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APPLICATION OF THE GAUSSIAN QUADRATURE TO TRANSPORT EQUATIONS

EIGENVALUES FOR ONE-DIMENSIONAL GEOMETRIES

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APPLICATION OF THE GAUSSIAN QUADRATURE TO TRANSPORT EQUATIONS EIGENVALUES FOR ONE-DIMENSIONAL GEOMETRIES

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ABSTRACT

"Probability tables", established to describe neutron cross-sections in a given energy group, can be introduced directly into the multigroup system of equations: this is the principle of the subgroup or multiband method.

We proposed to decompose the flux on a complete eigenflux basis. For this purpose we consider the eigenvalue problem for one-dimensional geometries. Our aim is to establish space probability tables which will describe the eigenflux as accurately as required. So far we succeed in the cases of the infinite slab and the sphere.

These probability tables are the tables required by the GAUSS quadrature and are well suited to any integral calculation. We outline the application of this method to the treatment of the transport equation.

Neutronics present a rather particular feature amongst the various branches of physics: the absence of any fundamental unknown, i.e. the physics are perfectly described by the BOLTZMANN equation of transport (all cross-sections data being assumed known). The only problem is how to deal in practice with this equation. The classical method is to divide the energy range into "groups". The multigroup theory implies the calculation of energy averages of cross-sections weighted by the neutron flux $\phi(E,\vec{r},\vec{\Omega})$. As emphasized by Dermott CULLEN [1] the multigroup equations would be exact if the flux were independent of the position (r) and the direction (Ω). The usual procedure is to define a single value, averaged over space and angles by ad hoc weighting functions.

In heterogenous geometry the values of effective cross-sections so defined, which will be introduced in the system of multigroup equations, are also dependent on adjacent media, a factor taken into account by the concept of "equivalent cross-section". Furthermore the calculation of the effective cross-section of mixtures theoretically requires the calculation of $\sigma(E)$ for the mixture. This is rarely practised, the usual procedure being to calculate the effective cross-sections for a nucleus by including the effective cross-section of other nuclei in the background cross-section.

For all these reasons the concept of "effective cross-section" is somewhat ambiguous.

In the 1970's NIKOLAEV et al. proposed the sub-group method [2]: the neutron cross-sections were represented by sets of discrete values $(p_i, \sigma_{ti}, \sigma_{xi})$ to be directly injected into the neutronic equations: these authors showed that the relations governing the new system of equations were nearly the same as the usual multigroup equations, and therefore proposed the name "subgroup method" for this approach to the neutronics. Later on, since 1974, Dermott CULLEN developed a similar method which he called the "multiband method" [1,3]. Several authors have studied and applied these ideas [4].

As shown by CULLEN, the use of probability tables increases effectively the accuracy of calculations. The accuracy is further improved if probability tables are based on moments [4].

Nevertheless the method was not widely adopted, perhaps because potential users were afraid of the increase in the number of groups (which is multiplied by the number of sub-groups or bands). Some theoretical difficulties also arise: according to ZHANG and SALVATORES the boundary conditions are not the same. These authors proposed modified equations but the results are not very satisfactory [5].

For these reasons we tried to approach this question by a quite different method. Henceforth we shall assume isotropic scattering cross-sections.

1. THE EIGENVALUE PROBLEM - The transport equation can be written:

$$(1\text{-}1)\quad \begin{cases} q(\vec{r},E) = \int_{E'} s(\vec{r},E'\to E)\ \phi(\vec{r},\vec{E'})\ dE' + S(\vec{r},E) \\ \phi(\vec{r},E) = \int_{r'} T(t|\vec{r}-\vec{r}'|)\ q(\vec{r}',E)\ d\vec{r}' \end{cases}$$

where: s is the scattering cross-section,
t is the total cross-section,
T is the transport operator,
S stands for the sources.

For monoenergetic neutrons - or for a neutron flux treated as a monoenergetic group - we get:

$$(1\text{-}2)\quad \phi(\vec{r}) = \int_{r'} s(\vec{r})\ T(t|\vec{r}-\vec{r}')\ \phi(r')d\vec{r}' + \int_{r'} T(t|\vec{r}-\vec{r}'|)\ S(r')\ d\vec{r}'$$

The eigenvalue equation is obtained when the source term is suppressed and the integration domain D is restricted to a geometric cell (constant cross-section values)

$$(1\text{-}3)\quad \lambda\ \phi(\vec{r}) = \int_{D} T(t|\vec{r}-\vec{r}'|)\ \phi(\vec{r}')dr'$$

where λ, the eigenvalue, is equivalent to 1/s.

