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ON THE TWO-PHOTON WIDTH OF THE δ(980)

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ON THE TWO-PHOTON WIDTH OF THE $\delta(980)$ *

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ABSTRACT

We evaluate the two photon width of the $\delta(980)$ using three-point function sum rules which are able to predict accurately the anomalous $\pi^0 \to \gamma\gamma$ and non-anomalous $\delta \to \eta\pi$ decay rates. The prediction, though smaller than previous results based on vector meson dominance, is still higher than the present Crystal Ball data. An analysis of the three-point function with one gluon exchange cannot support previous successful explanation of the data within the four-quark scheme.

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It remains still an open problem to understand the exact quark content of the well-established $I = 1$, $J^{PC} = 0^{++}$ $\delta(980)$ resonance which just lies above the $K\bar{K}$ threshold.

Actually, we have two tendancies. The first one is the conventional quark model point of view, where the δ is interpreted as the lowest ground state associated to the divergence of the vector current and so it is the chiral partner of the Goldstone π-meson. Various properties and implications of the δ within such a scheme have been studied so far (hadronic widths[1], quark mass-difference[2], decay amplitude[3], quark condensate ratio[3], δ-mass[4] and couplings[5) 6)]), and all of them agree well with the data and with some other independent predictions. However, the degeneracy of the δ-S^* masses and the possible strong coupling of the S^* to $K\bar{K}$ pairs remain unanswered if the S^* is a $\bar{s}s$ state, as one actually expect to have a $\bar{s}s$ state to be around 1.3 GeV, due to $SU(3)_f$-breaking effects[4]. Among other possible solutions to such problems, we can have a low mass 0^{++} gluonium[7] which can mix with the $q\bar{q}$ meson in the isoscalar channel[8], or we can blame the quark model and propose a four-quark structure of the δ and S^* [9]:

$$|\delta> \equiv \frac{1}{\sqrt{2}}(\bar{s}s)(\bar{u}u - \bar{d}d) \quad ,$$

$$|S^*> \equiv \frac{1}{\sqrt{2}}(s\bar{s})(\bar{u}u + \bar{d}d) \quad , \tag{1}$$

which appears, at the first sight, to be unrelated to the notion of divergence of vector current (which vanishes for $m_u = m_d$) for the δ. However, Eq(1) has motivated a large amount of phenomenological analysis[9-11] but there has not yet been any convincing evidence of the true quark structure of the δ-meson as both schemes of the quark assignement for the δ reproduce quite well the δ-parameters. However, it has been claimed recently[11] that the Crystal Ball data[12]:

$$\Gamma(\delta \to \gamma\gamma)\, B(\delta \to \pi\eta) \simeq (0.19 \pm 0.07 {}^{+0.10}_{-0.07})\ \text{keV} , \tag{2}$$

provide a good source of information on the quark structure of the δ-meson. Ref 11) has extrapolated the charmonium result[13] in order to relate the wave functions and so the two-photon width of the δ and of the well-established tensor meson A_2. In this way, he concludes that the $\bar{u}d$ assignement for the δ gives a two-photon width of the order of 1.5 keV which is too high compared to the data in Eq(2). However, one can suspect the validity of such a method for the light quark systems. In this paper, we use three-point function sum rules, which have given some quite satisfactory predictions for the couplings of known hadrons[4a, 5, 14], for the study of the two-photon and hadronic couplings of the δ-meson. We choose to work with the three-point function subtracted at the Euclidian point[5] which is certainly free of mass singularities and where some eventual anomalous thresholds and analyticity problems are absent in the approximation of narrow resonance poles used in the analysis. Our paper will be organized as follows: Firstly, we estimate the $\pi^0 \to \gamma\gamma$ and $\delta \to \eta\pi$ decay-widths which are representative for an anomalous and a non-anomalous processes and which are well established experimentally. Once we obtain a control of the accuracy of our method, we study the open case of the $\delta \to \gamma\gamma$ width in the models where the δ is a $\bar{u}d$ "conventional" state or it is an exotic four-quark state of the type proposed in Ref 9). We also study, in this later case, the hadronic couplings of the δ which we shall confront with the data.

1. $\pi^0 \to 2\gamma$ and $\delta \to \eta\pi$ decay widths

For $\pi^0 \to 2\gamma$, we estimate first the $\omega\rho\pi$ coupling using the three-point function[5]:

$$T^{\mu\nu}(p,q) = \int d^4x \, d^4y \, e^{iqx} \, e^{ipy} \langle 0 | T \, J^{\mu}_{\rho}(x) \, J_{\pi}(0) \, J^{\nu}_{\omega}(y) | 0 \rangle ,$$

with: (3)

$$J_{\pi}(x) = \frac{1}{\sqrt{2}} : \{ 2 m_d \, \bar{d}(i\gamma_5)d - 2 m_u \, \bar{u}(i\gamma_5)u \} : ,$$

$$J^{\mu}_{\rho}(x) = \frac{1}{2} : (\bar{u}\gamma^{\mu} u - \bar{d}\gamma^{\mu} d) : , \qquad (4a)$$

