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AN EXTENDED FORMULATION OF MAXWELL'S EQUATIONS

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ABSTRACT

An extended Lorentz invariant formulation of Maxwell's equations is presented which both includes time dependent and steady-state solutions. In this approach the charge and current densities are treated as intrinsic properties of the electromagnetic field itself, in vacuo.

Two main results follow from such an approach. First, a longitudinal electric wave is predicted to propagate in vacuo. Second, an axially symmetric steady state can be outlined in which "self-confined" electromagnetic radiation circulates in closed orbits around the axis of symmetry. For this state values are obtained of the charge, the spin, and the product between magnetic moment and mass which are of the same order of magnitude as those observed for some elementary particles such as the proton and electron. Consequently, this may provide certain areas of conventional elementary particle analysis with some complementary ideas. Whether the predicted new phenomena also correspond to physical realities is so far an open question which requires further investigation.

1. Introduction

The present paper has its origin in some unpublished ideas by the author in the 1960's, in attempts to explain particles with a rest mass as various eigenmodes of "self-confined" electromagnetic relation. These ideas have some features in common with a theory on oscillating cavity modes established by Jennison and Drinkwater[1] and Jennison[2,3], as well as with an analysis on the electron presented by Yadava[4].

In this paper an extended Lorentz invariant formulation of Maxwell's equations is presented in Section 2 which does not only include time dependent phenomena but also steady-state solutions. In the approach to be described here the charge and current densities are treated as intrinsic properties of the electromagnetic field itself, in vacuo.

There are two main results deduced from this approach. First, a longitudinal electric wave is predicted as shown in Section 3. Second, an axially symmetric steady solution is presented in Section 4, giving a number of relations between particle charge, mass, magnetic moment, and spin. These lead to data being of the same order of magnitude as the data observed for some particles such as the proton and electron.

2. Basic Equations and their Extended Formulation

The present theory is based on Maxwell's equations

$$\mathrm{curl}\,\underline{B}/\mu_0 = \underline{j} + \varepsilon_0 \partial\underline{E}/\partial t \tag{1}$$

$$\mathrm{curl}\,\underline{E} = -\partial\underline{B}/\partial t \tag{2}$$

$$\mathrm{div}\,\underline{E} = \sigma/\varepsilon_0 \tag{3}$$

where SI units are used throughout this paper, $\underline{E}$ and $\underline{B}$ are the electric and magnetic fields, $\underline{j}$ is the current density, and σ the electric charge density. The divergence of eq. (1) leads to eq. (3) and the divergence of eq. (2) results in $\mathrm{div}\underline{B} = 0$. Conventional interpretation of the densities σ and $\underline{j}$ implies that these quantities are due to electrically charged particles and their motion. Such an interpretation implies that σ and $\underline{j}$ vanish in vacuo where there only remain transverse electromagnetic waves, propagating at the velocity $c = (1/\mu_0\varepsilon_0)^{1/2}$ of light.

Here the interpretation of σ and $\underline{j}$ will be extended to include intrinsic properties of the electromagnetic field in vacuo. Thus, eq. (3) is now understood also to include a situation where the divergence of the electric field by itself produces a space charge. Then, the motion of such a space charge should generate an electric current density

$$\underline{j} = \sigma\underline{V} \tag{4}$$

where $\underline{V}$ is an equivalent so far unspecified velocity in three-space. To preserve the Lorentz invariance of the system (1)-(4), the four-dimensional current density $\underline{J} \equiv (\underline{j}, ic\sigma) = \sigma(\underline{V}, ic)$ has to become a four-vector also in this extended formulation, i.e. $\underline{J}$ has to transform like coordinate differences $(\Delta x, \Delta y, \Delta z, ic\Delta t)$ in four-space. This results in the requirement that

$$j^2 - c^2\rho^2 = \rho^2(V^2-c^2) = \text{const.} = 0 \qquad V^2 \equiv \underline{V}^2 = c^2 \tag{5}$$

where the physically relevant condition is imposed for which $\underline{j} = 0$ when $\rho = 0$. This condition is analogous to a choice of the origin in four-space such as to make $x^2+y^2+z^2 = 0$ when $t=0$. In this way eqs. (1)-(5) provide an extended Lorentz invariant formulation of Maxwell's equations which includes all earlier treated electromagnetic phenomena, but also contains a new class of time dependent and steady solutions. The latter class will be illustrated by some examples given in the following sections.