Our aim is to introduce moments; hereafter we shall only consider the one-dimensional case. On multiplying by x^n and integrating we obtain:

$$(1\text{-}4)\quad \begin{cases} \lambda \int_D x^n\ \phi(x)dx = \int_D x^n\ dx \int_D K(t|x-x'|)\ \phi(x')dx' \\ \lambda\ M_n = \int_D \phi(x')dx' \int_D x^n\ K(t|x-x'|)dx = \int_D \phi(x')\ F_n(x')dx' \end{cases}$$

$K(t|x-x'|)$ is deduced from T by integration over angular directions: its expression depends on the geometry.

$$(1\text{-}5)\quad F_n(x') = \int_D x^n\ K(t|x-x'|)$$ is independent of the flux ϕ and will contain all information about the geometry of the medium for a given total cross-section. If there exists a serial expansion of $F_n(x')$:

$$(1\text{-}6)\quad F_n(x') = \sum_{m=0}^{\infty} a_{nm}\ x'^m$$

we get:

$$(1\text{-}7)\quad \lambda\ M_n = \sum_{m=0}^{\infty} a_{nm} \int_m x'^m\ \phi(x')dx' = \sum_{m=0}^{\infty} a_{nm}\ M_m$$

We are then faced with a classical eigenvalue problem for an infinite matrix A.

Resolution of system 1-7 always gives a mixture of real and complex values λ.

Since T is a positively definite symmetrical kernel, eigenvalues of 1-3 will be real positive values. On the other hand 1-4-a is equivalent to:

$$(1\text{-}8)\quad \begin{cases} \int_D x^n \left(\lambda\,\phi(x) - \int_D K(x,x')\,\phi(x')dx'\right)dx = 0 \\ \text{i.e.:} \qquad \langle x^n,\ \lambda\,\phi - \int_D K\,\phi \rangle = 0 \qquad \forall n \in N \end{cases}$$

which implies, for λ real,: $\lambda\,\phi = \int_D K\phi\,dx$

The real eigenvalues of 1-7 are then those we require.

Our aim is to generate a complete basis of orthogonal eingenflux ϕ_λ on which any flux ϕ can be expanded. For this we shall demonstrate that the transport operators are compact. Furthermore the sets of eigenvalues are enumerable series with values approaching 0, which justifies the truncation of the series of eigenvalues.

In numerical applications the matrix A will have to be truncated: this will have two effects:

- a limited number only of eigenvalues will be obtained (much lower than the matrix order),
- eigenvalues and eigenvectors will contain numerical errors.

Hereafter we shall consider four cases.

2. THE LINEAR CASE - The neutron can move on a line segment, either forwards or backwards: $\cos\Theta = \pm 1$.

This case is not physical but corresponds to the diffusion approximation, and can be solved analytically. It will then provide a reference.

The transport kernel is:

$$(2\text{-}1)\quad K(x,x') = \tfrac{1}{2}\, e^{-t|x-x'|}$$

2.1 - Compactness of the kernel - The space is finite; on this space the transport operator is definitly positive: the operator is then compact.

2.2 - Expansion of F - If we choose the space (-1,+1) we have:

$$(2\text{-}2)\quad \lambda M_n = \int_{-1}^{1} \phi(x')\,dx' \int_{-1}^{1} \frac{x^n}{2}\, e^{-t|x-x'|}\,dx = \int_{-1}^{1} \phi(x')\,F_n(x')\,dx'$$

$$(2\text{-}3)\quad \begin{cases} F_n(x') = \tfrac{1}{2} \int_{-1}^{x'} x^n\, e^{-t(x'-x)}dx + \int_{x'}^{1} x^n\, e^{-t(x'-x)}\,dx \\ \qquad = \tfrac{1}{2} \sum_{m=0}^{n} \frac{n!}{(n-m)!\,t^{m+1}} \left(1 + (-1)^m\right)x'^{n-m} - e^{-t}\left(e^{tx'} + (-1)^m e^{-tx'}\right) \end{cases}$$

