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Gauge Anomalies, Gravitational Anomalies, and Superstrings

by

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Abstract

The structure of gauge and gravitational anomalies will be reviewed. The impact of these anomalies on the construction , consistency, and application of the new superstring theories will be discussed.

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Introduction

During the past fifteen years gauge field theories have been spectacularly successful in providing the theoretical framework for describing elementary particles and their interactions. The Standard model based on $SU_3 \otimes SU_2 \otimes U_1$ gauge dynamics appears to explain, or accommodate, essentially all of the known data pertaining to the strong, weak, and electromagnetic interactions. In addition, the gravitational interactions seem to also be based on the gauge principles of general coordinate invariance and local Lorentz invariance. Anomalies have played a crucial role in the development and the application of gauge theories in particle physics.

Anomalies seem, at first, to be only a minor discrepancy in the analysis of the Ward-Takahashi identities for certain currents in spinor field theories. However anomalies have deeper significance in gauge field theories as they reflect the impossibility of maintaining the local gauge symmetry when certain matter fields are quantized. Hence, understanding the structure of anomalies is essential to the development of gauge theories. They have also proved to be important to analyzing to implications these theories from the low energy theorems of current algebra to the rules for chiral boundstate structure in composite models.

In this talk, I will review certain aspects of the structure of gauge anomalies in the context of quantum field theory. Gravitational anomalies are similar to gauge anomalies as they reflect the breaking of the local gravitational symmetries by the quantum properites of the matter fields, and their structure is simply related to the gauge case. I will also discuss the crucial role of anomalies in the recent development of consistent superstring theories and the use of the anomaly stucture to determine the effective low energy implications of the superstring.

Gauge Anomalies

The impact of anomalies on gauge field theories follows from the general analysis of the nonabelian anomalies. These anomalies were determined by a detailed calculation of the relevant spinor loop diagrams[1]. I will review the procedure used in the original calculation and relate it to the wide variety of alternate procedures subsequently developed.

The Ward-Takahashi identities for arbitrary nonabelian currents were examined for general vector, axial vector, scalar, and pseudoscalar external fields[2]. The external field couplings were defined by the interaction lagrangian,

$$\mathcal{L}_I = \bar{\psi}(x)[-G_0 P_+(x) + \gamma_\mu V_+^\mu(x)]\psi(x) \tag{1}$$

where the boson fields, $P_+ = \Sigma + i\gamma_5 \Pi$ and $V_+^\mu = V + \gamma_5 A$, are arbitrary matrices in the space of internal symmetries. A symmetric point-split method was used to compute the gauge dependence of the effective action in the presence of these external fields. The anomaly is determined from this anomalous gauge dependence.

We can define the renormalized effective action, S_R, as

$$e^{S_R(\Sigma,\Pi,V,A)} = \langle e^{i\int dx\ \mathcal{L}_I(x)} \rangle_0 \ e^{-R(\Sigma,\Pi,V,A)} \tag{2}$$

where $R(\Sigma,\Pi,V,A)$ are a set of local counter terms used to renormalize the effective action. Under a general gauge transformation, the external fields transform according to

$$\delta_\Lambda V_+(x) = \partial^\mu \Lambda_+(x) + i\Lambda_+(x)V_+^\mu(x) - V_+^\mu(x) i\Lambda_+(x),$$

$$\delta_\Lambda P_+(x) = i\Lambda_-(x)P_+(x) - P_+(x)i\Lambda_+(x) \tag{3}$$

where $\Lambda_\pm = \Lambda_V \pm \gamma_5 \Lambda_A$. The direct computation of the gauge dependence of the effective action yields the anomaly,

$$\delta_\Lambda S_R(\Sigma,\Pi,V,A) = D_R(\Lambda_+;\Sigma,\Pi,V,A). \tag{4}$$

The precise form of the anomaly, D_R, depends on the choice of the local counter terms, R, used to renormalize the effective action. A choice of R which preserves the global chiral structure yields the anomaly,

$$D_R = (1/6)(4\pi)^{-2} i \int dx\, i\varepsilon_{\mu\upsilon\sigma\tau} \mathrm{tr}[\{2i\Lambda_+(x)\partial^\mu V_+{}^\upsilon(x)\partial^\sigma V_+{}^\tau(x) - \partial^\mu \Lambda_+(x) V_+{}^\upsilon(x) V_+{}^\sigma(x) V_+{}^\tau(x)\}\gamma_5] \tag{5}$$

Note that the anomaly depends only on the vector and axial vector fields and that the left-handed fields, V_-, and right-handed fields, V_+, are decoupled.

