INSTITUTE FOR NUCLEAR STUDY
UNIVERSITY OF TOKYO
Tanashi, Tokyo 188
Japan

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H. Sato and Y. Okuhara*

Institute for Nuclear Study, University of Tokyo,
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Abstract:

The interaction cross sections of high energy nucleus-nucleus scattering have been studied with the Glauber Model and Hartree-Fock like variational calculation for the nuclear structure. It is found that the experimental interaction cross sections of the light unstable nucleus-stable nucleus scatterings measured by INS-LBL collaboration are well reproduceable.

*). Present address; Department of Physics, Queen's University, Kingston, Ontario, CANADA.

The determination of nuclear size is one of the most important problem in nuclear physics. Thus far various experimental methods (the Coulomb displacement energy, high energy electron scattering, X-rays from the muonic atom and pion, proton and alpha scatterings) have been employed to determine the nuclear size. The radii of proton, neutron and charge distributions have been determined with these experiments and compared with available theoretical nuclear structure calculations. However due to the experimental restriction on the choice of the target, these measurements are thus far limited to the case of stable nuclei. Recently Tanihata et al (INS-LBL Collaborations) have succeeded in determining the interaction cross sections for the He isotope and stable nucleus scatterings with the use of the secondary He isotope beams produced by the primary Bevalac ^{11}B beam[1]. Since it is a quite interesting question to ask whether the interaction cross sections measured indicate the nuclear matter radii of stable and unstable nuclei, in this paper, we study the interaction cross section by performing the Glauber model calculation with the employment of the density dependent Hartree Fock (DDHF) type variational calculation of the nuclear density distribution, and show that the interaction cross sections measured by Tanihata et al are reasonably reproduceable.

Nuclear properties are well reproduced by the Hartree-Fock calculation for closed shell nuclei and by the shell model calculation for nuclei in the middle of shell. However since the mass number dependence of the core and single particle properties is not negligible in the shell model study of the light nuclei, in this work, we employ the Hartree-Fock type variational method to derive the nuclear density distribution of both closed shell and the middle of shell nuclei, which have been undertaken by Yazaki in the study of the systematics of the core and single particle properties of the sd shell nuclei and Ca isotopes[2]. Yazaki have derived the Hartree-Fock like equation for the single particle wave functions of the core (ν) and valence (α) nucleon states by minimizing the average shell model energy as a functional of single particle wave function, such as,

$$(h + U_c + \frac{n}{\Omega} U_v) \phi_{\alpha(\nu)} = \tilde{\varepsilon}_{\alpha(\nu)} \phi_{\alpha(\nu)} , \tag{1}$$

where h is the kinetic energy operator. The non-local potentials U_c and U_v for the core and valence nucleons are given by the two-body interaction $\mathcal{U}$, respectively, by

$$U_{c(v)}(x,x') = \int dx_1 dx_2 \left\{ \mathcal{U}(x,x_1:x',x_2) - \mathcal{U}(x,x_1:x_2,x') \right\} \times \rho_{c(v)}(x_1,x_2) , \tag{2}$$

with the density matrices ρ_c and ρ_v defined by

$$\rho_c(x,x') = \sum_{\nu} \phi_{\nu}(x) \phi^{*}_{\nu}(x') \quad \text{and} \quad \rho_v(x,x') = \sum_{\alpha} \phi_{\alpha}(x) \phi^{*}_{\alpha}(x') . \tag{3}$$

Here Ω denotes the number of the valence single particle states and n is the number of valence nucleons. The root mean square (rms) proton, neutron and matter radii for several nuclei are calculated in such a way that $r_{rms} = \sqrt{\langle r^2 \rangle - \frac{3}{2}b^2/A}$ [3,4] with the single particle wave functions obtained by the Skyrme III (SKIII) and Skyrme V (SKV) interactions[5]. And those are tabulated in Table 1. (The numerals obtained are quite similar to those obtained by the calculation performed by just plugging the proton and mass numbers in the DDHF program[6].) The neutron and matter radii of the ^{6}He nucleus calculated show almost equivalent or sometimes rather larger values than that for the ^{8}He nucleus. This stems from very delicate balancing between the binding energy dependence and mass number dependence of the rms radii. Comparing with calculations performed with the other Skyrme interactions, we find that the interaction, which produces a smaller neutron (or matter) radius for the ^{8}He nucleus than for the ^{6}He nucleus, predicts the existence of the bound ^{10}He nucleus. We also find that the interaction having rather weak three-body interaction term predicts the bound ^{10}He nucleus. It is also interesting to note that the proton radii for isotopes are almost equivalent or getting smaller with increasing neutron number. These phenomena are peculiar property of the light nuclei, and we usually do not observe such effects in other heavier nuclear systems.

