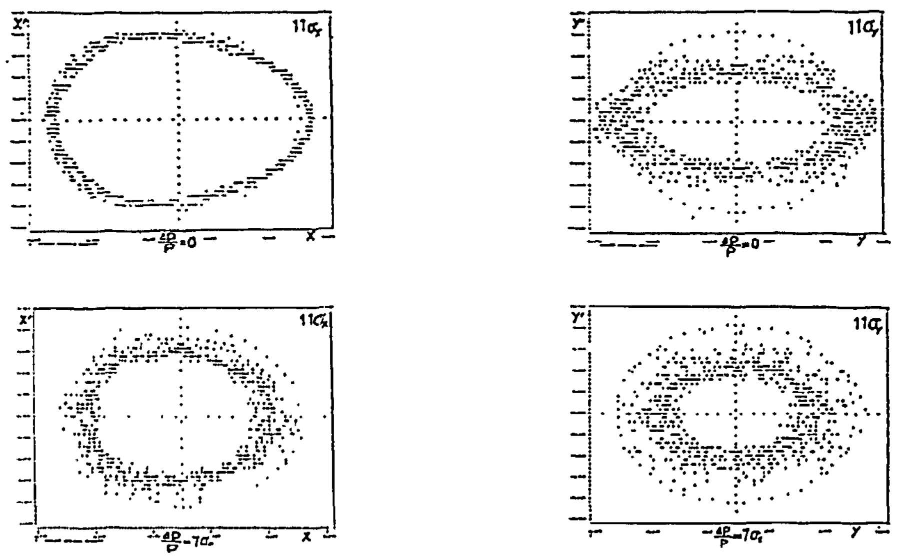


Fig. 4 Phase space diagram when tracking with the program PATRICIA. Particles are shown with 11 standard deviations of transverse amplitude and 0 and 7 momentum deviation. (Low-Beta)

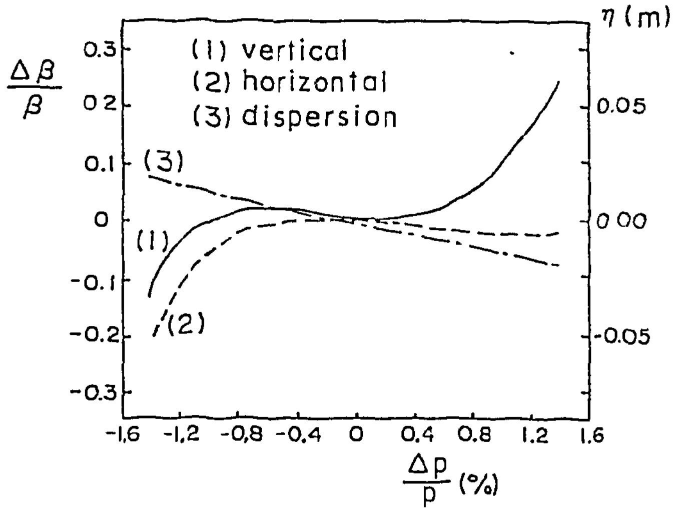



Fig. 5(a) Variations of $\beta$ functions and $\eta$ function with $\Delta p/p$. (Mini-Beta I)

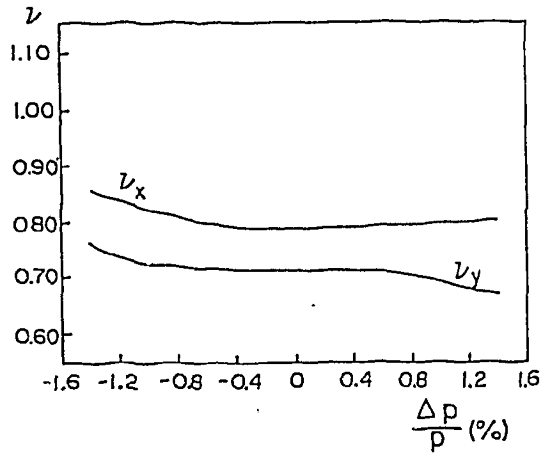



Fig. 5(b) Variation of non integral part of the tunes with momentum. (Mini-Beta I)

