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Abstract

The linear analysis by the six-moment method proposed by Shirazian for calculating the heat flux in a steep temperature gradient plasma is extended to the second order. The heat flux is expressed in terms of a ratio (inverse of Knudsen number) L/λ, L and λ are the scale length of the temperature gradient and the mean free path. In the collisional and small temperature gradient limit, the second order heat flux is $q=0.1416[1-(9/52)(1-R_t^2)][2/(3-R_t)][2/(1+R_t)]^{5/2}q_L$, (Shirazian's first order result is $0.1416\ [2/(3-R_t)][2/(1+R_t)]^{5/2}q_L$), where q_L is the exact result derived by Landshoff for the Lorentzian plasma and $R_t=T_c/T_h$, where T_h and T_c are the temperatures of hot and cold walls. The discrepancy between q and q_L comes from the lack of number of moments and the assumed form of the electron velocity distribution function. A new trial distribution function including eight moments is proposed which may be more appropriate for the Lorentzian plasma. A criterion for the choice of good trial function is proposed and tested.

§1. Introduction

In laser produced plasmas, steep temperature gradient exists of which scale length L is of the same order to the mean free path λ. Usual heat flux derived in the limit $L/\lambda \to \infty$ [1], is not valid. For arbitrary values of L/λ, many attempts have been made for calculating heat flux. They can be grouped into two. There are two ways of attacking the problem. One is the microscopic method[2] in which a deviation of the electron velocity distribution function from the equilibrium one is derived . Because of the large temperature gradient, velocity distribution function can become negative. This is the difficulty of the microscopic method.

The other one is the moment method[3,4] . This has no such problem, but it can have a lack of information in some case, because it describes plasma by a finite number of moments. Shirazian and Steinhauer[3] considered the plasma which exists between hot and cold heat baths separated by the distance L. (Fig.1) They employed the velocity distribution function

$$f = \begin{cases} n_1\left(\frac{m}{2\pi T_1}\right)^{3/2}\exp\left(-\frac{mV^2}{2T_1}\right)\left[1+a^{+}\frac{V_z}{V_{T_1}}\left(5/2 - v^2/v_{T_1}^2\right)\right], & v_z>0, \\ n_2\left(\frac{m}{2\pi T_2}\right)^{3/2}\exp\left(-\frac{mV^2}{2T_2}\right)\left[1+a^{-}\frac{V_z}{V_{T_2}}\left(5/2 - v^2/v_{T_2}^2\right)\right], & v_z<0. \end{cases} \quad (1)$$

The six unknowns (six moments) $n_1, n_2, T_1, T_2, a^+, a^-$ and one more unknown E, the electric field, can be determined by solving moment equations derived from the Boltzmann equation. Linearizing the moment equations with respect to small deviation from the hot elements, Shirazian et al. obtained the heat flux q (Ref.3 p.848). We will extend Shirazian and Steinhauer's linear treatment to the second order in the next section. Further we will develop a linear treatment by using eight-moment method and will discuss on the choice of the trial function.

§2. The Second Order Treatment

The six moments are assumed $n_1=n_h(1+\eta_{11}+\eta_{12})$, $n_2=n_h(1+\eta_{21}+\eta_{22})$, $T_1=T_h(1+\tau_{11}+\tau_{12})$, $T_2=T_h(1+\tau_{21}+\tau_{22})$, $a^+=a_1^++a_2^+$, $a^-=a_1^-+a_2^-$, $1\gg|\eta_{11}|\gg|\eta_{12}|$ etc. The normalization constants n_h and T_h are the number density and the temperature of the hot heat bath (Fig.1). The moment equation derived from the Boltzmann equation is given by

$$\frac{\partial}{\partial z} M[v_z Q(\mathbf{v})] + \frac{eE}{m} M[\frac{\partial Q(\mathbf{v})}{\partial v_z}] = C[Q(\mathbf{v})] , \qquad (2)$$

where $Q(\mathbf{v})$ is an arbitrary function of velocity v and the moment $M[Q(\mathbf{v})]=\int d\mathbf{v}Q(\mathbf{v})f(\mathbf{v})$ and $C[Q(\mathbf{v})]$ is the collision integral. Substituting (1) into (2) and choosing $Q(\mathbf{v})=1, v_z, v^2, v_z v^2, v_z^2, v_z^3$, we can obtain six moment equations. We solved these moment equations to the second order and obtained the second order correction q_2 to the heat flux

$$q_2=[K_1 \sinh x + (K_2 \sinh x + K_3 \cosh x + K_4) L/\lambda$$

$$- \{(R \sinh x - \cosh x)/(P \cosh x - Q \sinh x)\}\{K_5 \sinh x$$

$$+ K_6 \cosh x + (K_7 \sinh x + K_8 \cosh x) L/\lambda + K_9\}] q_1/2 , \quad (3)$$