$$J^{\nu}_{\omega}(x) = \frac{1}{6} : (\bar{u}\gamma^{\nu} u + \bar{d}\gamma^{\nu} d) : ,$$

where the quark currents are normalized as:

$$\langle 0 | J_{\pi}(x) | \pi \rangle = \sqrt{2} f_{\pi} m_{\pi}^2 \; : \; (f_{\pi} \simeq 93.3 \text{ MeV}),$$

$$\langle 0 | J^{\mu}_{V}(x) | V \rangle = \frac{M_V^2}{2\gamma_V} \epsilon^{\mu} \; : \; (V \equiv \rho, \omega \; ; \; \gamma_{\rho} = \frac{\gamma_{\omega}}{3} \simeq 2.57). \qquad (4b)$$

The leading-QCD contribution to Eq(3) is due to the ones in Fig 1a. We parametrize the phenomenological part of Eq(3) by the narrow resonance poles. Therefore, we apply the Laplace ↔ Borel operator[16] in order to improve the duality between the two sides of Fig. 1. In this way, we depress the effects of unknown high dimension condensates and

the ones of higher radial excitations. The effects of the latter have been estimated to give an increase of 60 %[5] *) in the LHS of the following leading-order sum rule:

$$g_{\omega\rho\pi} \, m_{\pi}^2 \, f_{\pi} \left(\frac{M_{\rho}^2}{2\gamma_{\rho}}\right) \left(\frac{M_{\omega}^2}{2\gamma_{\omega}}\right) \simeq \frac{1}{2} (m_u + m_d) \langle \bar{u}u \rangle \, \tau^{-1} e^{-M_{\rho}^2 \tau} , \qquad (5)$$

which stabilizes for τ between 1 to 2 GeV^{-2} ($\tau \equiv 1/M^2$ is the Laplace ↔ Borel sum rule variable). At such values of τ the effects of the mixed condensate $mg \langle \bar{u} \sigma^{\mu\nu} \lambda_a u F^a_{\mu\nu} \rangle$ evaluated in Ref 5) is negligible and where we have used the phenomenological parametrization $g \langle \bar{u} \sigma^{\mu\nu} \lambda_a F^a_{\mu\nu} \rangle \equiv M_0^2 \langle \bar{u}u \rangle$ with $M_0^2 \simeq (0.2 - 0.5) \text{ GeV}^2$ proposed in Ref 18). Taking into account all above possible effects, we obtain the numerical value:

$$g_{\omega\rho\pi} \simeq (2\gamma_{\rho})(2\gamma_{\omega}) \; (1.6 \, f\pi/\text{GeV}^2) \simeq 11.8 \text{ GeV}^{-1} , \qquad (6a)$$

which can be translated to $g_{\pi\gamma\gamma}$ through vector meson dominance:

$$g_{\pi\gamma\gamma} \simeq 2 e^2 g_{\omega\rho\pi} \left(\frac{1}{2\gamma_{\rho}}\right) \left(\frac{1}{2\gamma_{\omega}}\right) \simeq 2.5 \; 10^{-2} \text{ GeV}^{-1} \qquad (6b)$$

and which corresponds to the two-photon width:

$$\Gamma(\pi^0 \to \gamma\gamma) \simeq | g_{\pi\gamma\gamma} |^2 \frac{m_{\pi}^3}{64\pi} \simeq 8.5 \text{ eV} , \qquad (7)$$

in good agreement with the data (8 ± 0.6) eV. Now, let us study the $\delta \to \eta\pi$ width. We shall use the result in Ref 5) for the $\delta K\bar{K}$-coupling from the same method and by combining the three-point function constraints with the two-point function ones which help, in fact, to eliminate the radial excitation effects though not explicitly said in Ref 5). One obtained:

$$g_{\delta K\bar{K}} \simeq 2 \text{ GeV} , \qquad (8)$$

*) One could also incorporate in a systematic way the effects of all higher radial excitations by using a Veneziano-inspired dual model[17] where the parameters have been fixed from low energy data and current algebra constraints. We found that such effects give an increase less than 90 % in the LHS Eq(5) at $Q^2 \simeq 1 \text{ GeV}^2$.

which with the help of the $SU(3)_F$ relation :

$$g_{\delta\eta\pi} = \sqrt{\frac{2}{3}}\, g_{\delta K\bar{K}} \quad , \tag{9}$$

gives :

$$\Gamma(\delta \to \eta\pi) \simeq \frac{1}{16\pi} |g_{\delta\eta\pi}|^2 \frac{1}{M_\delta} \left(1 - \frac{M_\eta^2}{M_\delta^2}\right) \approx 37 \text{ MeV} \quad , \tag{10}$$

which compares favourably well with the data of 55 MeV. Our tests in Eqs (7) and (10) indicate that our approach can be credible within at most a factor two like we have already emphasized earlier[5,15]. However, some other authors[4a, 14] expect that the agreement between the data and the sum rule predictions are within 10 - 20 %, which in fact are the accuracy of our results in Eqs(7) and (10). Armed by the above remarks, let us study the $\delta \to \gamma\gamma$ width.

