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COMPACTIFICATIONS AND θ-STRUCTURES IN STRING THEORIES *

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ABSTRACT

Possible topological invariant terms in the first quantization of strings associated with nonzero elements of the second cohomology group of space-time are investigated. The direct result of such terms is C-violation.

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Recent research on string theories(1,2) **is due to the fact that SO(32), or $E_8 \times E_8$ version of superstrings provide a unified theory of all four forces. Especially, a realistic compactification** $M_{10} \longrightarrow M_4 \times K$ in which K is a 6 dimensional Calabi-Yau manifold with SU(3) holonomy was presented(2) and attracted much attention.

However, it is still worth doing something about constraints on compactifications. On one hand, there exist many choices of Calabi-Yau manifolds(2,4,5,6); on the other hand, as pointed out in ref.(2), it is possible to choose Ricci flat O(6) holonomy instead of SU(3) holonomy manifolds as the internal space. Unfortunately, a fully understood interacting covariant formulation is not available at present, so we have to start with the first quantization formalism. On this level, we previously considered θ-structure of strings due to the multiplicity of connection of space-time(3). Such θ-terms introduced, in the special case $M = M_4 \times T^d$, the first level excitations are all massive and all supersymmetries are broken. This needs us to require $\pi_1(M) = 0$. For the above mentioned case, we can also substitute 1-forms as θ-terms. This fact amounts to that if the internal space is connected and its fundamental group is commutative, then $\pi_1(M) = H^1(M,Z)$.

As for additional 2-form terms, many authors have considered them both on classical and quantum levels(7). But none of them investigated their global properties, although these authors stressed that it is a generalization of Wess-Zumino term. Besides the normal Nambu-Goto action

$$I_{N-G} = \frac{1}{2\alpha'}\int d^2\sigma \sqrt{g}\, g^{\alpha\beta} G_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu \tag{1}$$

one can introduce a additional term of strings coupling to an asymmetric tensor

$$I_1 = \int d^2\sigma\, \epsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu \tag{2}$$

where $B_{\mu\nu} = -B_{\nu\mu}$. Taking eqs.(1)and (2) together as another action we can of course generalize it to a supersymmetric one[7]. The authors of ref.(7) then considered its properties of perturbative quantization.

Now let us seek what global aspects **are lost in the previous perturbative** investigations. If we want to **keep the classical motion equations** unchanged (unchanged also in the sense of perturbative quantization, see below), the 2-form $b = B_{\mu\nu}\, dx^\mu \wedge dx^\nu$ must be closed for closed strings. To see this, let σ denote the closed space of parameters (σ^1, σ^2), it may be a sphere S^2 or a sphere with h handles T_h. Then the string configuration $x(\sigma)$, i.e., the world sheet, is a map from σ to space-time. If we make an infinitesimal variation of configuration $x(\sigma) + \delta x(\sigma)$. $(x, -(x+\delta x))$ in M can be always the boundary of a 3-d submanifold of M, say $x(\sigma)xI$. Now

$$\delta\int b = \int_{x} b - \int_{x+\delta x} b = \int_{x\times I} db \tag{3}$$

so $\delta\int b = 0$ if and only if $db = 0$. b must not be exact, if it is, $b = da$, then $\int_x b = \int_{\partial x} a = 0$.

The above argument shows that $b \in H^2(M,R)$, therefore we cannot find such a term if $H^2(M,R) = 0$, i.e., the second Betti number is zero. Thus, we assume $H^2(M,R) \neq 0$ and choose a basis $b_i \in H^2(M,R)$, $i = 1 \longrightarrow b_2$. According to de Rham theorem, there are fundamental circles $c_i \in H_2(M,R)$, $i = 1 \longrightarrow b_2$, such that we can normalize b_i and $\int_{c_i} b_i = \delta_{ij}$.

For a general map $x: \sigma \longrightarrow M$, $x(\sigma)$ is always closed in the sense of chain formalism, so $x(\sigma) \in H_2(M,Z)$. Note that here we deal with the second homology group with coefficients in Z, usually $H_2(M,R) = H_2(M,Z)xR$ and $H_2(M,Z)$ may have torsion. Let c_i and c_a be generators of $H_2(M,Z)$, where c_a are generators of the torsion subgroup of $H_2(M,Z)$. For $x(\sigma) \in H_2(M,Z)$, there are integers n_i and n_a ($n_i: -\infty \rightarrow +\infty$; $n_a: 0 \longrightarrow N_a$) so that

$$x(\sigma) = \sum_i n_i c_i + \sum_a n_a c_a + \partial N \tag{4}$$

where N is a 3d chain.

Thus we introduce an action $\sum_i i\,\theta_i \int b_i$

$$I_1 = i\int_{x(\sigma)} \sum_i \theta_i b_i = i\sum_i n_i\theta_i \int_{c_i} b_i + i\sum_{a,i} n_a\theta_i \int_{c_a} b_i + \int_N d\, i\sum_i \theta_i b_i = i\sum_i (n_i + \sum_a n_a f_{ia})\theta_i \tag{5}$$

where $f_{ia} = \int_{c_a} b_i$. We shall introduce a factor i in the action, otherwise when some $|n_i| \rightarrow \infty$, $\exp(-I)$ becomes divergent. We see that the additional term is topological invariant, so it is a θ-term.