where P,Q,R are the numerical constants and $K_1, K_2, \ldots, K_9$ are functions of $x=(5\pi/2)^{1/2}(9/32)(L/\lambda)$. (See APPENDIX A) The first and the second order heat fluxes $[q]_1=q_1$, and $[q]_2=q_1+q_2$ are plotted in Fig.2. Landshoff's value[1] $q_L=(128/9)(2/m\pi)^{1/2} n_e (T^{5/2}/T_h)(1-R_t)(\lambda/L)$, which is exact in the limit $L/\lambda \to \infty$ and $T_h-T_c \to 0$ is given as a reference. In this limit,

$$[q]_1 = 0.1416[2/(3-R_t)][2/(1+R_t)]^{5/2} \, q_L ,$$

$$[q]_2 = 0.1416[1-(9/52)(1-R_t^2)][2/(3-R_t)][2/(1+R_t)]^{5/2} \, q_L , \quad (4)$$

where $R_t=T_c/T_h$. Our result valid to the second order does not approach to q_L. This may be due to the assumed electron velocity distribution function (1) including only six moments.

§3. Eight-Moment Method

By Landshoff[1], an exact velocity distribution for the

Lorentzian plasma in the collisional limit is given by $f=f_M(1+v_z h)$,

$$h = \frac{v^3}{c}\left[-\frac{eE}{T} + \frac{1}{T}\frac{dT}{dz}\left(\frac{5}{2} - \frac{mv^2}{2T}\right)\right] , \tag{5}$$

where c is a quantity related to the collision frequency. Then the following velocity distribution function must be more appropriate than (1).

$$f=\begin{cases} f_{M_1}\,[\,1 + (v_z v^3/v_{T_1}^4)\{\,b^+ + a^+(5/2 - mv^2/2T_1)\}],\ v_z>0, \\ \\ f_{M_2}\,[\,1 + (v_z v^3/v_{T_2}^4)\{\,b^- + a^-(5/2 - mv^2/2T_2)\}],\ v_z<0. \end{cases} \tag{6}$$

The factor v^3 comes from the collision time τ which is proportional to v^3 in Lorentzian plasma as seen in (5). We can have eight moment equations by taking $Q(v) = 1,\ v_z,\ v^2,\ v_z v^2,\ v_z^2,\ v_z^3,\ v_z^4,\ v^4$. Normalizing and linearizing eight moments $n_{1,2}(z)$, $T_{1,2}(z)$, $a^{\pm}(z)$ and $b^{\pm}(z)$, we can develop the linear theory to obtain the heat flux q

$$\frac{q}{q_f} = - \frac{C_5\cosh X + C_6\sinh X}{\{C_1 + C_2(L/\lambda)\}\cosh X + \{C_3 + C_4(L/\lambda)\}\sinh X} , \tag{7}$$

where $C_1, C_2, \ldots, C_6$ are numerical constants (See APPENDIX B),

$X=(9/4)(10/1059)^{1/2}(L/\lambda)$ and the free streaming heat flux $q_f=(2/\pi m)^{1/2}(n_h T_h^{3/2}-n_c T_c^{3/2})$. In Fig.3, q/q_f is plotted as a function of the collisionality parameter L/λ. In the collisional limit, (7) reduces to $q=0.0184[2/(3-R_t)][2/(1+R_t)]^{5/2}q_L$. Contrary to our expectation the trial function (6), which has the exact dependence on velocity in the linear collision regime, does not give Landshoff's value q_L in the collisional limit. Shirazian et al. used the trial function (1) which does not have the exact velocity dependence in the collisional limit, but obtained much closer value (which is given by $[q]_1$ in (4)) to q_L. Then the exact velocity dependence of the trial function in the linear theory does not always give the exact collisional limit within the framework of the nonlinear theory. In the linear theory, $|v_z h|<<1$ is assumed and this term is neglected in the left hand side of the Boltzmann equation. This corresponds to approximate $(d/dz)[\int d\mathbf{v}Q(\mathbf{v})f_M(1+v_z h)]$ by $(d/dz)[\int d\mathbf{v}Q(\mathbf{v})f_M]$. But in the nonlinear formulation, we do not neglect $v_z h$ term. Then if we choose the trial function $f=f_M(1+v_z h)$ such that $|\int d\mathbf{v}Q(\mathbf{v})f_M v_z h|$ has a value as small as possible, we will obtain closer value to q_L in the collisional limit. In fact the function (6) has greater contribution to $|\int d\mathbf{v}Q(\mathbf{v})f_M v_z h|$ than (1). To illustrate the above statement, we employ a simple function

$$f = \begin{cases} f_{M_1}[\,1 + (v_z/v_{T_1})a^+\,]\;, & v_z>0, \\ f_{M_2}[\,1 + (v_z/v_{T_2})a^-\,]\;, & v_z<0. \end{cases} \tag{8}$$

This gives smaller integral $\left|\int d\mathbf{v}\ Q(\mathbf{v}) f_M v_z h\right|$ than (6). The six-moment formulation based on (8) gives the heat flux $q=0.049[2/(3-R_t)][2/(1+R_t)]^{5/2} q_L$ in the collisional limit, which is larger than the one obtained by using (6).