Next let's calculate the interaction cross section of the nucleus-nucleus scattering. Here we employ the Glauber model[7] which have been extensively studied by Franco and Varma[8,9]. Following their studies, we perform the Glauber model calculation by taking account of the Pauli principle, center of mass (c.m.) corrections and effects of higher order collisions. The scattering amplitude for collision between the nucleus A_1 and nucleus A_2 can be written as[9]

$$F(q) = \frac{ik}{2\pi}\int d^2b\, e^{i\vec{q}\vec{b}}\left(1 - e^{i\bar{\chi}_{opt}(\vec{b})} \right), \tag{4}$$

where the c.m. corrected optical phase-shift function $\bar{\chi}_{opt}(\vec{b})$ is related to the c.m. uncorrected phase-shift function $\chi_{opt}(\vec{b})$ as

$$e^{i\bar{\chi}_{opt}(\vec{b})} = (2\pi)^{-2}\int d^2q\, d^2b'\, e^{-i\vec{q}(\vec{b}-\vec{b}')}\, e^{i\chi_{opt}(\vec{b}')}K(q). \tag{5}$$

Here the c.m. correction K(q) is given by the function

$$K(q) = \exp\left\{q^2(R_1^2/4A_1 + R_2^2/4A_2)\right\}, \tag{6}$$

where the parameter R_i is related to the calculated rms matter radius $r_{rms}(A_i)$ as follows

$$R_i^2 = \frac{2A_i}{3(A_i-1)}\, r_{rms}^2(A_i). \tag{7}$$

The optical phase-shift function $\chi_{opt}(\vec{b})$ is given by

$$i\chi_{opt}(\vec{b}) = \ln\langle \Psi_{A_1}\Psi_{A_2}\Big|\prod_{i=1}^{A_1}\prod_{j=1}^{A_2}\left\{1-\Gamma_{ij}(\vec{b}-\vec{s}_i+\vec{s}_j')\right\}\Big|\Psi_{A_1}\Psi_{A_2}\rangle, \tag{8}$$

where Ψ_{A_i} are the ground state wave function of the nucleus A_i, $\vec{s}_i$ and $\vec{s}_j'$ are the projections of nucleon coordinates on the impact parameter plane, and Γ_{ij} are the nucleon-nucleon (NN) profile functions. Both phase-shift functions $\bar{\chi}_{opt}(\vec{b})$ and $\chi_{opt}(\vec{b})$ are obtainable by expanding those in powers of Γ_{ij} such that[9],

$$i\bar{\chi}_{opt}(\vec{b}) = i\sum \bar{\chi}_j(\vec{b}) \quad \text{and} \quad i\chi_{opt}(\vec{b}) = i\sum \chi_j(\vec{b}). \tag{9}$$

In this work, we employ the terms upto second order, where both first and second order terms are evaluated with the Slater determinant of the single particle wave functions generated with the Hatree-Fock like equation (1). The NN profile function $\Gamma(b)$ is related to the experimentally measured NN scattering amplitude $f(k_N;\vec{q})$ as follows,

$$\Gamma(b) = (2\pi i k_N)^{-1}\int d^2q \; e^{-i\vec{q}\vec{b}} f(k_N;\vec{q}). \tag{10}$$

Here for the NN scattering amplitude, we employ the usual high energy parameterization

$$f(k_N;\vec{q}) = \frac{k_N\sigma(i+\rho)}{4\pi} \, e^{-aq^2/2}, \tag{11}$$

where parameters are taken to be[10],

$\sigma_{pp} = \sigma_{nn} = 46.9$ mb, $\sigma_{pn} = 38.0$ mb, $\rho_{pp} = \rho_{nn} = 0.14$,
$\rho_{pn} = -0.34$ and $a = 5$ $(\text{GeV/c})^{-2}$ at $E = 0.79$ GeV/N.

