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J.Chýla

ON FIXING THE SCALE IN QCD FROM LATTICE CALCULATIONS

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Substitute this table for table 1 on page 3.

n_f	0	I	2	3	4
W	28,8	33,9	40,8	50,5	64,7
KS	28,8	35,2	43,6	56,7	76,3

The purpose of this note is to help answer a simple but important question : what is the value of the basic QCD scale parameter $\Lambda_{\overline{MS}}$? Precise knowledge of this parameter is necessary for testing the internal consistency of perturbative QCD and even more essential for grand unified theories, where a factor of two uncertainty in $\Lambda_{\overline{MS}}$ results in a factor of 16 ambiguity in the proton life-time.

This kind of sensitivity of the theoretical predictions to the value of $\Lambda_{\overline{MS}}$ is unfortunately absent in perturbative calculations which have so far been the main source of information on it. A conservative conclusion from analyses of hard scattering processes in lepton-hadron, hadron-hadron and electron-positron interactions is that all these processes are quantitatively consistent with QCD for $\Lambda_{\overline{MS}}$ in a rather broad range 100-400 MeV/c /1/. There are, however, data which prefer even smaller /2/ or higher /3/ value of $\Lambda_{\overline{MS}}$. This large spread in the extracted values of $\Lambda_{\overline{MS}}$ is not too alarming, as for different processes different theoretical assumptions must be made in order to extract this quantity from raw experimental data. The lack of sufficiently precise data (at, say, 1% level), the inherently weak dependende of perturbative calculations on $\Lambda_{\overline{MS}}$ and limited understanding of the higher twist contributions do not allow an accurate determination of $\Lambda_{\overline{MS}}$ from hard scattering processes alone.

The only type of calculations in QCD, which is sufficiently sensitive to $\Lambda_{\overline{MS}}$ and for which precise experimental data exist seems to be the lattice calculations of the hadron spectrum. Whether these calculations are also reliable is a difficult question to which we return at the end of this note. Given some quantity $\mathcal{O}$ with dimension $(\text{energy})^{d}$,the results of such calculations take (for massless quarks) the form

$$\sigma = k(\Lambda_L)^2, \tag{1}$$

where k is a calculable number and Λ_L is the dimensional parameter appearing in the construction of the continuum limit of the lattice approximations. This is defined as the simultaneous limit of the lattice spacing a and the bare coupling g(a), corresponding to this spacing, according to

$$a \to 0 \;;\; g^2(a) = \frac{4\pi^2}{b \ln(1/a\Lambda_L)} - \frac{4\pi^2 c}{b} \frac{\ln \ln (1/a\Lambda_L)^2}{\ln^2(1/a\Lambda_L)}. \tag{2}$$

or equivalently

$$g^2 \to 0 \;;\; a(g^2) = \frac{1}{\Lambda_L}\left(\frac{b g^2}{8\pi^2}\right)^{-\frac{c}{b}} \exp\left(-\frac{4\pi^2}{b g^2}\right) = \frac{1}{\Lambda_L} f(g^2). \tag{3}$$

where $b=(33-2n_f)/6$, $c=(153-19n_f)/(66-4n_f)$ and n_f denotes the number of light (effectively massless) quarks taken into account. The notion "light" and "heavy" are defined with respect to the natural lattice scale, given by the inverse lattice spacing a^{-1}. The form of (2) and (3) is given by the asymptotic freedom property of the continuum QCD. If we identify lattice spacing a with the cut-off in the conventional cut-off regularization of the continuum QCD, the relation (2) is just the usual dependence of the bare coupling constant g on that cut-off. Renormalizability of QCD dictates the shape of (2), but does not fix the value of Λ . Different renormalization schemes correspond to different values of Λ and thus also to different $g^2(a)$. The question whether perturbation expansion - when summed to all orders - gives a unique and well defined result is a difficult problem, which has so far no definite answer /4/. For our purpose it is important only to realize that there exists simple relation between Λ-parameters associated with different renormalization schemes. As the lattice regularization together with the limiting procedure defined in (2),(3) represents just one of the possible renormalization schemes, there is also a relation between Λ_L from (2) and the parameters $\Lambda_{\overline{MS}}, \Lambda_{MOM}$ corresponding to the usual $\overline{\text{MS}}$ and MOM schemes in the continuum space-time.