A different choice of **R** which preserves vector current conservation can be found. Now only the axial vector gauge transformation is anomalous,

$$\delta_{\Lambda(V)}\, S_R = 0$$

$$\begin{aligned}\delta_{\Lambda(A)}\, S_R = D_R = -(4\pi^2)^{-1} i \int dx\, \varepsilon_{\mu\upsilon\sigma\tau}\, \mathrm{tr}_I[\Lambda_A(x) \\ \{(1/4)F_V{}^{\mu\upsilon}(x)F_V{}^{\sigma\tau}(x) + (1/12)F_A{}^{\mu\upsilon}(x)F_A{}^{\sigma\tau}(x) + \\ (2/3)iA^\mu(x)A^\upsilon(x)F_V{}^{\sigma\tau}(x) + (2/3)iF_V{}^{\mu\upsilon}(x)A^\sigma(x)A^\tau(x) + \\ (2/3)iA^\mu(x)F_V{}^{\upsilon\sigma}(x)A^\tau(x) - (8/3)A^\mu(x)A^\upsilon(x)A^\sigma(x)A^\tau(x)\}]\end{aligned} \tag{6}$$

where the trace tr_I is now only over the internal symmetry degrees of freedom and the field strengths are given by

$$\begin{aligned}F_V{}^{\mu\upsilon}(x) &= \partial^\mu V^\upsilon(x) - \partial^\upsilon V^\mu(x) - i[V^\mu(x), V^\upsilon(x)] - i[A^\mu(x), A^\upsilon(x)] \\ F_A{}^{\mu\upsilon}(x) &= \partial^\mu A^\upsilon(x) - \partial^\upsilon A^\mu(x) - i[V^\mu(x), A^\upsilon(x)] - i[A^\mu(x), V^\upsilon(x)].\end{aligned} \tag{7}$$

This result follows from the direct calculation of one fermion loop Feynman diagrams to all orders in the external fields.

The particular regularization procedure used to compute the diagrams is not relevant to the determination of the structure of the anomaly. Many other methods have been suggested in the literature, from the original Pauli-Villars method used by Steinberger[3] in 1949 to modern functional integral methods[4]. Any valid method must give an unambiguous definition to the effective action so that the gauge dependence can be properly determined.

Generally, any particular definition will produce many anomalous terms as in the case of the unrenormalized point-split method. However, the various renormalized effective actions can only differ by terms which are local products of the external fields, and every properly defined procedure can yield the same results with the appropriate renormalization counterterms. Also, there is no uniquely preferred definition or method of calculation of the spinor loops as different symmetries may be required for different theories, and the anomalies may not permit the simultaneous imposition of these different symmetries. For example, the conservation of the $SU_{2L} \otimes U_{1Y}$ symmetry of the Standard model conflicts with conservation of the vector baryon number symmetry[5] for the same set of fermion loops.

For gauge field theories, the anomaly equations derived above have three independent implications.

a) The dynamical currents must be fully gauge invariant and free of anomalies. This condition places a constraint on the fermion representations for certain gauge theories. It is necessary to require that $\mathrm{tr}(\lambda^a\{\lambda^b,\lambda^c\}) = 0$ where $\{\lambda^a\}$ are the dynamical coupling matrices of the gauge fields to the spinors written as lefthanded fields. If these conditions are satisfied, then all other dynamical anomalies will vanish due to the consistency conditions[6].