Here we note that, while we employ the usual method[3,4] for the c.m. correction in the calculation of the rms radii and the total binding energy, we may need much more consistent and accurate treatment of the c.m. correction in the nuclear structure calculation, because the c.m. correction plays very important role in the Glauber model calculation.

The total and interaction cross sections for He-isotopes - ^{4}He, He-isotopes - ^{12}C, He-isotopes - ^{27}Al and ^{12}C - ^{12}C scatterings calculated with the SKV interaction are tabulated in Table 2. The calculations upto first order ($\bar{\chi}_1$) and second order ($\bar{\chi}_2$) terms are also shown in Table 2. Assuming a simple formula πR^2 for the cross section, we compare those cross sections with the radial distribution of the matter density, then we find that the total cross sections correspond to the position at 10% of the central value of nuclear matter distributions except for the ^{4}He nucleus where the cross sections are determined by the position at about twice of the corresponding matter density. On the other hand, the interaction cross sections correspond to the position at about 40% of the central matter density of ^{4}He nucleus and at about 30% of those of other nuclei.

The interaction cross sections for nucleus-nucleus scatterings are calculated with the SKIII and SKV interactions and tabulated in Table 3. For comparison, the exparimental values obtained by Tanihata et al[1,11] are tabulated in the same Table. Those for the case of ^{12}C target are summarized in Fig.1. The experimental interaction cross sections are generally well reproduced. The SKV interaction reproduces the experimental tendencies better than the SKIII interaction. Here we note that the cross sections calculated upto second order $\bar{\chi}_2$ term give rather underestimated values (less than 5%[9]), because the power series expansion (eqs.(9)) of the $\bar{\chi}_{opt}(\vec{b})$ has alternating signs. In this sense, the overestimation of the cross sections for ^{4}He-^{4}He, ^{4}He-^{12}C and ^{4}He-^{27}Al scatterings is serious. This stems from the fact that rms radii of the ^{4}He nucleus obtained by the DDHF calculation is usually much larger than the experimental value. Therefore, as far as the ^{4}He nucleus is concerned, we need to employ a much more realistic structure calculation. On the other hand, the underestimation of the cross section for ^{11}Li-^{12}C scattering seems also hard to be understood within this kind of calculations. We need more information and study on the ^{11}Li nucleus. We examine the additivity relationship of the interaction cross sections, which have been suggested by Tanihata et al [1] in such a way that,

$$\sigma_{int}(p,t) = \pi(R_p + R_t)^2, \qquad (12)$$

where R_p and R_t are the effective interaction radii of the projectile and the target nuclei, respectively. Then we find the additivity relationship is numerically well satisfied .

In summary, we conclude that the experimental interaction cross sections are generally well understandable by the realistic Glauber model calculation with the employment of the realistic nuclear wave functions. This nice agreement between the experimental and theoretical interaction cross sections may give a significant one step to determine the matter radius of the unstable nucleus and to study the consistency between normal nuclear physics and nuclear physics for nuclei far from stability.

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Figure Captions.

Fig.1. The interaction cross sections for the nucleus (A_p) - ^{12}C scatterings at 0.79 GeV/N calculated with the SKIII and SKV interactions and the experimental values measured by INS-LBL Collaboration[1,11]. The solid line with open circles is for the experimental values, the dotted line with solid squares is for the SKIII interaction, and the dash-dotted with solid circles is for the SKV interaction.