A simple, but essential conclusion from the formula (2) is that for different n_f we get different dependence of $a(g^2)$ on g^2. Not only is the local slope n_f-dependent, but more significantly, the absolute normalization changes dramatically with n_f going from 0 to more realistic n_f=3,4(see the figure 1).For $\beta=6/g^2$ around 6,which is the typical value used in current Monte-Carlo simulations, $a(g^2)$ decreases by an order of magnitude between $n_f=0$ and $n_f=4$! Consequently, given the value of $a(g^2)$ at some particular g^2 (by comparing MC calculations with experimental data), the value of Λ_L extracted by means of the formula (3) will significantly depend on the chosen value of n_f. Moreover, the closer we get to the continuum limit, the bigger the difference.

The relevance of the above, in fact quite simple observation for lattice calculations in QCD is connected with the fact that there are complications, both principal and practical, with putting quarks on the lattice. There are essentially two different approaches how to include quarks while avoiding the notorious species doubling problem, in one way or another [6]. I shall not discuss merits and shortcomings of either the Wilson fermions[7] or a family of three in the spirit related but in detail distinct formulations, known under the name Kogut-Susskind fermions[8-10]. What is essential is to realize that each of these four formulations represents different regularization procedure and is thus associated with different Λ -parameter. Table 1 contains the value of the ratio $\Lambda_{\overline{MS}}/\Lambda_L$ as a function of n_f for Wilson fermions[11] and for Kogut-Susskind fermions in the momentum space[9,12]. Analogous results for other two variants of the latter are to my knowledge so far not available. This is unfortunate especially in the case of Kogut-Susskind fermions in the configuration space[10] as there is recent large statistics Monte-Carlo calculation[13] of the hadron spectrum using this formulation.

Table 1.

The ratio $\Lambda_{\overline{MS}}/\Lambda_L$ as a function of n_f for Wilson (W) and Kogut-Susskind (KS) fermions.

n_f	0	1	2	3	4	5
W	83.42	89.24	96.36	105.24	116.27	131.73
KS	83.42	92.42	103.05	117.97	137.33	164.17

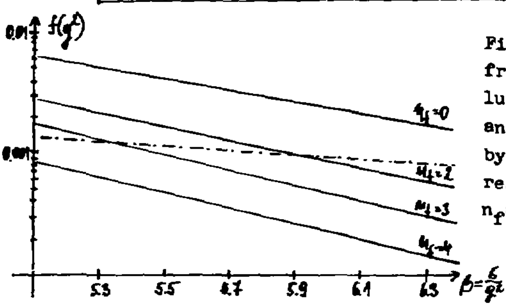

Figure. The function $f(g^2)$ from (3) for various values of n_f (solid lines) and the ratio, multiplied by 10^{-2}, of the curves corresponding to $n_f=0$ and $n_f=4$ (dash-dotted line).

In the following I am going to discuss results of three recent Monte-Carlo calculations of the hadron spectrum[13-15] and their implications for the extraction of $\Lambda_{\overline{MS}}$. All these calculations were performed in the quenched approximation, which amounts to neglection of all quark loops on the lattice. For Wilson fermions there exists in principle (though not in practice) straightforward procedure, known as hopping parameter expansion, how to include quark loops of arbitrary size[16]. The required computer time grows, however, linearly with the volume of the lattice and therefore only lattices of moderate size (up to 8^4) could so far be employed.

The question I want to address is the following : do the lattice Monte-Carlo calculations in the quenched approximation give us an estimate of $\Lambda_{\overline{MS}}$, or if not, at least some bound on it ? To answer this question, we need to know how to define the continuum limit in the quenched approximation and which physical quantities are most suitable for comparison with lattice calculations in this approximation. I consider the mass of the ρ-meson as particularly suitable for fixing $\Lambda_{\overline{MS}}$ by adjusting its value to give the measured m_ρ=0.770 GeV/c^2, but the following conclusions would change by only something like 10-15% if other low lying hadrons (proton, Δ, lambda) were used for the same purpose. We cannot use the pion, as it is sensitive to the current quark mass (contrary to the particles mentioned above) and serves in fact to fix the value of the so-called critical hopping parameter. Table 2 shows the values of the lattice spacing a, required to reproduce the experimental value m_ρ=0.770 GeV/c^2, at particular β, together with the corresponding linear lattice size and the resulting values of Λ_L and $\Lambda_{\overline{MS}}$ for n_f=0,3,4. The authors of ref. [13-15] claim to be already in the continuum region, in which the physical quantities should not depend on the lattice spacing, or equivalently, β.

ref.		[13]	[14]	[15]
	a [fm]	0.25	0.10	0.15
	d [fm]	2.5	1	2.4
Λ_L	n_f=0	2.6	4.5	4.5
	n_f=3	0.62	0.98	0.98
[MeV/c]	n_f=4	0.28	0.45	0.45
$\Lambda_{\overline{MS}}$	n_f=0	75	130	130
	n_f=3	32	50	50
[MeV/c]	n_f=4	21	29	29

Table 2.