b) Flavor currents have charges which commute with the gauged, dynamical currents. However, the flavor currents may have dynamical anomalies. By an appropriate choice of counter terms, it is always possible to define the flavor currents to be gauge invariant with respect to the dynamical symmetries[7]. The anomalous divergence of this covariant symmetry current is given by

$$\partial^\mu J_{s\mu}(x) = (32\pi^2)^{-1}\varepsilon_{\mu\upsilon\sigma\tau}\, \mathrm{tr}_I\{\lambda_s T^{\mu\upsilon}(x) T^{\sigma\tau}(x)\} \qquad (8)$$

where λ_s is the coupling matrix of the symmetry current and the trace is over the internal degrees of freedom of the spinors. This divergence is proportional to a topological index. For nonabelian gauge theories which have fluctuations in the topological charge (instantons, etc), the anomaly may imply that certain symmetry charges are not conserved as implied by the solution to the U(1) problem in QCD[8]. If there exist background field configurations with nontrivial topological structure, the anomalies will determine the spectrum of light fermions through the index theorems[9].

c) Flavor currents may be free of dynamical anomalies but still have flavor anomalies. If these anomalies are unrenormalized (Adler-Bardeen theorem[10]), then they impose certain constraints on the possible infrared structure of the theory. The anomalies of the effective low energy theory must match the anomalies of the fundamental theory. These constraints are known as the 't Hooft conditions[11] when applied to composite models. That the anomaly implies certain infrared constraints can be seen by examining the AVV triangle amplitude,

$$\langle 0 | J_{5\lambda}(k) J_\mu(p) J_\upsilon(q) | 0 \rangle = k_\lambda \varepsilon_{\mu\upsilon\alpha\beta} p^\alpha q^\beta \; A(k,p,q) + \ldots\ldots \qquad (9)$$

If we use the form of the axial anomaly which preserves vector current conservation, then this amplitude must be proportional to **p** and to **q**. The divergence amplitude is then kinematically of order (momentum)^3 (really of order (momentum)^4). However the anomaly matrix element is only of order (momentum)^2,

$$k^\lambda \langle 0 | J_{5\lambda}(k) J_\mu(p) J_\upsilon(q) | 0 \rangle = \varkappa (2\pi^2)^{-1} \varepsilon_{\mu\upsilon\alpha\beta} p^\alpha q^\beta \, . \qquad (10)$$

Hence the invariant functions of the three current amplitude, such as **A(k,p,q)**, must be infrared singular. This infrared singularity can be produced by massless fermions, as in the fundamental theory, or by the Goldstone poles generated by Wess-Zumino terms[12] of an effective action, or by a combination of these mechanisms.

All of the above implications have been extensively exploited for their phenomenological consequences. However, their significance is only important if the structure of the anomaly goes beyond the perturbative calculation of the spinor loop.

Although anomaly was established by direct calculation, the form of the anomaly can be determined indirectly by observing that the anomaly must satisfy a set of consistancy conditions. Since the anomaly is the gauge variation of an effective action its gauge transformation properties are constrained. The commutator of two gauge transformations is again a gauge transformation,

$$\delta_\Lambda \delta_\Omega - \delta_\Omega \delta_\Lambda = \delta_{i[\Omega,\Lambda]} \tag{11}$$

The constraint on the anomaly follows by applying this relation to the effective action

$$\begin{aligned}(\delta_\Lambda \delta_\Omega - \delta_\Omega \delta_\Lambda) S_R &= \delta_\Lambda D_R(\Omega) - \delta_\Omega D_R(\Lambda) \\ = \delta_{i[\Omega,\Lambda]} S_R &= D_R(i[\Omega,\Lambda]).\end{aligned} \tag{12}$$

This constraint is called the Wess-Zumino consistancy condition[12]. Up to the freedom embodied by the addition of local counterterms to the effective action, the anomaly found by explicit calculation is the unique particular solution to the consistancy conditions with only the overall scale undetermined. However, the overall scale can be determined by an index theorem for the Dirac operator[9]. In higher dimensions, there may be additional independent solutions which are again determined by index theorems. The consistency conditions imply a structure to the anomaly which is independent of perturbation theory.