From an expansion of exponentials we obtain:

$$(2\text{-}4)\quad \begin{cases} a_{nm} = \tfrac{1}{2}\,\frac{n!}{m!}\,\frac{(1+(-1)^{n-m}}{t^{n-m+1}}\,S_n(t) & \text{if } m < n \\ a_{nm} = \tfrac{1}{2}\,\frac{n!}{m!}\,\frac{(1+(-1)^{n-m}}{t^{n-m+1}}\,R_n(t) & \text{if } m > n \end{cases}$$

with: $(2\text{-}5)\quad R_n(t) = e^{-t} \sum_{j=0}^{n} \frac{t^j}{j!} = 1 - S_n(t)$

Eigenvalues and eigenvectors are calculated with the matrix A so defined.

2.3 - Analytical calculations of eigenvalues and eigenvectors. By twice deriving the basic equation:

$$(2\text{-}6)\quad \lambda\,\phi(x) = \tfrac{1}{2}\int_{-1}^{1} e^{-t|x-x'|}\phi(x')dx'$$

we get:

$$(2\text{-}7)\quad \lambda\,\phi''(x) = \frac{t^2}{2}\int_{-1}^{1} e^{-t|x-x'|}\,\phi(x')dx' - t\phi(x) = (\lambda t^2 - t)\,\phi(x)$$

i.e.:

$$(2\text{-}8)\quad \begin{cases} \phi''(x) + \omega^2\,\phi(x) = 0 \\ \text{with:}\quad \omega^2 = t\left(\dfrac{1}{\lambda} - t\right) \end{cases}$$

We always obtain $1/\lambda > t$. This corresponds to physical facts: eigenvalues with $s < t$ cannot exist ($s > t$ implies the presence of sources). ω^2 is then positive.

Solutions of 2-8 are:

$$(2\text{-}9)\quad \begin{cases} \ell = 2k \qquad \phi_{2k}(x) = \sin \omega_k x & \omega_k = \dfrac{k\pi}{1+d_\ell} \\ \ell = 2k+1 \qquad \phi_{2k+1}(x) = \cos \omega_k x & \omega_k = \dfrac{(k+\frac{1}{2})\pi}{1+d_\ell} \end{cases}$$

with an extrapolation length d_ℓ. By carrying back these results to 2-6 we obtain:

$$(2\text{-}10)\quad \begin{cases} \ell = 2k \qquad \operatorname{tg}\,\omega = -\omega_k/t \\ \ell = 2k+1 \qquad \omega_k \operatorname{tg}\,\omega_k = t \end{cases}$$

It can be shown directly that the ϕ_ℓ are orthogonal.

2.4 - Accuracy of eigenvalues. The analytical values provide references, which allow the value accuracy to be determined from 1-3 with a_{nm} given by 2.4. Figure 1 represents the error on the eigenvalues.