According to conventional electromagnetic theory, the present equations result in

$$\underline{B} = \text{curl } \underline{A} \tag{6}$$

$$\underline{E} = -\underline{\nabla}\phi - \partial\underline{A}/\partial t \tag{7}$$

Poynting's theorem further yields the total electromagnetic field energy

$$W_f = (\varepsilon_0/2) \iiint_{-\infty}^{+\infty} (E^2 + c^2B^2)dxdydz \tag{8}$$

3. Plane Field Geometry

In plane geometry, where all quantities become functions of (x,t) in a frame (x,y,z), the basic equations (1)-(4) reduce to $B_x = 0$ and

$$V_x(\partial E_x/\partial x) + \partial E_x/\partial t = 0 \tag{9}$$

$$- c^2(\partial B_z/\partial x) = V_y(\partial E_x/\partial x) + \partial E_y/\partial t \tag{10}$$

$$c^2(\partial B_y/\partial x) = V_z(\partial E_x/\partial x) + \partial E_z/\partial t \tag{11}$$

$$\partial E_z/\partial x = \partial B_y/\partial t \tag{12}$$

$$\partial E_y/\partial x = -\partial B_z/\partial t \tag{13}$$

3.1. An Example on Steady Equilibrium

In their extended form eqs. (1)-(5) also allow for steady solutions. A simple illustration is given by $\underline{V} = (0, \pm c, 0)$, $\underline{E} = (E, 0, 0)$ and $\underline{B} = (0, 0, B)$ in a geometry where the fields $\underline{E}$ and $\underline{B}$ are symmetric or antisymmetric with respect to the plane $x = 0$. Then equations (9)-(13) lead to

$$E(x) = \pm cB(x) \tag{14}$$

This solution can be interpreted as an equilibrium where the Lorentz force $\underline{j} \times \underline{B} = \varepsilon_0(\mathrm{div}\underline{E})\underline{V} \times \underline{B}$ balances the magnetic field "pressure gradient" represented by $(\mathrm{curl}\underline{B}) \times \underline{B}/\mu_0 = -\nabla(B^2/2\mu_0)$.

3.2. Plane Waves

We next consider plane waves where all field quantities vary as $\exp[i(\omega t + kx)]$. Among the possible solutions of eqs. (9)-(13) the following are used here as illustrations:

(i) When $E_x \equiv 0$, the set of eqs. (9)-(13) reduces to the conventional equations for transverse electromagnetic waves in vacuo.

(ii) When $E_x \neq 0$, and we limit ourselves to the particular case $\underline{V} = (\pm c, 0, 0)$, the set of eqs. (10)-(13) becomes decoupled from eq. (9). The former set still describes the conventional transverse electromagnetic mode. In addition to this, eq. (9) leads to the dispersion relation $\omega = \pm ck$ for a purely longitudinal wave which only includes the electric field component E_x and has no magnetic field components. That the phase velocity of this wave is equal to the velocity c of light is consistent with the assumption of Lorentz invariance made in Section 2. An attempt to understand this type of wave can be based on physical arguments being analogous to those which are often used to explain the origin of the transverse wave from the acceleration of an electric point charge. First a homogeneous electric field $E_{xo} = -d\phi_o/dx = \text{const.}$ is assumed to be directed along x in a plane static configuration. Then a sudden change of the potential $\phi(x)$ is assumed to take place at some plane $x = x_1$. This would result in a plane disturbance of the potential pattern, propagating at a finite velocity c along x. Therefore there must arise a "kink" on this pattern, leading to $\partial^2\phi/\partial x^2 \neq 0$ and to an electric space charge σ at the wave front. In this connection the changes in the potential at x_1 and the corresponding redistribution of electric charge must take place in a way which does not at the same time produce current components being able to generate a transverse wave mode.

4. Axially Symmetric Steady State

Turning now to the axially symmetric steady state, a cylindrical frame (r, φ, z) of reference is chosen which is at rest with respect to the field configuration and has z along the axis of symmetry, with the point $(r=0, z=0)$ being chosen in the plane of symmetry. The analysis is further restricted to poloidal electric and magnetic fields, $\underline{E} = (E_r, 0, E_z) = -\nabla\phi$ and $\underline{B} = (B_r, 0, B_z) = \mathrm{curl}\,\underline{A}$, where $\underline{A} = (0, A, 0)$. According to eqs.(1)-(5) we then have $\underline{V} = (0, \pm c, 0)$ and

$$D(r\phi) + (\phi/r) = \pm cD(rA) \tag{15}$$

where

$$D = (\partial^2/\partial r^2) - (1/r)(\partial/\partial r) + (\partial^2/\partial z^2) \tag{16}$$

Eqs, (15) and (16) show that, for a given magnetic field configuration determined by $A(r,z)$, there is a corresponding electric field configuration determined by the solution $\phi(r,z)$ of eq. (15), and vice versa. The relation (14) is an example of this balance in plane geometry which can be considered as a limiting case of vanishing curvature for which the operator (16) turns into the form $\partial^2/\partial x^2$.