§4. Conclusions

1. Shirazian and Steinhauer's first order treatment of the nonlinear heat flux based on the six-moment method is extended to the second order. The expression of the second order correction to the first order heat flux is given by (3). In Fig.2, normalized heat flux is given versus L/λ. The second order heat flux is smaller than the first order one. Especially in the collisional limit, i.e., $L/\lambda \to \infty$, the heat flux is smaller than the exact value q_L. This is thought to be originated from the choice of the poor trial function (1) with the six moments.

2. Eight-moment trial function (6) which has the exact dependence on velocity in the collisional limit is used and the expression (7) of the nonlinear heat flux is derived and drawn in Fig.3 . This time, we have smaller heat flux than the one derived by the poor six moments trial function (1), though we employed "exact" trial function with eight moments given by (6). In order to obtain macroscopic quantities such as the heat flux which is the averaged quantity over the velocity, the "exact" velocity dependence of the trial function is not always considered to be important.

3. A criterion for choosing the trial function which will give better value of the heat flux in the collisional limit is proposed and tested. When the trial function is written as $f=f_M(1+\tilde{v}_z h(\tilde{v}))$, $\tilde{v}$ and $\tilde{v}_z$ being normalized velocity and its z component, the criterion is expressed by $|\int d\mathbf{v} f_M Q(\mathbf{v}) \tilde{v}_z h(\tilde{v})| \ll |\int d\mathbf{v} f_M Q(\mathbf{v})|$. As an illustration, a simple trial function (8) which gives actually smaller integral than (6) is shown to give a better heat flux of which collisional limit is 2.7 times as large as the one derived from the trial function (6). Since $h(\tilde{v})$ includes moments, the above integral depends on the moments. In the comparison made above between two cases based on (6) and (8), we assumed implicitly that the moments in both cases have same order of magnitude.

4. Though we have a guide for the choice of the trial function, we still do not get to the best trial function at present.

Acknowledgement

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Appendix A Expressions of $K_1, K_2, \ldots, K_9$

$K_1 = 2[(4PQ/\pi + 6Q/\pi) C_1 C_4 + (112PQ/15\pi - 15Q/\sqrt{\pi} - R) C_1 C_5]$,

$K_2 = (9/8)\sqrt{5\pi/2}\, C_1 C_4$, $K_3 = 2(27/16 - 78/25\pi) C_1 C_4$,

$K_4 = (27/50\pi) C_3 C_4 - (6/25\pi) C_3 C_5$, $K_5 = 2QC_1 C_4$, $K_6 = -2PC_1 C_4$,

$K_7 = -(9/8)\sqrt{5\pi/2}\ PC_1C_4$, $\quad K_8 = (9/8)\sqrt{5\pi/2}\ QC_1C_4$,

$K_9 = (P/2)C_3(C_4 + 2C_5)$,

$C_1 = P(1-R_t)/2D$, $\quad C_3 = (P\cosh x - Q\sinh x)(1-R_t)/D$,

$C_4 = R_t - 1$, $\quad C_5 = P(1-R_t)\{(Q/\sqrt{\pi})\sinh x - (1/2)\cosh x\}/D$,

$D = P(1+(39/25\pi)(L/\lambda))\cosh x - (PR + (39Q/25\pi)(L/\lambda))\sinh x$,

$P = 8/5\sqrt{\pi}$, $\quad Q = \sqrt{2/5}$, $\quad R = (416/75\pi - 3)\sqrt{2/5\pi}$,

$R_t = T_c/T_h$

Appendix B Values of $C_1, C_2, \ldots, C_6$

$C_1 = 384W + 3V - 2YV$, $\quad C_2 = (92-64Y)U$, $\quad C_3 = 16PW - 3RV - 2SV$,

$C_4 = -(92R + 64S)U$, $\quad C_5 = 2(2Y+1)/5$, $\quad C_6 = 2(2S-R)/5$,

$P = (9/4)\sqrt{1059/10} + 1365\sqrt{10/1059}$,

$R = (1632/35)\sqrt{10/1059}$, $\quad S = (752/35)\sqrt{10/1059}$

$U = 9/320$, $\quad V = 4/5$, $\quad W = 1/80$, $\quad Y = 19/2$.