Lattice spacing a, corresponding linear lattice size d and the resulting Λ-parameters as defined in the text. Based on calculations in ref. [13-15]. The calculations were performed at β=5.7 in[13,15] and β=6[14].

Their claim is based on the observation that the measured dependence of the product $m_\rho a(\beta)$ on β is fitted well by the formula (3) with $n_f=0$. I shall come to this point when discussing the reliability of the used calculations at the end of this note. There is some discrepancy between the results of ref. [13] and those of [14,15] which should for $n_f=0$ give the same value of $\Lambda_{\overline{MS}}$.

The extracted values of $\Lambda_{\overline{MS}}$ depend significantly on the assumed number of light quarks. I consider $n_f=3$ or 4 realistic as the lattice spacing $a(\beta)$ required to give the correct mass m_ρ corresponds to momenta well above u,d,s quark masses and just at the threshold of the charmed quark mass. Clearly finer analysis of quark mass effects would be worthwhile. I took $n_f=3,4$ just to show the magnitude of the effect of including light quarks. For Kogut-Susskind fermions used in[13] the ratio $\Lambda_{\overline{MS}}/\Lambda_L$ is as yet not known, so I took as an estimate the value valid for its momentum space variant[9].

Table 2 shows clearly that for realistic values of n_f the resulting values of $\Lambda_{\overline{MS}}$ are significantly lower than those obtained from most hard scattering processes. This may be somewhat disquieting, but it is on the other hand not excluded that the effects of higher twists and/or orders are in fact more important than usually assumed.

Several objections can be raised againts the above reasoning. First, one might question the justification of using $n_f=3,4$ within the quenched approximation, when the latter amounts to neglecting quark loops on the lattice while the n_f-dependence of $a(\beta)$ in (3) is just the consequence of the existence of quark loops. However, what is neglected in the quenched approximation are <u>large</u> quark loops that is loops explicitely visible on the lattice. From the point of view of the continuum QCD, fluctuations due to quark loops smaller than the lattice spacing can easily be included by using formula (3) with the correct, realistic $n_f=3$ or 4. There is no obvious inconsistency in neglecting large while including small quark loops. Moreover, once we go beyond the quenched approximation , within the hopping parameter expansion in our case, progressively larger and larger quark loops are taken into account. As there is no sharp distinction between "large" and "small" loop, there is also no basic difference between the quenched approximation and that of finite order hopping parameter expansion. In the latter, one would probably not hesitate to take $n_f=3$ or 4 when constructing the continuum limit. I therefore think the same can, and in fact should, be done in the quenched approximation, too.

Secondly, one might argue that it is difficult to see how the quenched approximation could, even on the infinite lattice, "feel" the existence of quark loops smaller than the lattice spacing, as would be necessary if the dependence of the measured quantity $m_\rho a(\beta)$ on β should take the form of (3) with $n_f \neq 0$. This objection holds, however, even in the case of finite order hopping parameter expansion. Only if quark loops of all sizes are included, one can hope to get such a dependence. As the rational behind the quenched approximation relies on neglecting large while retaining small quark loops, one should, when relating lattice spacing to Λ_L and β use the formula (3) with $n_f=3$ or 4 and accept the fact that there will be no scaling region in the strict sense of the word. In a limited region of β values, where on a finite lattice scaling is usually observed with $n_f=0$, the difference in the shape between $a(\beta, n_f=0)$ and $a(\beta, n_f=3)$ or $a(\beta, n_f=4)$ is small and the only consequence of using $n_f=3$ or 4 instead of $n_f=0$ is thus a big decrease in the resulting Λ_L.

If no scaling region exists, even on a infinite lattice, then the results for m_ρ will depend on the chosen β, contrary to the conventional procedure. This is, however, hardly surprising, as by increasing β we decrease the lattice spacing $a(\beta)$ and thus also the amount of quark loops taken into account in the quenched approximation. In this situation the optimal choice of β is then a compromise between two opposite requirements : high values of β are necessary to get into the continuum limit, while small β is prefered from the point of the validity of the quenched approximation. Thus the following procedure suggests itself :

1) scaling region is identified using (3) with $n_f=0$,
2) in this, usually narrow, window of β values, $\Lambda_L(\beta)$ is determined from comparison of experimental data with Monte-Carlo calculations using (3) with $n_f=3$ or 4,
3) β-dependence of $\Lambda_L(\beta)$ is investigated. The "optimal" value of Λ_L is to be identified with $\Lambda_L(\beta)$ for β in the lower part of the scaling region defined in 1). This adds some ambiguity to the error on Λ_L coming from the Monte-Carlo calculations themselves.