In the configuration assumed here, the velocity $\underline{V}$ and the current density $\underline{j}$ become directed along φ, and

$$j = \pm c\sigma \tag{17}$$

according to eqs. (4) and (5). This can be imagined as a kind of "self-confined" electromagnetic radiation which circulates in closed orbits around the axis of symmetry at the velocity of light.

In the same model it is further assumed that σ and $\underline{j}$ decrease at increasing large distance from the origin (r=0, z=0). Thus, σ and $\underline{j}$ should have a negligible influence on eqs. (1)-(3) at points being outside of a closed surface S which surrounds the origin. The surface S is defined in the frame of the steady configuration and is not rigid. Therefore the definition of S does not contradict the Lorentz invariance. In the region outside of S the basic equations (1) and (2) reduce to those which determine the conventional electrostatic and magnetostatic fields.

We finally use eq. (16) in a crude estimate of the relative magnitude of E^2 and B^2 in the region inside of S. Thus, by replacing $\underline{E}$ and $\underline{B}$ (or ϕ and A) by their maximum values inside of S, and by replacing the derivatives by expressions including the corresponding characteristic lengths, the result becomes

$$E^2/c^2B^2 = k_f \qquad \text{(inside S)} \qquad (18)$$

where k_f is a dimensionless constant of order unity. This result is analogous to that of eq. (14) in the plane case.

4.1. Application to Elemantary Particle Analysis

There is no ambition here to replace the conventional theory on elementary particles by the present analysis. The purpose of the latter is merely to provide some complementary ideas which could possibly apply to certain areas of elementary particle physics. In terms of the present analysis, a steady equilibrium should have the form of "self-confined electromagnetic radiation" which bends its own paths of propagation into closed orbits. The energy flux would then circulate around the axis of symmetry. In a general case the equilibrium may include more complicated field geometries than those considered here, as well as effects from self-gravity and "centrifugal" forces due to the circulatory energy.

We now turn to first order estimates of some of the parameters of a particle which would result from the steady state solution obtained here.

The average radius R of the surface S is defined in the frame where the configuration is at rest. This radius should not be interpreted as a physical particle radius, but merely be considered as a mathematical auxiliary tool of the analysis. The field configuration is namely continuous in space and may even have a form for which the definition of a particle radius does not make sense. This implies that only the final results obtained later in this paper, in which the radius R has been eliminated, should become suitable for physical interpretation and comparison with measured data.

Introducing spherical polar coordinates (r,θ,φ), the electric charge of the particle becomes

$$q_0 = 2\pi\int_0^\infty\int_0^\pi \bar{\sigma} r^2 \sin\theta d\theta dr \tag{19}$$

and the magnetic moment

$$M_0 = + \pi\int_0^\infty\int_0^\pi c\bar{\sigma} r^3 (\sin\theta)^2 d\theta dr \tag{20}$$

according to eq. (17). Consequently eqs. (19) and (20) combine to

$$M_0 = k_M c q_0 R \tag{21}$$

where k_M is a dimensionless constant of order unity.

When estimating the particle rest mass m_0, Einstein's relation $W_f = m_0 c^2$ is further applied in combination with expression (8). Here the fields $\underline{E}$ and $\underline{B}$ are estimated by means of eq. (18) in the region inside of S. In the region outside of S, there are static fields $E_r = q_0/4\pi\varepsilon_0 r^2$ $B_r = \mu_0 M_0(\cos\theta)/2\pi r^3$ and $B_\theta = \mu_0 M_0(\sin\theta)/4\pi r^3$. Taking the integral (8) over entire space, eqs. (3) and (18) then yield

$$m_0 = k_m \mu_0 q_0^2 / R \tag{22}$$

in a first approximation, where k_m is a dimensionless constant of order unity.

The azimuthally circulating energy flux corresponds to an angular momentum (spin) which can be estimated in an analogous way to

$$s_0 = k_s m_0 c R = k_s k_m \mu_0 c q_0^2 \tag{23}$$

where k_s is a dimensionless constant of order unity.