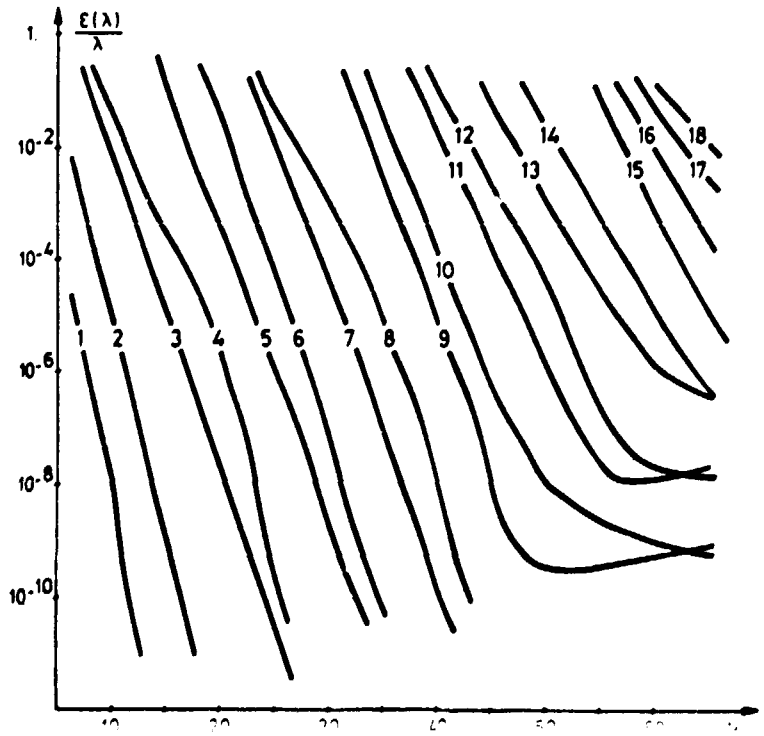


Fig.1 - Error on the real eigenvalues as a function of N, the matrix A order. For the first eight real eigenvalues the accuracy can be better than 10^{-6}. Numerical problems appear for N>50. About 30% of the eigenvalues are real.

It is known that the infinite sums of λ^2 is equal to the integral of T^2. That allows a check of calculations and an estimation of the rate of convergence. Figure 2 represents this comparison.

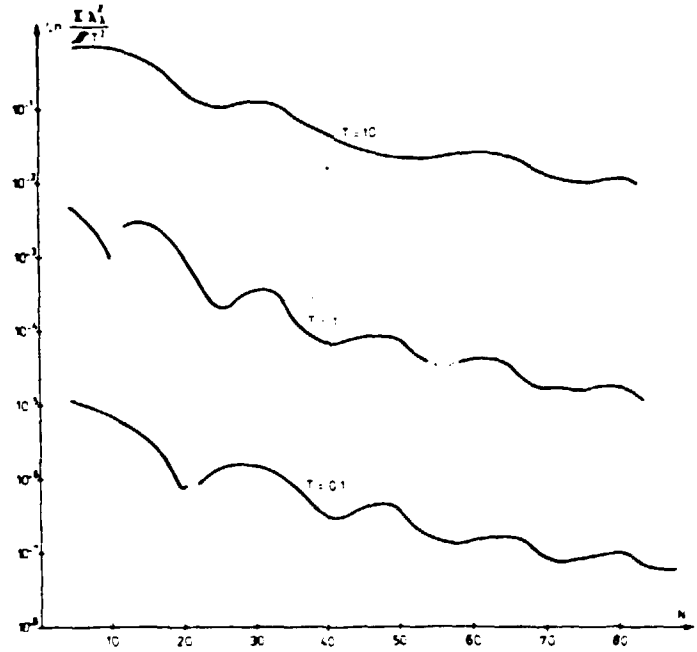


Fig.2 - Ratio of the sum of the square of real eigenvalues to the integral of the square of the operator K as a function of N, the A matrix order. This ratio converges to 1, but rather slowly when t is large.

2.5 - Eigenvectors. The eigenvectors are the flux $\phi_\lambda(x)$, not the moments. If the calculation is performed with another space - for instance in the space (0,+2)- we obtain different values of the moments $M_{\lambda,n}$, but the same $\phi_\lambda(x)$. In this case we know the analytical expressions of $\phi_\lambda(x)$.

From the moments we calculate spatial probability tables $[p_i, x_i]$ as explained in [4]. These tables are the result of the eigenflux calculation ; they constitute the information which must be calculated just once as a function of total cross-section and is to be recorded.

Accuracy of eigenflux - The errors on moments behave similarly to those on eigenvalues as given by fig.1.

2.6 - Complex eigenvalues. As remarked, these complex eigenvalues are introduced by the change from eq. 1.3 to eq. 1.7. and lack any kind of regularity: no apparent relation exists between the complex values calculated for order N and those obtained for orders N+1 or N+2. To look for a clue we considered the value of:

$$S_N = N * \sum_{i=1}^{N} |\lambda_c|^2$$

S_N decreases as N increases, then on average, $|\lambda_c|^2$ decreases faster than 1/N. Similar results are obtained for the other cases.

The main disadvantage of the existence of these complex eigenvalues is to increase the order of the A matrix.