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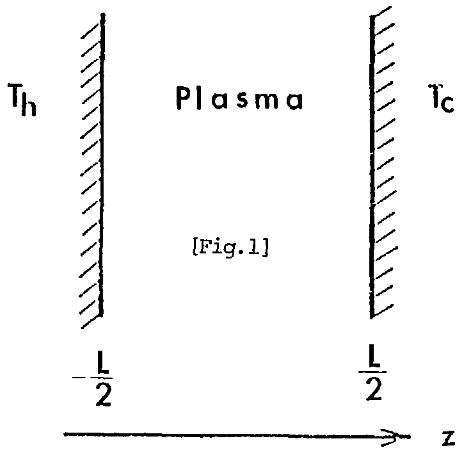
Figure Captions

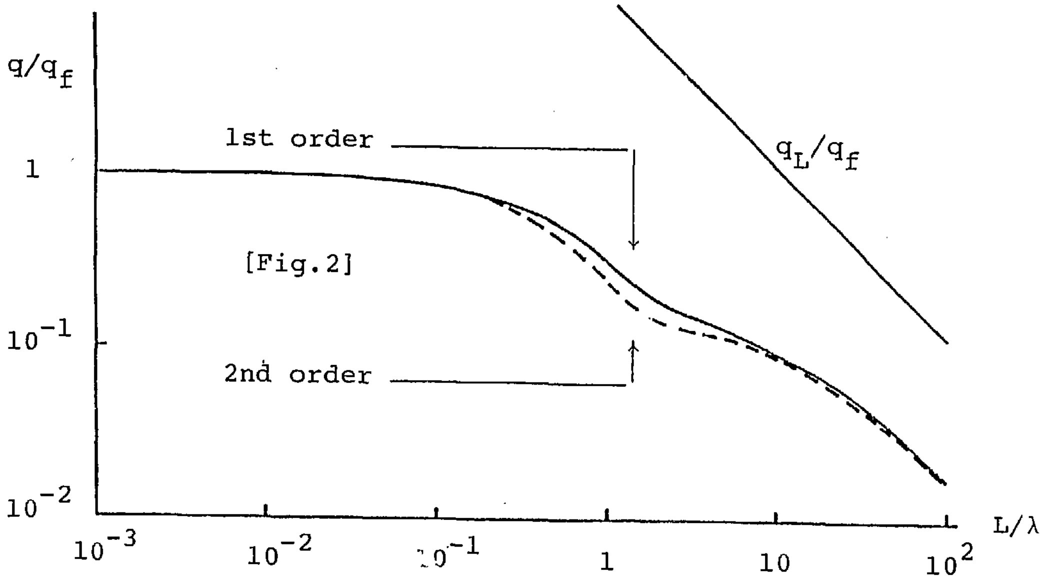
Fig.1 Situation considered

Fig.2 First and second order heat fluxes derived by Shirazian et al.'s six-moment theory. Landshoff's value q_L is given which is exact in the linear collisional limit. Heat flux q is normarized by the free streaming value q_f.

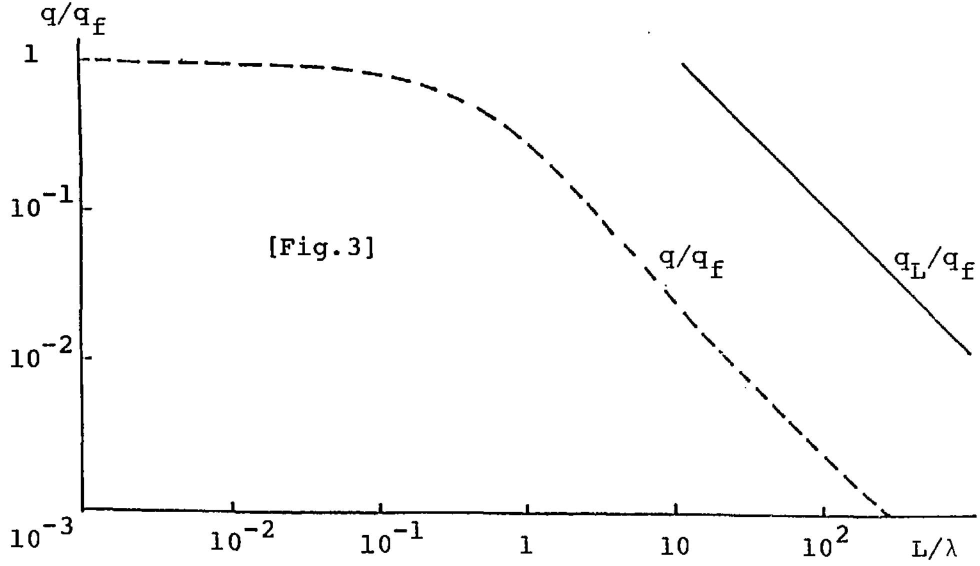
Fig.3 Heat flux q derived by the eight-moment theory.



[Fig.1]



[Fig.2]



[Fig.3]