For simplicity, we assume the torsion of $H_2(M,Z)$ is zero, therefore

$$I_1 = I(\theta) = i\sum_i n_i\theta_i \tag{6}$$

Note that, in the above discussion, we have assumed $x(\sigma)$ may n times bind the **imagine set of σ**, hence n_i may have a common divisor n. For all n_i integer in eq.(6), the space of parameters θ_i must be $U(1)^{b_2}$ or T^{b_2}.

To quantize this system, the path integral in partition function Z must sum all (n_i) classes of configuration, then

$$e^{-W} = \sum_{(\sigma)}\sum_{(n_i)} \int DXDg_{\alpha\beta} \exp(-I_{N-G} + i\sum_i n_i\theta_i) \tag{7}$$

where $\sum_{(\sigma)}$ means suming over sphere S^2 and T_h, $\sum_{(n_i)}$ means suming over all possible classes (n_i) ($x(\sigma) = \sum_i n_i c_i + \partial N$). Path integral (7) is very difficult to evaluate for usually M is complicated, it is hard even to classify map $x(\sigma)$.

We further point out that term (6) only depends on the topology of manifold M, while not on geometry of M. So unlike Nambu-Goto term and general asymmetric term, it is more natural.

As for Calabi-Yau compacitification, because C-Y manifold is Kähler, the Kähler form $\omega = ig_{a\bar{b}}\, dz^a \wedge d\bar{z}^b$ is real and closed. It is not exact, for $\int_K \omega^3$ is proportional to the volume of internal space K, if $\omega = da$ for some 1-form a, $\int_K \omega^3 = 0$. Thus ω is a nonzero element of $H^2(K,R)$, here there arises a natural θ-term in Calabi-Yau compactification.

Here we must point out that, if $H^2(M,R) \neq 0$, it turns out $H_2(M,Z) \neq 0$, so there is some fundamental 2-circle belonging to $H_2(M,Z)$ which is not the boundary of a 3-d submanifold of M, such a circle can always be chosen to be $x(\sigma)$ then we cannot have a derivation of Wess-Zumino term(7,8). In this case, the asymmetric term thus is naturally θ-term rather than a generalization of Wess-Zumino term, we will mention this problem again later.

We turn to the discussion for a general 2-form $b = B_{\mu\nu}\, dx^\mu \wedge dx^\nu$ not necessarily being closed. Action

$$I = \frac{1}{2\alpha'} \int d^2\sigma \sqrt{g}\, (g^{\alpha\beta} G_{\mu\nu} + \epsilon^{\alpha\beta} B_{\mu\nu})\, \partial_\alpha X^\mu \partial_\beta X^\nu \tag{8}$$

can be easily extended to be supersymmetric For simplicity, assume σ is flat (it is actually possible for $\sigma = T_1$) so that the following action(7)

$$I_S = \frac{1}{2\alpha'} \int d^2\sigma\, d^2\theta\, [\, G_{\mu\nu}(X) - B_{\mu\nu}(X)]\, \bar{D} X^\mu (1+\gamma_3) D X^\nu \tag{9}$$

is (1,1) type supersymmetric generalization of action (8), where

$$X^\mu = x^\mu + \bar{\theta}\psi^\mu + \frac{1}{2}\bar{\theta}\theta F^\mu \tag{10}$$

Integrate θ and eliminate the auxiliary fields F^μ, eq.(9) reads

$$\begin{aligned} I_S = \frac{1}{2\alpha'} \int d^2\sigma\, [\, & G_{\mu\nu} \partial_\alpha x^\mu \partial_\alpha x^\nu + i G_{\mu\nu} \bar{\psi}^\mu \not{\partial} \psi^\nu + B_{\mu\nu} \epsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu \\ & - \frac{1}{2} \partial_\alpha (i B_{\mu\nu} \bar{\psi}^\mu \gamma_3 \gamma_\alpha \psi^\nu) + \frac{1}{8} R_{\mu\nu\rho\sigma} \bar{\psi}^\mu (1+\gamma_3) \psi^\nu \bar{\psi}^\rho (1+\gamma_3) \psi^\sigma \end{aligned} \tag{11}$$

in above equation, we have included torsion coming from $B_{\mu\nu}$ in curvature $R_{\mu\nu\rho\sigma}$. The torsion is

$$T_{\mu\nu\rho} = 3\, B_{[\mu\nu,\rho]} \tag{12}$$

We now only quote some results from refs.(7). The one loop on-shell ultraviolet counterterms are

$$\begin{aligned} G^{(1)}_{\mu\nu} &= -\frac{\alpha'}{2\pi} \frac{1}{2-d} R_{(\mu\nu)} \\ F^{(1)}_{\mu\nu} &= -\frac{\alpha'}{2\pi} \frac{1}{2-d} R_{[\mu\nu]} \end{aligned} \tag{13}$$

and the renormalization-group equations

$$m \frac{d}{dm} \left(\frac{1}{\alpha'} G_{\mu\nu} \right) = \frac{1}{2\pi} R_{(\mu\nu)} \tag{14}$$

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