2. $\delta(\bar{u}u - \bar{d}d) \to \gamma\gamma$ width

Therefore, we analyze the $\delta\gamma\gamma$-coupling like we have done for the $\pi^0\gamma\gamma$ one. We start with the δ-current normalized as :

$$J_\delta = \frac{i}{\sqrt{2}} (m_d - m_u) : (\bar{d}d - \bar{u}u) : , \tag{11a}$$

in order to have a renormalization group invariant operator.
We define the δ-decay amplitude as :

$$< 0 \mid J_\delta \mid \delta > = \sqrt{2} f_\delta M_\delta^2 \quad , \tag{11b}$$

with $f_\delta \simeq 1.3$ MeV[2,3]. The QCD contribution to the amplitude is due to the one in Fig. 1a. We obtain :

$$T^{\mu\nu} = (p^\nu q^\mu - g^{\mu\nu}(p.q))\,(m_d - m_u) \frac{< \bar{u}u >}{\sqrt{2}\,(Q^2)^2} \left\{1 + \mathcal{O}\left(\frac{m_u^2}{Q^2}\right)\right\} \quad , \tag{12}$$

where the mixed condensate effect is zero, to leading order. We parametrize Fig. 1b by the δ, ω, ρ poles and we use a duality between the two sides of Fig. 1. Such a constraint is improved using the Laplace ↔ Borel operator. Therefore, we obtain the sum rule :

$$g_{\omega\rho\delta} \simeq (m_d - m_u) <\bar{u}u> \tau^{-1} e^{M_V^2 \tau} \left(\frac{2\gamma_\rho}{M_\rho^2}\right)\left(\frac{2\gamma_\omega}{M_\omega^2}\right) \frac{1}{f_\delta M_\delta^2} \quad , \tag{13}$$

where we have used $M_V \equiv M_\omega \approx M_\rho \approx M_\delta$. The analysis of the τ behaviour of Eq(13) shows that there is a stability for $\tau \simeq 1 - 2$ GeV^{-2}. We use the value :

$$(\bar{m}_u + \bar{m}_d)\,(1 \text{ GeV}^2) \simeq 15 \text{ MeV}^{19,3)} \quad ; \quad m_d = 1.8\, m_u{}^{19)} \quad , \tag{14a}$$

which gives with the help of pion PCAC :

$$< \bar{u}u > (1 \text{ GeV}^2) \simeq -0.011 \text{ GeV}^3 \quad . \tag{14b}$$

We can consider the effects of higher radial excitations to the LHS of the sum rule to be of the order of (60 - 90) %. The first number is based on the good realization of the asymptotic $SU(2)_L \times SU(2)_R$ symmetry which implies the same strength of continuum in the analysis of the $\omega\rho\pi$ and $\omega\rho\delta$ couplings. The second number comes from the dual model in Ref 17). Therefore, we obtain :

$$g_{\omega\rho\delta} \simeq (8 - 9.4) \text{ GeV}^{-1} \quad . \tag{15a}$$

Using vector meson dominance, we deduce :

$$\Gamma_{\delta \to \gamma\gamma} \simeq \frac{1}{64\pi} |g_{\delta\gamma\gamma}|^2 M_\delta^3 \simeq (1.5 - 2.2) \text{ keV} \quad . \tag{15b}$$

We can also work for the obtention of $g_{\omega\rho\delta}$ with the ratio of the $\delta\omega\rho$ and $\pi\omega\rho$ vertices. In this way, one can expect a much less effect of radial excitations in the analysis[20]. We obtain :

$$\left(\frac{g_{\omega\rho\delta}}{g_{\omega\rho\pi}}\right) \simeq \left(\frac{Q^2 + M_\delta^2}{Q^2 + m_\pi^2}\right)\left(\frac{m_\pi}{M_\delta}\right)^2 . \left(\frac{f_\pi}{f_\delta}\right)\left(\frac{m_d - m_u}{m_d + m_u}\right) \quad . \tag{16a}$$

The validity of Eq(16) should be in the range of Q^2-values where both the lowest pole dominance and the leading QCD expression make sense. We expect that such values of Q^2 is of the order of $M_V^2 - 1$ GeV^2. We obtain :

$$g_{\omega\rho\delta} \simeq (0.7 - 0.9)\, g_{\omega\rho\pi} \quad . \tag{16b}$$

Using the phenomenological value of $g_{\omega\rho\pi} \simeq 1.05/f_\pi$, we deduce the two-photon width :

$$\Gamma_{\delta\to\gamma\gamma} \simeq (1.6 - 2.6)\ \mathrm{keV} \quad . \tag{17}$$