Table 1. The rms radii (in fm) of neutron, proton and matter calculated with SKIII and SKV interactions.

	neutron		proton		matter		charge		
Nucleus	SKIII	SKV	SKIII	SKV	SKIII	SKV	SKIII	SKV	Exp.
^{4}He	1.78	1.75	1.79	1.76	1.79	1.76	1.93	1.90	1.63±0.04[a)]
^{6}He	2.77	2.75	1.80	1.80	2.49	2.47	1.94	1.95	
^{8}He	2.80	2.60	1.83	1.81	2.59	2.43	1.91	1.89	
^{10}He	2.87		1.88		2.70		1.92		
^{6}Li	2.19	2.44	2.22	2.52	2.21	2.48	2.35	2.65	2.50±0.10[a)]
^{7}Li	2.35	2.53	2.13	2.34	2.26	2.45	2.24	2.45	2.43±0.04[a)]
^{8}Li	2.44	2.54	2.10	2.27	2.32	2.44	2.20	2.36	
^{9}Li	2.50	2.53	2.09	2.22	2.37	2.43	2.18	2.31	
^{11}Li	3.00	2.74	2.13	2.24	2.79	2.61	2.21	2.31	
^{9}Be	2.35	2.47	2.24	2.41	2.30	2.44	2.35	2.51	2.519±0.012[b)]
^{10}Be	2.42	2.48	2.23	2.36	2.34	2.43	2.33	2.46	
^{12}C	2.38	2.42	2.40	2.45	2.39	2.44	2.50	2.55	2.472±0.002[c)]
^{27}Al	2.98	3.00	2.98	3.00	2.98	3.00	3.06	3.08	2.91±0.10[a)]

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Table 2. The total and interaction cross sections (in mb) for nucleus-nucleus scatterings at 0.79 GeV/N calculated with the SKV interaction.

Nuclei	σ_{tot}(mb) upto $\bar{\chi}_1$	σ_{tot}(mb) upto $\bar{\chi}_2$	σ_{int}(mb) upto $\bar{\chi}_1$	σ_{int}(mb) upto $\bar{\chi}_2$	Exp.[a)]
$^4He-^4He$	418	377	302	280	262 ± 19
$^6He-^4He$	588	523	421	386	
$^8He-^4He$	707	642	487	454	
$^{10}He-^4He$	845	772	576	538	
$^4He-^{12}C$	876	785	571	529	503 ± 5
$^6He-^{12}C$	1175	1018	768	689	722 ± 6
$^8He-^{12}C$	1335	1203	843	781	817 ± 6
$^{10}He-^{12}C$	1547	1398	971	899	
$^4He-^{27}Al$	1421	1270	873	809	780 ± 13
$^6He-^{27}Al$	1843	1556	1144	1004	1063 ± 8
$^8He-^{27}Al$	2025	1810	1222	1123	1197 ± 9
$^{10}He-^{27}Al$	2301	2054	1384	1266	
$^{12}C-^{12}C$	1545	1394	940	875	

a). Ref. 1) and Ref. 11).

Table 3. The interaction cross sections (in mb) for nucleus-nucleus scatterings at 0.79 GeV/N calculated with the SKIII and SKV interactions.

Target		^{4}He		^{9}Be		^{12}C		^{27}Al	
Beam		Theor.	Exp.[a)]	Theor.	Exp.[a)]	Theor.	Exp.[a)]	Theor.	Exp.[a)]
^{4}He	(SKIII)	282		453		521		801	
	(SKV)	280	262±19	473	485± 4	529	503± 5	809	780±13
^{6}He	(SKIII)			592		672		984	
	(SKV)			624	672± 7	689	722± 6	1004	1063± 8
^{8}He	(SKIII)			704		790		1136	
	(SKV)			713	757± 4	781	817± 6	1123	1197± 9
^{10}He	(SKIII)								
	(SKV)			825		899		1266	
^{6}Li	(SKIII)			567		645		952	
	(SKV)			622	651± 8	688	688±12	1001	1010±11
^{7}Li	(SKIII)			609		690		1008	
	(SKV)			666	686± 4	735	736± 6	1059	1071± 7
^{8}Li	(SKIII)			652		735		1065	
	(SKV)			703	727± 6	773	768± 9	1108	1144± 8
^{9}Li	(SKIII)			695		780		1123	
	(SKV)			737	739± 5	807	796± 6	1153	1135± 8
^{11}Li	(SKIII)			823		918		1289	
	(SKV)			829		902	1056±30	1269	
^{9}Be	(SKIII)			673		758		1093	
	(SKV)			731	756± 6	802	807± 9	1144	1176±11
^{10}Be	(SKIII)			709		796		1141	
	(SKV)			760	766± 8	831	825±10	1181	1180±16

a). Ref. 1) and Ref. 11).

Fig. 1