The consistency conditions are essentially a integrability condition for the existence of an effective action. A natural language for studying the anomaly, particularly in higher dimensions, is that of differential geometry and the use of differential forms[13]. The abelian anomaly is directly a saturated form involving the field strength forms, and the nonabelian anomaly is related to invariant polynomials of the field strength form in two dimensions higher than the space-time dimension. While the methods of differential geometry can be used to determine the structure of the anomaly, the overall strength must again be directly computed or, for the spinor loop, determined from index theorems.

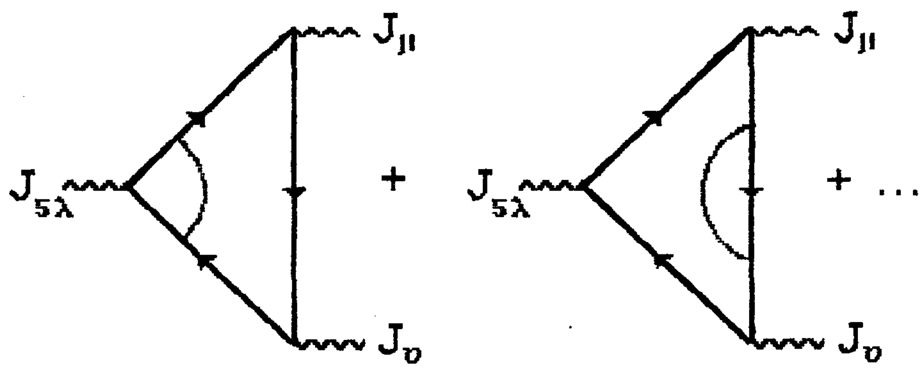
Index theorems[9] can be derived for the Dirac operator with nontrivial background gauge fields. The index theorem relates the number of zero modes of the Dirac operator to a topological property of the background gauge field. These theorems are consistent with the anomaly equations even though the anomaly equations were derived for perturbative external fields with no nontrivial stucture. However, it is not clear that the index theory can be used beyond lowest order as the quantum structure of the gauge field may be essential, and the anomaly may be affected by renormalization.

The importance of the anomaly cancellation conditions or the 't Hooft consistency conditions depends on the fact that the anomaly conditions are not affected by renormalization. The choice of fermion representations to cancel anomalies should not be affected by a correction which is a power series in the coupling constant. The absence of higher order radiative corrections to the anomaly is known as the Adler-Bardeen Theorem[10].

The original calculation showed the absence of these higher order corrections in QED and in a model with PCAC. The axial vector current divergence equation was shown to have the form,

$$\partial^\lambda J_{5\lambda}(x) = J_5(x) + (\alpha_0/4\pi)F^{\mu\upsilon}(x)F^{\sigma\tau}(x)\,\varepsilon_{\mu\upsilon\sigma\tau} \tag{13}$$

where α_0 is the bare electromagnetic coupling constant which is renormalized only by the external line wavefunction renormalization of the photon. The result is also estabished in second order by an explicit calculation of the radiative corrections to the triangle diagram.



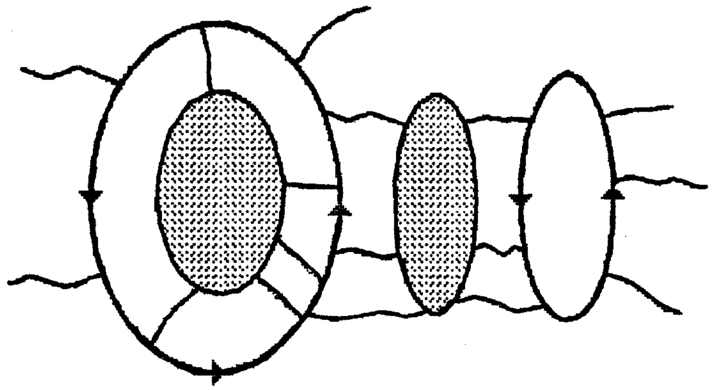
RADIATIVE CORRECTIONS

The sum of these diagrams is shown to vanish. The result implies an exact low energy theorem for the two photon matrix element of the naive divergence of the axial current,

$$\langle 0|J_5(0)|\gamma_1(p)\,\gamma_2(q)\rangle \to -(\alpha/\pi)\,F_1^{\mu\upsilon}(p)\,{}^*F_{2\mu\upsilon}(q) \tag{14}$$

as $p,q \to 0$.