The results in Eqs(15b) and (17) are surprisingly much higher (by a factor ranging from 4.5 to 52) than the data quoted in Eq(2). Such a disagreement appears to be a common feature of the $\bar{q}q$ model predictions (see some previous works in Ref 21) and the recent paper of Ref 11) which are mostly based on the vector meson dominance of the photon and scalar propagators. Our results might indicate that the δ-meson cannot be treated like other conventional mesons. That might be due to the fact that the narrowness of the δ can be accidental because it stays just above the $\bar{K}K$-threshold. In the same way, if the S^* is a $(\bar{u}u + \bar{d}d)$ state, we obtain a two-photon width which is outside the upper limit given by the Crystall Ball and Jade experiments[22] :

$$\Gamma(S^* \to \gamma\gamma) < 0.8\ \mathrm{keV} \quad . \tag{18}$$

Now, let us discuss the implications of the four-quark scheme of Ref 9). As before, we shall test the accuracy of the method from the analysis of the $\delta\to\eta\pi$ width and study later on the $\delta\to\gamma\gamma$ process.

3. $\delta\to\eta\pi$, $K\bar{K}$ in the four-quark scheme

We can form, in principle, many operators which have the quantum number in Eq(1) in the Dirac and colour matrices basis. For definiteness, we describe the δ-meson by the following colour singlet interpolating fields :

$$\mathcal{O}_\delta = \mathcal{O}_1^\pm + t\,\mathcal{O}_2^\pm \quad , \tag{19a}$$

with

$$\mathcal{O}_1^\pm = \frac{1}{\sqrt{2}} \sum_{\Gamma=\mathbb{1},\pm\gamma_5} \bar{s}\Gamma s\,(\bar{u}\Gamma u - \bar{d}\Gamma d) \quad , \tag{19b}$$

$$\mathcal{O}_2^\pm = \frac{1}{\sqrt{2}} \sum_{\Gamma=\mathbb{1},\pm\gamma_5} (\bar{s}\,\Gamma\,\lambda_a s)(\bar{u}\Gamma\lambda^a u - \bar{d}\Gamma\lambda^a d) \quad , \tag{19c}$$

and t is some coupling coefficient which mixes $\mathcal{O}_1^\pm$ and $\mathcal{O}_2^\pm$. λ_a is the colour matrices and the combinations in Eq(19) have been shown to be free of the non-local $\frac{1}{\epsilon}\log\frac{Q^2}{\nu^2}$ divergence[23) *)] ($4-\epsilon$ is the space-time dimension). We shall treat the quark in Eq(19) as "current" quark so in the chiral limit we have

*) The relative sign between the $\mathbb{1}$ and γ_5 contributions cannot be fixed from the two-point function analysis of Ref 23).

$m_u \simeq m_d \simeq m_s \simeq 0$. However, we are aware of the fact that the four-quark scheme might only be consistent with a "quark constituent" treatment. If so, such a feature will be signaled by the dominance of the quark condensate in our approach as it is known[24)] that this quantity provides the dynamical part of the quark constituent mass.

We describe the η by the octet current :

$$J_8 = \sum_{u,d} :2\,m_u\,\bar{u}(i\,\gamma_5)\,u\; - 4\,m_s\,\bar{s}\,(i\,\gamma_5)s: \quad , \tag{20a}$$

with :

$$\langle 0\,|\,J_8\,|\,\eta\rangle = \sqrt{2}\, f_\eta M_\eta^2 \qquad (f_\eta \simeq \sqrt{6}\ f_\pi) \quad , \tag{20b}$$

and the $\eta - \eta'$ mixing angle $\theta \simeq 17°$[25)]. Therefore, the leading contribution to the $\delta\eta\pi$ coupling is the diagram in Fig 2 a,b due to the $\mathcal{O}_1^\pm$ operator. It is easy to show that the "good" operator which is free of $\frac{1}{\epsilon}\log\frac{Q^2}{\nu^2}$ in the chiral limit is $\mathcal{O}_1^-$ but not $\mathcal{O}_1^+$, while $\mathcal{O}_2^\pm$ cannot contribute in our leading-order approximation. The contribution of Fig 2a,b is

$$\Gamma_{\eta\pi} \simeq (m_u+m_d)\, m_s \langle\bar{s}s\rangle \left(\frac{3}{2\pi^2}\right) \left\{ m_s \log\frac{Q^2}{\nu^2} + \frac{8\pi^2}{3Q^2}\langle\bar{u}u\rangle \right\} . \tag{21}$$

Taking the Laplace transform of $\Gamma_{\eta\pi}$, we obtain the sum rule :

$$g_{\delta\eta\pi} \simeq \cos\theta\, \frac{1}{f_E M_E^2} \left(1 - \frac{M_\eta^2}{M_E^2}\right) \frac{\sqrt{3}}{8\pi^2} \left(\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}\right) f(\tau) \quad , \tag{22a}$$

where f_E is the δ-decay amplitude (E will denote in the following the δ-meson in the four-quark scheme). The function $f(\tau)$ is :

$$f(\tau) \equiv \tau^{-1} \cdot \frac{\bar{m}_s\left(\bar{m}_s - \frac{8\pi^2}{3}\langle\bar{u}u\rangle\,\tau\right)}{\left[\left(1 - e^{-M_\eta^2\tau}\right) - \frac{M_\eta^2}{M_E^2}\left(1 - e^{-M_E^2\tau}\right)\right]} \quad , \tag{22b}$$

