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WHY IS THE ENTROPY OF BLACK HOLES SO BIG?

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Viki is the master of order of magnitude estimates. I shall avail myself of his art for some considerations pertaining to the question posed in the title. In this way I hope to arrive at a more direct understanding of the number 10^{76} which is usually quoted in this context [1]. Since I always considered the entropy as a local notion and never thought that there was too much sense in talking about the entropy of a black hole as a whole my knowledge of the relevant literature is rather scanty. It is therefore likely that the following trivialities have appeared in some papers and I have to apologize to whoever might be the author. In any case they do not seem to be widely known and illustrate how such considerations, in the absence of a complete theory, give the feeling that one has understood something.

1. The Simple Argument

When people quote this outrageous 10^{76} for the entropy of a black hole of the mass of the sun they sometimes blame the wild situation inside [2] for the increase by the factor 10^{20} over the entropy of a star. One talks about many degrees of freedom being excited, all sorts of quarks and perhaps preons around etc. I shall adopt the point of view that the question about what happens inside a black hole is illegitimate and for us the only data of a black hole is its energy Mc^2 and its radius $\kappa M/c^2$ (forgetting for the moment e and J). Furthermore I join those who believe that it will eventually evaporate [3]. As entropy I shall ascibe to it the entropy coming out in the form of radiation. Now Viki has told us that one particle has got one unit of entropy and since the only length in the problem is $\kappa M/c^2$ the emerging photons (or other mass zero particles) will have an

$$\text{energy} = c\cdot\text{momentum} = c\hbar/\text{wavelength} = c^3\hbar/\kappa M \ .$$

Thus I have

entropy = number of particles = total energy/energy per particle =

$= Mc^2/(c^3\hbar/\kappa M) = (\kappa M/c^2)^2/(\kappa\hbar/c^3)$ = surface [measured in

Planck length $= (\kappa\hbar/c^3)^{1/2}$] . (1)

Since the surface (measured in $(cm)^2$ as is still dear to the heart of Viki) is 10^{10} and the Planck length 10^{-33} I get the fabulous 10^{76}.

2. A More Mathematical Argument

One might object that in the previous argument it was too simple to give all photons the wavelength of the original $\kappa M/c^2$ since upon radiation the black hole will shrink. Though this is correct it does not matter too much since most of the entropy is in the radio waves and the final flash involves little entropy. To see this I call $N(\nu)$ the frequency spectrum of the emerging particles and E the energy of the black hole after particles with a frequency $\leq \nu$ have been radiated. ν will again be related to E by

$$\text{wavelength} = \frac{c}{\nu} = \text{radius} = \kappa E/c^4 \; .$$

Thus

$$E = Mc^2 - \hbar \int_{\nu_0}^{\nu} d\nu' \, N(\nu') \, \nu' = \frac{c^5}{\nu\kappa} \; . \tag{2}$$

Differentiating with respect to ν gives

$$N(\nu) = \frac{c^5}{\hbar\kappa\nu^3} \tag{3}$$

and ν_0 is determined by

$$Mc^2 = \hbar \int_{\nu_0}^{\infty} d\nu \, N(\nu') \, \nu' = \frac{c^5}{\nu_0\kappa}$$

or

$$\frac{c}{\nu_0} = \kappa M/c^2 = \text{original radius.}$$

Thus

$$\text{entropy} = \int_{\nu_0}^{\infty} d\nu\, N(\nu) = \frac{c^5}{2\hbar\kappa}\frac{1}{\nu_0^2} = \frac{c^3}{2\hbar\kappa}\frac{\kappa^2 M^2}{c^4} = \frac{1}{2} \text{ of } (1) \;. \qquad (4)$$

Thus all this mathematical effort only tells us that the original argument overestimated the entropy by a factor 2 which in face of 10^{76} is not so bad.

3. The Complete Theory

A serious investigation would have to incorporate the reaction of the emitted radiation onto the gravitational field. Since the radiation is of quantum origin one will have to decide what to do about the quantization of the gravitational field. A few decades ago it was still claimed [4] that there is no logical necessity for quantizing gravity because one can construct consistent theories of a classical field interacting with a quantum system. However, this claim has never been substantiated by exhibiting and exploring fully such a theory (see Ref. [5] for a reasonable discussion of this matter). For gravity no experimental fact has so far indicated whether it should be quantized or not. However there is the empirical fact that the people who predict that there should be a particle associated with a field, starting from Yukawa up to Glashow, Salam and Weinberg, usually end up with the Nobel prize. This may have changed the psychology so that nowadays nobody seriously questions that gravity should be quantized. Unfortunately, nobody really knows how to do that. Thus, since one neither knows how to quantize nor how not to quantize gravity the section on a complete theory ends right at its beginning. One can only hope that there is something to the arguments à la Viki even if one does not really understand the matter.

References

[1] J. Beckenstein, in: Jerusalem Einstein Centennial Symposium,
Y. Ne'eman Ed., Addison-Wesley, 1981;
P.C.W. Davies, Rep. Prog. Phys. 41, 1313 (1978).

[2] G. Sewell, Contribution to the Ludwig Boltzmann Seminar, Feb. 1986.

[3] S. Hawking, Commun. Math. Phys. 43, 199 (1975).

[4] L. Rosenfeld, Nucl. Phys. 40, 353 (1963);
A. Peres, Nucl. Phys. 48, 622 (1963).

[5] R.M. Wald, General Relativity, Chicago University Press, 1984.
See discussion after (14.1.3).