A general argument for the cancellation of higher order corrections to the anomaly can be made by constructing a regularization procedure where the boson lines are given a gauge invariant regularization. One possibility is to



General Higher Order Diagram

use dimensional regularization for the boson fields but keep the fermions in four dimensions, and another is to remain completely in four dimensions but use higher derivative, gauge invariant terms in the action to provide the regularization. In both cases, only the small spinor loops remain divergent and are possible sources for the anomalous behavior. These anomalies have already been completely classified by the spinor loop calculation. Because the boson lines are regulated, the fermion loop diagrams with the radiative corrections can be viewed as fermion loops with more external lines. However, fermion loops with many external lines are free of anomalies. Hence there can be no additional source for the breaking of the gauge Ward-Takahashi identities in the regulated theory. The theory must still be renormalized and sufficient care must be taken to preserve the Ward identities in the renormalization process[14].

Another proof of the nonrenormalization theorem which avoids the expicit use of any regularization procedure was advocated by A. Zee. This approach makes use of renormalization group techniques through the application of the Callan-Symanzik equation to the anomaly to prove the nonrenormalization theorems[15].

Perhaps the most controversial application of the nonrenormalization theorems concerns supersymmetry. The supersymmetry transformation properties of the anomaly and the related scale and supercurrent anomalies are involved. The regularization methods discussed above can not be directly applied as they seem to break supersymmetry (although a version of the higher derivative method can work). The naive application of the theorem seems to imply the the two loop beta function is exact with no higher order correction, a statement which clearly is renormalization prescription dependent.

The application of the nonrenormalization theorem to the anomaly cancellation condition and to the 't Hooft conditions are obvious. In the first case we arrange for the anomalies to cancel in lowest order, and the nonrenormalization theorem implies that no further anomalies arise in higher order, zero stays zero. In the second case the anomaly of the flavor currents occurs only in the lowest order, and there are no dynamical corrections. The mixed case where the flavor currents have dynamical anomalies, as in the U(1) problem, is more subtle as one must be careful to properly define all terms in the divergence equations[16].

In addition to the anomalies which can be studied using the infinitesimal diagram analysis, there are also discrete versions of the anomaly[17]. Here there exist finite gauge transformations which do not leave the effective action (or fermion determinant) invariant even though there are no infinitesimal anomalies. An example is an SU_2 gauge theory with an odd number of Weyl doublets. These anomalies may be studied using index theory and spectral flow in one dimension higher than the space-time dimension.

Gravitational Anomalies

Gravitational interactions are also subject to anomalies. The local gauge symmetries are supplemented by the local symmetries of general coordinate invariance and local Lorentz invariance. The study of the loop diagrams for matter fields yields gravitationally induced chiral anomalies.

In the presence of background gauge fields, the singlet axial current has anomalous divergence which is proportional to a topological density,

$$\partial^\mu J_{5\mu}(x) = J_5(x) + (1/16\pi^2)\, \varepsilon_{\mu\upsilon\sigma\tau}\, \mathrm{tr}\{F^{\mu\upsilon}(x)F^{\sigma\tau}(x)\}. \qquad (15)$$

For nontrival gauge field configurations (instantons, etc), the anomaly is related to the index of the Dirac operator and predicts the chiral structure of the zero modes in the limit of exact chiral symmetry.