On the self-confined circulating radiation we finally impose the periodicity condition

$$2\pi k_\nu R = nc/\nu \tag{24}$$

where ν is the frequency of the n-th state of an equivalent electromagnetic oscillator having the energy $W_f = nh\nu$, h denotes the Planck constant, and k_ν is a dimensionless constant of order unity. Combination with the expression $W_f = m_0 c^2$ yields

$$R = n^2 h / 2\pi k_\nu m_0 c \tag{25}$$

4.2. Comparison with Some Elementary Particle Data

The conceptual difficulties in defining R as a particle radius,and the uncertainties in the experimental data of the measured particle radius in certain cases such as that of the electron, suggest that the results of eqs. (21)-(23) and (25) are rearranged in a way being independent of R.

The result then becomes

$$q_o = n(h/2\pi\mu_o c k_\nu k_m)^{1/2} \tag{26}$$

$$M_o m_o = n^3(k_M^2 h^3/8\pi^3\mu_o c k_\nu{}^3 k_m)^{1/2} \tag{27}$$

$$s_o = n^2 k_s h/2\pi k_\nu \tag{28}$$

The proton and electron data given by Fermi[5] and Schiff[6] among others are now taken as examples where $M_o = 1.41 \times 10^{-26}\,Am^2$ and $m_o = 1.67 \times 10^{-27}$ kg for the proton, $M_o = 0.93 \times 10^{-23}\,Am^2$ and $m_o = 9.11 \times 10^{-31}$ kg for the electron, and $s_o = h(3/16\pi^2)^{1/2}$ for both these particles. Putting $n = 1$, eqs. (26)-(28) then yield $(k_\nu k_m)^{1/2} \simeq 3.3$ and $k_\nu/k_s \cong 1.6$ for both particles, $k_M/k_\nu \simeq 0.8$ for the proton, and $k_M/k_\nu \simeq 0.5$ for the electron. Consequently, the values of q_o, $M_o m_o$ and s_o given by eqs. (26)-(28) can be made to agree with the measured data for coefficients k_ν, k_m, k_M and k_s which all turn out to be rather close to unity, as assumed in the present theoretical model.

It has finally to be pointed out that the present system of eqs. (21)-(28) is not fully determined, in the sense that the dimensionless coefficients k_M, k_m, k_s and k_ν do not always have to be of order unity, e.g. in cases where the fields $\underline{E}$ and $\underline{B}$ have strongly inhomogeneous profiles in space. As an example, the values of q_o, M_o, m_o and s_o can be kept close to those observed for the electron, even when the "radius" R becomes very small. This is possible by keeping $(k_\nu k_m)$, (k_ν/k_s) and (k_M/k_ν) constant and letting k_ν approach values being much larger than unity.

5. Discussion and Conclusions

The present Lorentz invariant extended formulation of Maxwell's equations both includes earlier treated electromagnetic phenomena and leads to a new class of time dependent and steady-state phenomena. Here the following should be especially mentioned:

(i) A longitudinal purely electric wave in vacuo is predicted to arise, in addition to the transverse electromagnetic wave. The ways by which the existence of the former wave could be verified are not straight-forward, however, because there are non-trivial problems how to generate and detect such a wave, and how to separate it from the transverse wave.

(ii) Steady states in vacuo are predicted to be formed, where the charge density becomes an intrinsic property of the electric field itself. This makes steady states possible in which electromagnetic radiation becomes subject to a kind of self-confinement. Such states, if they exist, could provide elementary particle analysis with some complementary features. Thus, an estimation of the field quantities in the steady states described in this paper leads to values of the particle charge, of the product between the particle magnetic moment and mass, and of the spin, being of the same order of magnitude as those observed for some particles such as the proton and electron.

At this stage is not clear whether or not the existence of one of the predicted phenomena, (i) or (ii), depends on the existence of the other. Thus, it cannot for certain be excluded that the extended formulation given by eqs. (4) and (5) applies only to a steady state and not to plane waves, or vice versa.

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6. References

[1]Jennison, R.C. and Drinkwater, A.J., "An approach to the understanding of inertia from the physics of the experimental method", J. Phys. A:Math.Gen., 10(2), 167-179(1977).

[2]Jennison, R.C., "Relativistic phase-locked cavities as particle models", J.Phys. A:Math. Gen., 11(8), 1525-1533(1978).

[3]Jennison, R.C., "What is an electron?", Wireless World, June 1979, p. 42-47.

[4]Yadava, K.S., "On the stability and the genesis of the electron", Il Nuvo Cimento, 32B(2), 273-283(1976).

[5]Fermi, E., Nuclear Physics, pp.6, 13, 242, Univ. of Chicago Press, Revised Edition, Chicago 37(1951).

[6]Schiff, L.I., Quantum Mechanics, pp. 74, 145, McGraw-Hill Book Comp., New York(1949).

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Key words: Electromagnetic field, elementary particles.