which we show in Fig. 2e. The stability of $f(\tau)$ is obtained for $\tau \simeq 0.7 - 1.1\ \mathrm{GeV}^{-2}$. Considering the 60 - 90 % effects of the higher radial excitation states which we had in the previous analysis of the $\bar{q}q$ assignement, we deduce the value of the coupling constant :

$$g_{\delta\eta\pi} \simeq (2.1 - 2.5)\ \text{GeV} \cdot \frac{1}{(1+\frac{32}{9}t^2)^{1/2}} \qquad (23)$$

where we have rescaled the decay amplitude $f_E \simeq 2.7$ MeV by taking into account the contribution of $\mathcal{O}_2^{\pm}$ in the two-point function analysis of Ref 23). Eq(23) suggests that :

$$\Gamma(\delta \to \eta\pi) \simeq (62 - 88)\ \text{MeV} \cdot \frac{1}{(1+\frac{32}{9}t^2)}, \qquad (24a)$$

which for the data of the order of 55 MeV corresponds to the value of t :

$$|t| \simeq 0.2 - 0.4 \quad . \qquad (24b)$$

For the $\delta \to K\bar{K}$ process both $\mathcal{O}_1^-$ and $\mathcal{O}_2^{\pm}$ can contribute. The evaluation of Fig 2c,d shows that $\mathcal{O}_2^-$ is the operator which is free of $\frac{1}{\varepsilon}\log\frac{Q^2}{\nu^2}$ singularity. The deviation from the $SU(3)_f$-relation in Eq(9) is of the order of $(1+\frac{16\,t}{3})$ which for the values of t in Eq(24) gives :

$$g_{\delta K^- K^0} \simeq (5 - 8)\ \text{GeV} \quad , \qquad (25)$$

which is still acceptable phenomenologically. Actually, an eventual significant deviation of the data from the $SU(3)_f$-relation should be due to the presence of the $\mathcal{O}_2$-operator. So it is interesting to also have an accurate measurement of this coupling. Now, let us turn our attention to the $\delta \to \gamma\gamma$ width.

4. $\delta \to \gamma\gamma$ in the four-quark scheme.

If the $\mathcal{O}_2^-$ operator is not inside the δ-wave function, as we might interpret from Eq(1) and Refs 9), then the $\delta \to \gamma\gamma$ width should come from the diagrams in Fig 3a-c. Due to the Lorentz structure of the first super-Zweig allowed diagram in Fig 3a, one can show that it does not contribute to the $\delta \to \gamma\gamma$ amplitude, once a summation over the polarization of the outgoing real photons is done. The non-vanishing effects of $\mathcal{O}_1^-$ should come from the ones in Fig 3b-c where even number of gluon lines should be exchanged between the two-quark blobs. The QCD evaluation of such contributions is technically complicated because we have to deal with two and three-loop calculations.

In the chiral limit ($m_s \simeq 0$), we expect to have the order of magnitude estimate of the vertex :

$$A^1_{\gamma\gamma} \approx \left(\frac{\alpha_s}{\pi}\right)^2 \langle\bar{s}s\rangle\ \langle\bar{u}u\rangle\ \frac{1}{(Q^2)^2}\log\frac{Q^2}{\nu^2} \quad , \qquad (26)$$

where we have taken into account the $16\pi^2$ factor coming from the loop momentum integration and the factor due to the traces over the Dirac and colour matrices. Then, we can deduce from Eq(26), the Laplace sum rule of $(Q^2)^2\ A^1_{\gamma\gamma}$:

$$g_{E\phi\rho} \approx \left(\frac{2\gamma_\rho}{M_\rho^2}\right)\left(\frac{2\gamma_\varphi}{M_\varphi^2}\right)\frac{1}{\sqrt{2}}\ f_E\ M_E^4\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 \langle\bar{u}u\rangle\langle\bar{s}s\rangle\tau^{-1}\ e^{M_V^2\tau}\left(1-M_V^2\tau+\frac{M_V^4}{2}\tau^2\right), \quad (27)$$

where we have taken $M_E \approx M_\rho \approx M_\phi \equiv M_V$. We find that it is much more informative to take the ratio of Eq(13) and Eq(27) and to use vector meson dominance in order to see the relative strength of the $\delta\gamma\gamma$ coupling in the two and four-quark schemes. We deduce

$$\frac{g_{E\gamma\gamma}}{g_{\delta\gamma\gamma}} \approx \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\langle\bar{s}s\rangle}{(m_d-m_u)\ M_E^2}\left(\frac{f_\delta}{\sqrt{2}\ f_E}\right)\left(1 - M_V^2\tau + \frac{M_V^4}{2}\tau^2\right), \qquad (28a)$$

which stabilizes for $\tau \approx 2/M_V^2$ at which corresponds the optimal value :

$$g_{E\gamma\gamma} \approx 0.4\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 g_{\delta\gamma\gamma} \ . \qquad (28b)$$