3 - INFINITE SLAB - As in the linear case we shall normalize the space to (-1, +1). The transport is:

(3-1) $K_p(t,|x-x'|) = \frac{1}{2} E_1(t|x-x'|)$

3.1 - Expansion of F - we have:

(3-2) $$F_n(x) = \frac{1}{2}\int_{-1}^{1} x'^n E_1(t|x-x'|)dx' = \frac{1}{2}\int_0^1 \frac{du}{u}\int_{-1}^{1} x'^n e^{-t\frac{|x-x'|}{u}} dx'$$

(3-3) $$I_n = \int_{-1}^{1} x'^n e^{-t\frac{|x-x'|}{u}} dx' = e^{-\frac{tx}{u}}\int_{-1}^{x} x'^n e^{\frac{tx'}{u}} dx' + e^{\frac{tx}{u}}\int_{x}^{1} x'^n e^{-\frac{tx'}{u}} dx'$$

by successive integrations by parts, we obtain:

$$(3\text{-}4)\quad I_n = n! \sum_{i=0}^{n} \left(\frac{u}{t}\right)^{i+1} \frac{(1+(-)^i)x^{n-i} + \left((-)^{n-1} e^{-t\frac{(1+x)}{u}} - e^{-t\frac{1+x}{u}}\right)}{(n-i)!}$$

$F_n(x)$ can be expressed as expansions of $E_n(t|1\mp x|)$ and $E_n(t|\&-x|)$. Finally we get:

$$(3\text{-}5)\quad \begin{cases} m \leq n & a_{nm} = \dfrac{n!}{2m!}(1+(-)^{n-m})\, t^{m-1}\left(\dfrac{1}{(n-m+1)t^n} - \sum_{j=0}^{n} \dfrac{E_{j-m+2}(t)}{(n-j)!t^j}\right) \\ m > n & a_{nm} = -\dfrac{n!}{2p!}(1+(-)^{n-m})\, t^{m-1} \sum_{j=0}^{n} \dfrac{Ej-m+2(t)}{(n-j)!t^j} \end{cases}$$

3.2 - The operator K_p is compact - The integral of K^2 is:

$$I = \int_D \int_D K_1^2(x,x')dx\,dx' \qquad D = [-1, +1]$$

with the variable change: $u = x-x'$ $\qquad v = x+x'$

we get:

$$I = 2\int_0^2 E_1^2(-tu)du$$

$E_1(-tu)$ is infinite for u=0. However:

$$E_1(x) = \sum_{m=1}^{\infty} \frac{(-x)^m}{(-m)m!} - (\gamma+Lnx) \underset{x\to\infty}{\approx} -Lnx$$

Since $\int_0^1 Ln^2(x) = 2$, the integral I is finite.

Hence the square of K is integrable, the kernel is then compact and the eigenflux basis is complete.

3.3 - Results - Table 1 shows results for a few eigenvalues, compared with those of the literature.

Table 1 - Values of eigenvalues obtained for a slab; our values were obtained by extrapolation and an estimation of the error is given.

		ℓ order of eigenvalues				
t	reference	1	2	- - -	7	8
0.1	This work	0.2610764	0.0844687	- - -	0.0158857	0.0136941
	[5]	8	6		6	7
		0.2610757	0.0744677	- - -	0.0158848	0.0136932
1.	This work	0.783025	0.502926	- - -	0.148198	0.129250
	[5]	2	4		6	6
		0.783023	0.502921	- - -	0.148189	0.129241
10.	This work	0.992916	0.972627	- - -	0.773449	0.731568
	[5]	1	3		21	25
		0.992915	0.972622	- - -	0.773413	0.731529

4 - SPHERE - We normalize the radius to 1, and calculate the moments relative to $r = \frac{1}{2}$. The kernel is:

$$(4\text{-}1)\quad K_S(r,r') = \frac{r'}{2r}\left(E_1(t|r-r'|) - E_1(t|r+r'|)\right)$$

This kernel is not symmetrical. Since we need symmetry in order to have a complete basis we change the variable:

(4-2) $\psi(r) = r\,\phi(r)$

by writing: (4-3) $S(r,r') = \frac{r}{r'} K(r,r')$

we get:

(4-5) $\lambda\,\psi(r) = \int_0^1 S(r,r')\,\psi(r')\,dr'$

(4-6) $S(r,r') = \frac{1}{2}\left(E_1(t|r-r'|) - E_1(t|r+r'|)\right)$

4.1 - Expansion of F - we get (4-7):

$$m<n \quad a_{nm} = \frac{n!}{2t\,m!}\,\frac{1+(-)^{n-m}}{(n-m+1)t^{n-m}} + t^m \sum_{i=0}^{n} (-)^m \frac{E_{i-m+1}(\frac{3t}{2}) - (1+(-)^{n-m}+(-)^{n-m-i})E_{i-m+2}(\frac{t}{2})}{2^{n-i}(n-i)!\;t^i}$$

$$m>n \quad a_{nm} = \frac{n!}{2m!}\,t^{m-1} \sum_{i=0}^{n} (-)^m \frac{E_{i-m+1}(\frac{3t}{2}) - (1+(-)^{n-m}+(-)^{n-m-i})E_{i-m+2}(\frac{t}{2})}{2^{n-i}(n-i)!\;t^i}$$

4.2 - Compactness of operator S - We have:

(4-8) $S^2(r,r') = \frac{1}{4}\left(E_1^2(r+r') + E_1^2(r-r') - 2\,E_1(r+r')\,E_1(r-r')\right)$

We show, for the slab, that $\int E_1^2(r+r')$ is finite; the same demonstration holds for $\int E_1^2(r-r')$; The SCHWARZ inequality is used to show that $\int E_1(r+r')E_1(r-r')$ is finite. Then S is compact on $L^2(0,1)$.

4.3 - Relation between slab and sphere - If φ is a solution for the slab:

(4-9) $\lambda\varphi(x) = \int_{-1}^{+1} K_p(x,x')\varphi(x')\,dx'$

If φ is odd, we define ϕ as: $\varphi(x) = x\,\phi(x)$. Let us calculate:

$$I = \int_0^1 K_S(r,r')\,\phi(r')\,dr'$$

$$= \frac{1}{r}\int_0^1 S(r,r')\varphi(r')\,dr' = \frac{1}{2r}\int_{-1}^{1} S(r,r')\,\varphi(r')\,dr'$$

$$= \frac{1}{4r}\left(\int_{-1}^{1} E_1(t|r-r'|)\varphi(r')dr' - \int_{-1}^{1} E_1(t|r-r'|)|\varphi(r')dr'\right)$$

$$= \frac{1}{2r}\int_{-1}^{1} E_1(t|r-r'|)\varphi(r')\,dr' = \frac{1}{r}\int_{-1}^{1} K_p(r,r')\varphi(r')dr'$$

$$= \lambda\frac{\varphi(r)}{r} = \lambda\,\phi(r)$$

Thus any eigenflux of the sphere corresponds to an odd eigenflux of the infinite slab, with the same eigenvalues.

4.4 - Results - Table 2 represents few eigenvalues obtained directly by solving the sphere problem. Clearly we have a problem of accuracy due to a lack of exactness of elements a_{nm}, which can be solved either by writing another formulation of these elements or by using the slab results.

Table 2 - Eigenvalues obtained directly for the sphere

t	reference	ℓ order of eigenvalues 1	2	3	4
0.1	This work	0.74598	0.301		
	[5]	0.74468	0.301646		
1.0	This work	0.50259	0.26188	0.1731	0.126
	[5]	0.50292	0.26189	0.17349	0.12924
10.	This work	0.097266	0.090343	0.081761	0.07323
	[5]	0.097262	0.090333	0.081693	0.07315

4.5 - Remark - We began the study of the case by calculating moments relative to the origin, 0. But, as for the slab, we could not obtain an expansion of F, and the calculation was performed in relation to a value for the abscissa of 0.5.