We may consider the analogous situation for background gravitational fields. Explicit calculation[18] of the triangle and related diagrams yield an anomalous divergence for the singlet axial vector current,

$$D^{\mu}J_{5\mu}(x) = J_5(x) + N(1/768\pi^2)\,\varepsilon_{\mu\upsilon\sigma\tau}\,R^{\mu\upsilon\alpha\beta}(x)R^{\sigma\tau}{}_{\alpha\beta}(x) \qquad (16)$$

where $R^{\mu\upsilon\alpha\beta}$ is the curvature tensor. The anomalous divergence is again related to a topological index for the background manifold. Although the anomaly is a total divergence, it can not be canceled if the axial vector current is required to be general coordinate covariant. As in the gauge case, the existence of the anomaly reflects the conflict between axial gauge invariance and general coordinate invariance. This anomaly must be canceled for realistic gauge models. The Standard model does satisfy this cancelation condition as the U(1) hypercharge current is traceless for each generation. Since the anomaly has infrared significance for the three point function, it cannot be viewed as a purely short distance problem, and its cancelation is required for the infrared consistency of the theory.

Chiral fields can also induce purely gravitational anomalies. Gravity has a nonabelian structure similar to gauge fields. The anomaly stucture of the gravitational interactions can be analyzed by direct calculations[19] or by use of the methods of differential geometry and the consistency conditions[20].

The connection between the gauge and gravitational anomalies can be established through the examination of the transformation properties under general coordinate invariance. We have

$$\delta_{\xi}x^{\mu} = \xi^{\mu}(x),$$

$$\delta_{\xi}A(x) = \xi^{\mu}\partial_{\mu}A(x) \quad \text{- for a scalar field,} \qquad (17)$$

$$\delta_{\xi}g_{\mu\upsilon}(x) = \xi^{\lambda}\partial_{\lambda}g_{\mu\upsilon}(x) + \partial_{\mu}\xi^{\lambda}g_{\lambda\upsilon}(x) + \partial_{\upsilon}\xi^{\lambda}g_{\mu\lambda}(x)$$
- for the metric tensor

with similar transformation properties for other tensors. The connection transforms inhomogenously like the gauge potential,

$$\delta_\xi \Gamma^\tau{}_{\mu\sigma}(x) = \xi^\lambda \partial_\lambda \Gamma^\tau{}_{\mu\sigma}(x) + \partial_\mu \xi^\lambda \Gamma^\tau{}_{\lambda\sigma}(x) + \partial_\sigma \xi^\lambda \Gamma^\tau{}_{\mu\lambda}(x) \\ -\Gamma^\lambda{}_{\mu\sigma}(x)\partial_\lambda \xi^\tau - \partial_\mu \partial_\sigma \xi^\tau \tag{18}$$

and the curvature tensor transforms like the field strength,

$$R_{\mu\upsilon\sigma}{}^\tau = \partial_\mu \Gamma^\tau{}_{\upsilon\sigma} - \partial_\upsilon \Gamma^\tau{}_{\mu\sigma} + \Gamma^\lambda{}_{\mu\sigma}\Gamma^\tau{}_{\upsilon\lambda} - \Gamma^\lambda{}_{\upsilon\sigma}\Gamma^\tau{}_{\mu\lambda}.$$

$$\delta_\xi R_{\mu\upsilon\sigma}{}^\tau = \xi^\lambda \partial_\lambda R_{\mu\upsilon\sigma}{}^\tau + \partial_\mu \xi^\lambda R_{\lambda\upsilon\sigma}{}^\tau + \partial_\upsilon \xi^\lambda R_{\mu\lambda\sigma}{}^\tau \\ + \partial_\sigma \xi^\lambda R_{\mu\upsilon\lambda}{}^\tau - R_{\mu\upsilon\sigma}{}^\lambda \partial_\lambda \xi^\tau. \tag{19}$$

The nonabelian structure of the general coordinate transformation can be seen by considering the commutator of two transformations, $\delta_\xi\ \delta_{\xi'} - \delta_{\xi'}\ \delta_\xi = \delta_{[\xi,\xi']}$ where $[\xi,\xi'] = \xi'^\lambda \partial_\lambda \xi - \xi^\lambda \partial_\lambda \xi'$. The gravitational transformations are just those of a local gauge symmetry, but where the group and field representations are specified.