The $SU(3)_f$- breaking effects due to the non-zero value of the strange quark mass can increase Eq(28) by at most a factor two which is a typical size of such effects. However, the result in Eq(28b) suggests that the δ-photon width due to $\mathcal{O}_1^-$ is 10^{-4} times smaller than the data. So, if the δ is indeed a four-quark state its wave function should contain other operators than $\mathcal{O}_1^-$. So, let us study now the effect of the $\mathcal{O}_2^-$ operator which has the advantage to contribute to the $\delta \to \gamma\gamma$ amplitude by a (α_s/π) stronger factor than $\mathcal{O}_1^-$ because one needs only one gluon exchange here (Fig 3d). In the chiral limit the leading effect is due to the double $\langle\bar{s}s\rangle\ \langle\bar{u}u\rangle$ condensate. We get the gauge invariant result of the amplitude :

$$\Gamma^{\mu\nu}_{\gamma\gamma} = (g^{\mu\nu}(p.q) - q^\mu p^\nu) \cdot \frac{8}{27\sqrt{2}}\left(\frac{\alpha_s}{\pi}\right)\langle\bar{s}s\rangle\langle\bar{u}u\rangle \cdot \frac{t}{(Q^2)^2}\log\frac{Q^2}{\nu^2} \qquad (29)$$

where we have normalized the Φ-meson current as :

$$J^{\mu}_{\Phi} = :\frac{1}{3}\ \bar{s}\gamma^{\mu} s\ : \quad , \langle 0 | J^{\mu}_{\Phi} | \Phi \rangle = \epsilon^{\mu} \frac{M^2_{\Phi}}{2\gamma_{\Phi}} \quad . \qquad (30)$$

We take the Laplace transform of $(Q^2)^2\ \Gamma^{\mu\nu}_{\gamma\gamma_1}$. This will allow us to eliminate the ν-dependence and the effects of unknown $\frac{\gamma\gamma_1}{(Q^2)^2}$ terms in the sum rule. We obtain, by taking $M_{\delta} \approx M_{\rho} \approx M_{\Phi} = \frac{1}{3}(M_{\delta} + M_{\rho} + M_{\Phi}) \equiv M_V$ and by normalizing with the result in Eq(13), the ratio :

$$\frac{g_{E\gamma\gamma}}{g_{\delta\gamma\gamma}} \approx \frac{4t}{27} \left(\frac{\alpha_s}{\pi}\right) \cdot \frac{\langle\bar{s}s\rangle}{(m_d - m_u)\ M^2_E}\ \frac{f_{\delta}}{f_E}\ \left(1 - 2M^2_V\tau + \frac{M^4_V}{2}\tau^2\right) \quad , \qquad (31)$$

which stabilizes for $\tau \approx 2/M^2_V$. We expect that the $SU(3)_F$-breaking terms affect Eq(31) by at most a factor two which we have seen explicitly for the simplest cases of the $\delta\eta\pi$ and $\delta K\bar{K}$ couplings. Therefore we obtain the estimate for $t = 0.2 - 0.4$:

$$\Gamma(E \to \gamma\gamma) = (2 - 5)\ 10^{-4}\ \text{keV} \ , \qquad (32)$$

where we have taken into account the effects of $\mathcal{O}^-_2$ in the two-point function analysis of Ref 2) by rescaling f_E. We need a factor at least hundred in order to restore the agreement between Eqs(2) and (32). Here, we comment briefly on the result of Ref 10) who also noticed that the leading-contribution to the two-photon width of the scalar meson in the four-quark scheme is due to the three-point function where one gluon is exchanged between the quark blobs. Then, they obtain the very crude estimate :

$$\Gamma(E \to \gamma\gamma)_{(\bar{q}q)^2} \approx \alpha^2_s\ \Gamma(\delta \to \gamma\gamma)_{\bar{q}q} \qquad (33)$$

which in, our opinion, might not take properly the π^2 factor which emerges from the momentum integration and which is actually the factor which renders our result in Eq(32) too small. Therefore, an improvment of the crude estimate in Eq(33) will be necessary for clarifying the conclusion of Ref 10).

Concluding Remarks

We have studied the hadronic and two-photon widths of the $\delta(980)$ using three-point function sum rules. We have shown that the $\delta \to \eta\pi$ width are well predicted in the $\bar{q}q$ and $(\bar{q}q)^2$ quark model-assignements of the δ-meson. [Eqs (10) and (24)].