Third, one might question the relevance and reliability of the quenched approximation calculations for the construction of the continuum limit. It remains to be shown quantitatively, how important the contributions of quark loops to the hadron masses really are. Some recent calculations/16/ going beyond the quenched approximation indicate that, hopefully, not very much. The results of 16th order hopping parameter expansion give (on moderately large lattice

8^4) for the product $m_\rho a(\beta=5.7)$ the value higher by only 20-30% then that of the quenched approximation at the same β and the same distance from the estimated position of the critical hopping parameter, which means, roughly, for the same current quark mass. This trend is to be expected, as the energy associated with the fluctuactions due to the quark loops should increase the mass of any hadron, provided Λ_L (and the masses of quarks) are held fixed. Turning this argument around, in order to reproduce the known masses of hadrons, lower Λ_L is to be expected from full, unquenched calculations, compared with that of the quenched approximation. The numbers obtained for $\Lambda_{\overline{MS}}$ in the quenched approximation are therefore probably upper bounds on this parameter. A rather small difference between the quenched and unquenched calculations is encouraging, but so far no scaling behavior analysis has been performed to establish whether this accuracy characterizes also the continuum limit. Also, extensive simulations will be needed to clearly locate the position of the critical hopping parameter.

As for the scaling behavior of the hadron masses in the quenched approximation, the evidence presented in ref. [13-15] is admittedly rather unconvincing /17/. This is disquieting especially in the light of the recent discovery /18/ that the lattice β-function does not scale, in the region of β values investigated in /13-15/, according to the asymptotic scaling formula (3). Nevertheless, the general patern of deviations from it has been interpreted /19/ as the evidence for "scaling in general". This means that in extracting the physical quantities from lattice calculations one should use instead of (3) a slightly modified formula

$$a(g^2) = \frac{1}{\Lambda_L} \varphi(g^2) = \frac{1}{\Lambda_L} f(g^2)\left[\frac{\varphi(g^2)}{f(g^2)}\right] \qquad (4)$$

where $\varphi(g^2)$ is a universal function of g^2 (and possibly n_f, see below) which for $g^2 \to 0$ goes over into $f(g^2)$. The ratio $r(g^2)=f(g^2)/\varphi(g^2)$ has been extracted from theoretical lattice calculations in the quenched approximation, i.e. for $n_f=0$, in ref. [19]. One finds, for instance, $r(\beta=5.7)=0.68$, $r(\beta=6)=0.81$. Qualitatively the same situation is to be expected in the unquenched calculations, but the concrete numbers may be different. As we do not presently understand the reasons behind the validity of the "scaling in general" formula (4) it is difficult to speculate on its n_f-dependence. Keeping this reservation in mind, I shall assume the approximate validity of (4) even for $n_f=3,4$. Taking the above correction factors into account increases the extracted value of $\Lambda_{\overline{MS}}$ by a factor 1.5 at β=5.7 and by a factor 1.25 at β=6.

Summarizing all sources of uncertainty in the determination of $\Lambda_{\overline{MS}}$ from the available lattice calculations of the hadron spectrum I conclude that barring a complete failure of the quenched approximation in the continuum limit, the resulting value of $\Lambda_{\overline{MS}}$ lies below 100 MeV/c.

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Хила И. E2-85-741
Об определении Λ-параметра КХД при помощи решеточных расчетов

Последние результаты расчетов в КХД на решетке при помощи метода Монте-Карло используются для оценки параметра $\Lambda_{\overline{MS}}$. Особое внимание уделяется влиянию легких кварков на построение непрерывного предела решеточной КХД в приближении замороженных кварков. Показано, что в силу этого вытекающее значение параметра $\Lambda_{\overline{MS}}$ сильно зависит от числа легких кварков и находится в области $\Lambda_{\overline{MS}} < 100$ МэВ/с.

Работа выполнена в Лаборатории теоретической физики ОИЯИ

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Chýla J. E2-85-741
On Fixing the Scale in QCD from Lattice Calculations

Recent results of lattice Monte-Carlo calculations in QCD are used to estimate the value of $\Lambda_{\overline{MS}}$. Special attention is paid to the role played by the light quarks in the construction of the continuum limit of lattice approximations in QCD. The resulting value of $\Lambda_{\overline{MS}}$ turns out to be strongly dependent on n_f, the number of light quarks taken into account.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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