For the effective action to be general coordinate invariant, the energy momentum tensor should be covariantly conserved. However, the existence of anomalies will imply that the conservation of the energy momentum tensor cannot be maintained in the presence of certain chiral fields. The structure of the purely gravitational anomalies implies that they exist only in 4n+2 dimensions, ie 2, 6, 10, etc. The analysis of these anomalies is complicated by that fact that neither power counting or covariance can be used to limit the order of diagrams subject to anomalies.

The result of direct calculation or the use of differential geometry gives an expression for the gravitational anomaly which has exactly the same structure as the gauge anomaly with even the same functional dependence. If the nonabelian gauge anomaly is given as the gauge variation of the effective action and expressed in terms of differential forms as

$$\delta_\Lambda W = H_\Lambda = \int \Lambda \cdot G(A,F) \tag{20}$$

then the gravitational anomaly is given by

$$\delta_\xi W = H_\xi = - \int \partial_\upsilon \xi^\mu \, G^\upsilon{}_\mu(\Gamma,R) \tag{21}$$

where $G(x,y)$ is the same function in both cases and the forms are related through the correspondence given in Eq.18 and Eq.19. The pure gravitational anomaly can only exist in 2,6,10, etc because the corresponding gauge theories have anomalies only in these dimensions. Using the methods of differential forms, the nonabelian anomaly can be expressed in terms of invariant polynomials in two dimensions higher, ie 4,8,12, etc. Since the curvature matrix is antisymmetric, the invariant polynomials for the purely gravitational anomalies are nonvanishing only in these dimensions.

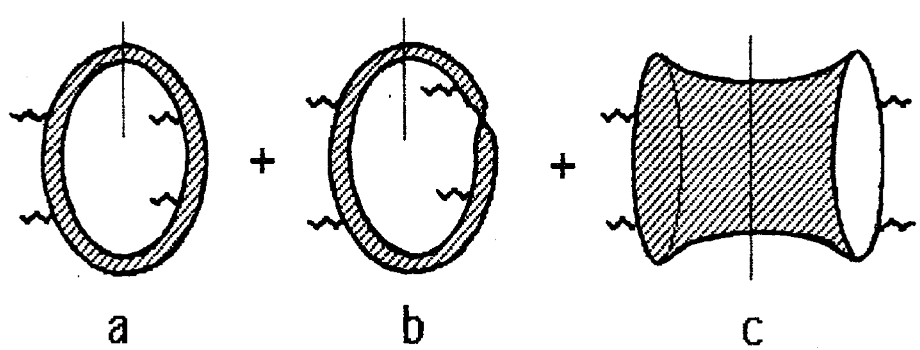
The explicit calculations of Alvarez-Gaumé and Witten[19] reveal the general form of the gravitational anomalies which come from loops involving chiral spin 1/2 and spin 3/2 fermion fields and self-dual tensor boson fields. This anomaly structure determines the criteria for gravitational anomaly cancellation. Witten[21] has also shown that there are also the global versions of the gravitational anomaly which must also be considered when constructing consistent theories involving gravity.

Superstrings

The "elementary" particles we see today made be composite structures involving still more "elementary" particles as has been the case in the past. However this time, the more fundamental level of understanding may require a different kind of substructure, the superstring. The superstring is a fundamental one dimensional object which replaces zero dimensional point particle as the ultimate form of matter. In fact, superstring theory appears to be the only fundamental quantum theory which consistently unifies gravity with other interactions. Anomalies have played an important role in the development of these superstring theories and play an equally important role in their analysis.

Superstring theories live naturally in ten dimensions. At energies below the Planck scale, the ten dimensional superstring theory becomes a normal ten dimensional supergravity model. These supergravity models were analyzed for gauge and gravitational anomalies[19]. The type IIA supergravity model which corresponds to a nonoriented, closed superstring theory is nonchiral and has no anomalies; it also does not seem to produce an acceptable low energy phenomenology. The type IIB supergravity model which corresponds to an oriented, closed superstring theory is chiral and has anomalies for each loop that cancel in the sum; this theory also does not seem to have the necessary gauge and chiral structure to reproduce phenomenology. The other possibility is the type I supergravity model which corresponds to a nonoriented, closed and open, gauged superstring theory. Although it appears to have a sufficiently rich gauge structure, it also appears to have both gauge and gravitational anomalies for any gauge group.