The $\delta K\bar{K}$-coupling can deviate from the $SU(3)_F$-relation in Eq(9) by a factor two to three if there exists in the δ-wave function, operators of the type $(\bar{s}\,\Gamma\,\lambda_a s)(\bar{u}\,\Gamma\,\lambda^a u - \bar{d}\,\Gamma\,\lambda^a d)$. We are aware of the fact that the evaluation of the $\delta \to \gamma\gamma$ width using similar methods does not lead to a satisfactory agreement with the recent Crystal Ball data[12]. The $\bar{q}q$ assignement of the δ leads to a width higher (4 - 50) than the data [Eqs (15,17)] while a $(\bar{q}q)^2$ assignement implies a width at least hundred times smaller [Eq(31)]. Persumably, the marginal position of the δ just above the $K\bar{K}$-threshold might be the origin of the failure of the theoretical approach, but then, it is unclear why the approach is able to predict the correct hadronic width of the δ but not its two-photon one. One might, therefore, speculate that the underlying assumption of vector meson-dominance of the two-photon propagators might not be a good approximation for the case of the δ-meson. Vector-meson dominance of the photon propagator favours respectively the $\delta \to \rho\gamma$ process in the quark model and the $\Phi \to \delta\gamma$ one in the four-quark model. A study of these reactions should be of great importance for testing the substructure of the δ. However, if we insist to fit the data of $\gamma\gamma$ using the conventional approach used here based on vector meson dominance, we should assume a mixing scheme between the $\bar{q}q$ and $(\bar{q}q)^2$ assignements of the δ-wave function[26] :

$$|\delta\rangle = \cos\theta_M (m_d - m_u)\bar{u}d + \sin\theta_M\ \bar{s}s\ (\bar{u}d) \qquad (34)$$

where the $\delta \to \gamma\gamma$ width constrains θ_M to be greater or equal than 62°. In this case, one should notice that the $\delta \to \eta\pi$ prediction obtained earlier is almost unchanged and so it still agrees with the data. However, Eq(34) should affect notably the value of the quark mass-difference obtained in Refs 2,3) which is, however, unlikely. From the above analysis, we might conclude that either the δ-meson is much more exotic than naively expected or (and) the approach used here based on vector meson dominance of the photon and scalar propagators are inadequate for the δ-meson.

We might conclude from our analysis that :

1) The $\bar{q}q$ assignement of the δ can be doubtful if the vector meson dominance of the photon and scalar propagators are proved to be a good approximation or vice-versa. An experimental measurement of the $\delta \to \rho\gamma$ process should test simultaneously the above two assumptions[27]. It would be also desirable to analyze the two-photon width of the δ using an effective QCD lagrangian which is consistent with the quark model and with the realization of QCD chiral symmetry[28] which does not necessary need either the asumption of vector meson dominance or the picture of the K-$\bar{K}$ molecule for the δ[11]. However, we think that, at present, such a program cannot yet be done carefully because we do not have yet a complete lagrangean which incorporates properly the coupling of a scalar meson

with the non-linear pseudoscalar Goldstone mesons put in the electromagnetic field.

2) The $(\bar{q}q)^2$ assignement of the δ is not enough for explaining the smallness of the two-photon width of the δ contrary to previous result of Ref 10). If indeed, the δ is a four-quark narrow resonance, then we might expect a production of the δ through the Φ radiative decay. Also, if a large deviation of the $\delta K\bar{K}$-coupling from the $SU(3)_F$-relation is observed, we might expect the presence of the operator

$$\sum_{\Gamma=1,-\gamma_5} \bar{s}\,\Gamma\,\lambda_a s\,(\bar{u}\,\lambda^a\Gamma u - \bar{d}\,\lambda^a\Gamma d)$$

inside the δ-wave function. So we think that a careful measurement of the above parameters should help in answering the nature of the the δ.

Perhaps, the δ-meson is much more exotic than usually expected !

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FIGURE CAPTIONS

FIG 1 : a) $<\bar{u}u>$ and $<\bar{u}\,\sigma^{\mu\nu}\,\frac{\lambda_a}{2}\,F^a_{\mu\nu}\,u>$ quark and mixed condensates effects to the $\omega\rho\pi$ and $\omega\rho\delta$ coupling constants

b) Meson pole contributions to the above coupling constants .

FIG 2 a-d : Contributions of the four-quark operator to the $\delta \to \eta\pi$ (a,b) and $\delta \to K\bar{K}$ (c,d) decay amplitudes.

FIG 2 e : Behaviour of the function $f(\tau)$ which gouverns the $\delta \to \eta\pi$ coupling versus τ .

FIG 3 : Contributions to the $\delta \to \gamma\gamma$ decay amplitude due to the four-quark operator in Eq (19) :

a) Full-disconnected quark blobs due to $\mathcal{O}_1^\pm$

b-c) Quark blobs connected by two gluon lines due to $\mathcal{O}_1^\pm$

d) Quark blobs connected by one gluon line due to $\mathcal{O}_2^\pm$.