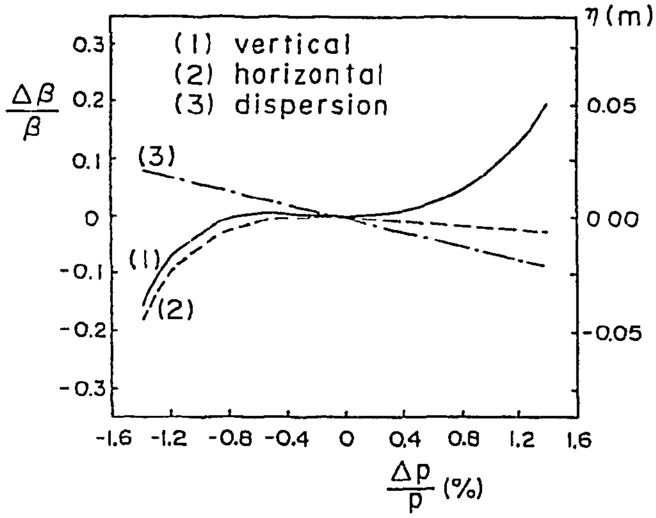



Fig. 6(a) Variations of β functions and η function with Δp/p. (Mini-Beta II)

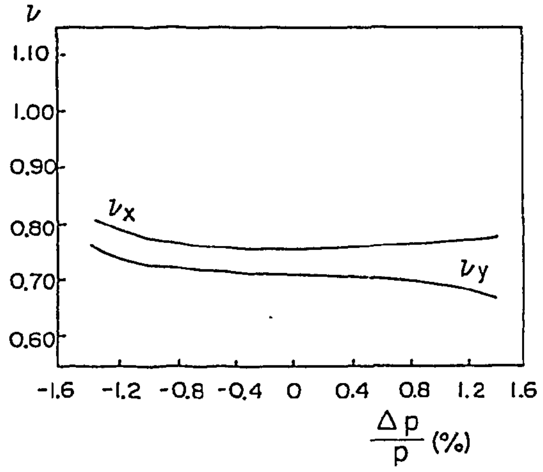



Fig. 6(b) Variation of non integral part of the tunes with momentum. (Mini-Beta II)

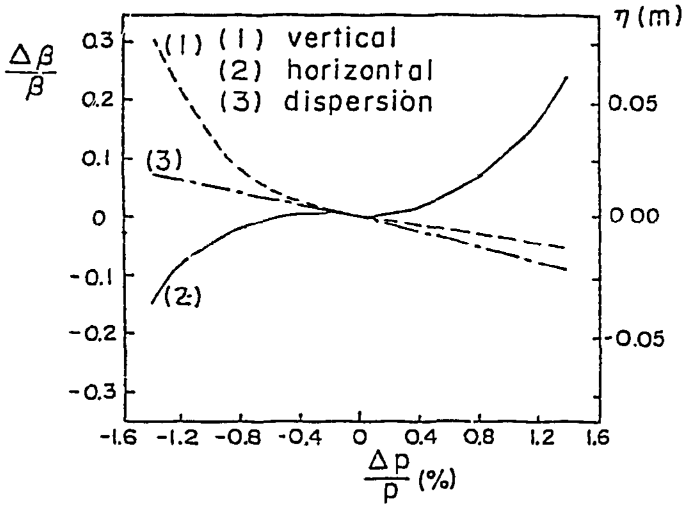



Fig. 7(a) Variations of β functions and η function with Δp/p. (Low-Beta)

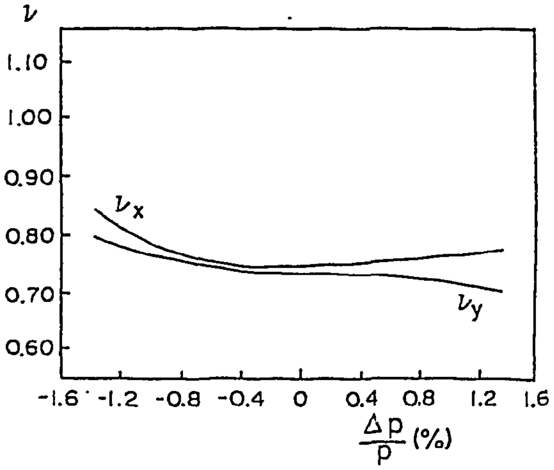



Fig. 7(b) Variation of non integral part of the tunes with momentum. (Low-Beta)