Schwarz and Green[22] have discovered a new mechanism for the cancellation of the anomalies found from the loop calculations of the supergravity model. They argue for the existence of new, anomalous interactions for the antisymmetric tensor field, a partner to the gravition field, and that these interactions are actually already a part of the superstring theory. These interactions are analogous to the Wess-Zumino terms of chiral models[12] and contribute to the anomaly in tree approximation precisely cancelling the loop anomalies. This cancellation appears artifical in the supergravity models but is natural in the superstring theories as both the loop contributions (**a**,**b**) and the anomalous tree contributions (**c**) are part of the low energy limit of the same string diagram (**a** = **c**). The cancellation of the remaining anomaly between the planar diagrams (**a**) and the twisted diagrams (**b**) works only for the gauge group SO_{32}. Apparently the anomaly structure permits only one possible unified theory of gravity with a sufficiently rich gauge and chiral structure. Actually the supergravity



Superstring diagrams

models allow one other solution based on the gauge group, $E_8 \otimes E_8$. An entirely new superstring theory, the heterotic string[23], was discovered which corresponds to this case. It is remarkable that the anomaly structure has dictated the existence of only two possible superstring theories. Of these two theories, the heterotic string seems to be the most promising from the phenomenological point of view.

To make connection with the low energy world we see around us, we must understand how the ten dimensional string theory reduces to a four dimensional effective field theory. As mentioned before, the naive low energy limit produces a supergravity theory in ten flat spacetime dimensions. However, if six dimensions are associated with a compact manifold, then the low energy limit produces a normal effective field theory of the other four dimensions. Since the string theory involves gravity, the determination of the compact and noncompact dimensions will be a complex dynamical problem.

Since the compact radii are expected to be of Planck size, a the typical energy scale for particle masses should be of order the Planck mass. The exceptions will be the "zero modes" or zero energy solutions to the field equations on the compact six dimensional manifold. These zero modes are to be identified with the low energy particle content of the theory and are determined by the topological structure of the manifold through the index theorems related to the anomalies.

The search for acceptable six dimensional manifolds has focused on manifolds which preserve an N=1 supersymmetry as required by solutions to the gauge heirarchy problem. The requirement of N=1 supersymmetry was shown to imply that the compact space should have SU_3 holonomy[24]. The gauge symmetry, $E_8 \otimes E_8$, of the heterotic string is reduced to $E_6 \otimes E_8$ on these spaces. The gauge dynamics associated with the E_6 gauge symmetry can be identified as the grand unification symmetry while the gauge dynamics of the E_8 group refer to a hidden sector or shadow world theory which may produce the desired low energy breaking of supersymmetry. The spectrum of light fermions is determined by the topology of the space and the number of generations, 27's of E_6, is related to the Euler characteristic of the manifold. The nontrivial structure of the manifold will also affect the symmetry breaking patterns of the E_6 gauge group through the existence of zero action gauge configurations on the extra space[25]. Of course, an intense effort is being made by many people to study the possible compactification manifolds and their impact on finding solutions to the superstring theory which predict the known low energy physics.

Summary

Anomalies began as a minor discrepency in spinor loop calculations. I have reviewed the systematic study of these anomalies through the use of Feynman diagram calculations. I have discussed how the results made be interpreted through the use of consistency conditions, differential geometry, and index theorems. The application of these results to physics such as the anomaly cancellation conditions and current algebra depend on the existence of nonrenormalization theorems which establish the results beyond perturbation theory. The anomaly structure of the gauge interactions has been generalized to include the gravitational interactions which can also be viewed through its local gauge properties. Both gauge and gravitational anomalies have had a significant impact on the development and the analysis of the new superstring theories which may provide the framework for the ultimate unified theory of all matter.

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