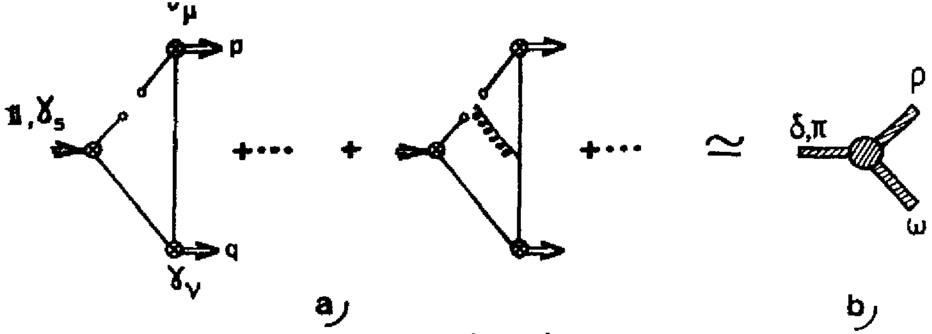

fig. 1

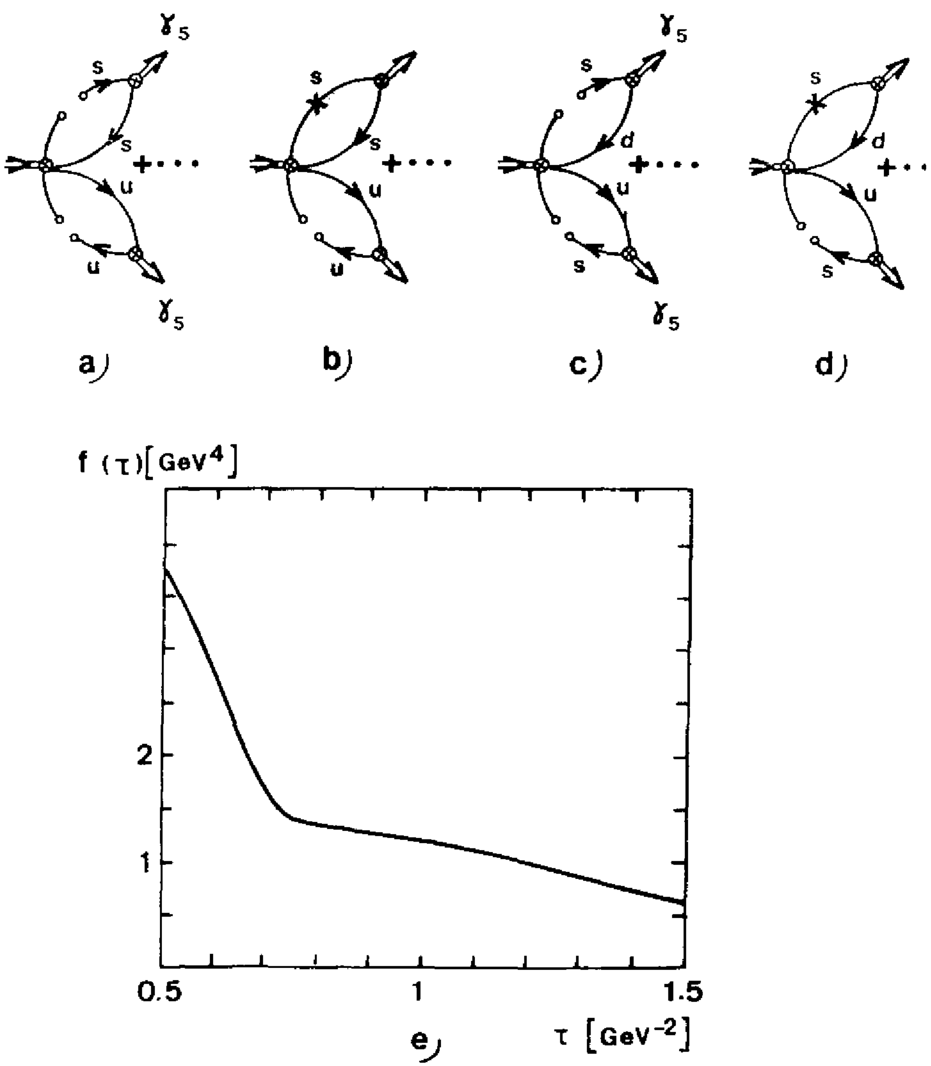

fig.2

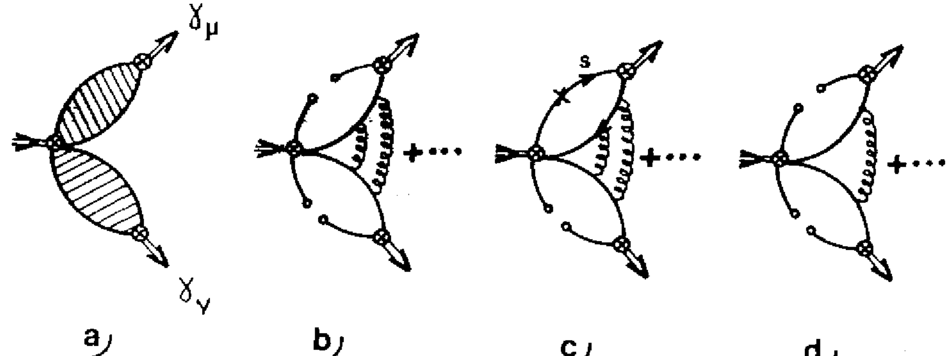

fig.3