5 - CASE OF THE CYLINDER - The kernel is:

$$(5\text{-}1)\quad K_C(r,r') = \frac{r'}{\pi}\int_0^{\frac{\pi}{2}} d\Theta \int_0^{\pi} \frac{e^{-\frac{t}{\cos\Theta}}}{\rho}\, d\varphi$$

with: $\rho^2 = r^2 + r'^2 - 2\, r\, r' \cos\varphi$

By changing of the variable φ to ρ and replacing $\frac{1}{\cos\Theta}$ by ch(u):

$$\int_0^{\frac{\pi}{2}} e^{-\frac{t\rho}{\cos\Theta}}\, d\Theta = \int_0^{+\infty} \frac{e^{-t\rho\,\mathrm{ch}u}}{\mathrm{ch}t}\, du = \int_{t\rho}^{\infty} K_0(n)\, du = K_{11}(t\rho)$$

we obtained:

$$K_3(r,r') = \frac{2r'}{\pi}\int_{(r-r')}^{r+r'} \frac{K_{11}(t\ell)}{\sqrt{\rho^2-(r-r')^2}\,\sqrt{(r+r')^2-\rho^2}}\, dP$$

but failed to obtain an expansion of F.

Compactness of the kernel - We have shown that Kc is compact on L^2 (0,1). We shall omit the demonstration here.

More details will be given in a report in preparation [7].

6 - NEUTRON FLUX - We shown that a complete basis can be generated for one dimensional geometries, at least for the slab and the sphere. Eigenfluxes are described by their moments, which are space dependant.

As we explained in another contribution to this conference the moments are used to determine "probability tables", i.e. sets of values (p_i, x_i) which:

- describe the moments,
- allow approximations of $\varphi_\lambda(x)$ by sums of the orthogonal polynomials P(x),
- provide easy estimations of any integrals by GAUSS quadratures:

$$\int_D F(x)\, \varphi_\lambda(x) = \sum_i p_i\, F(xi) + R_N$$

the rest R_N being as small as required (R_N decreases as N increases).

The probability tables (p_i, x_i) are independant of the abscissa origin. The eigenfluxes, of which analytical expressions are unknown, are described by the probability tables with the required accuracy.

For a given geometry, the probability table as a function of t, the total cross section, has to be calculated once and for all (assuming that an interpolation method has been developed).

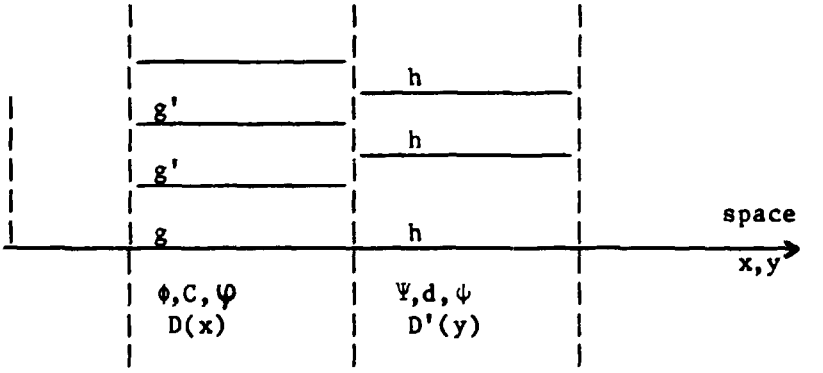
We shall demonstrate how the ease of integration can be applied to the treatment of the transport equation.

7 - APPLICATION TO THE TREATMENT OF THE TRANSPORT EQUATION - It will be shown schematically how the 4 terms of the transport equation can be treated.

The flux in group g will be given by:

$$\phi_g(x) = s_{gg} \int_D T(t|x-x'|)\ \phi_g(x')\,dx'$$

$$+ \sum_{g' \neq g} s_{g'g} \int_D T(E|x-x'|)\ \phi_{g'}(x')\,dx' + \sum_h s_{hg} \int_{D'} T(\tau|y-x|\ \psi_{g'}(y)\,dy$$



The second term represents the scattering of the group or sub-group inside itself, and s_{gg} is simply the scattering cross-section. The third term represents the contribution of the other groups or sub-groups to g-s_{gg}, being the "scattering" cross section of g' into g. The fourth term has the same meaning for the geometric medium D'.

The flux can be expanded on a complete basis:

$$\phi_g(x) = \sum_\lambda c_{g,\lambda}\, \varphi_{g,\lambda}(x)$$

$$\Psi_h(y) = \sum_\lambda d_{h,\lambda}\, \Psi_{h,\lambda}(x)$$

Replacing ϕ and Ψ by their expansion, multiplying by $\varphi_{g\lambda}$ and integrating we obtain:

1) First term:

$$\int_D \varphi_{g\lambda}(x) \sum_\lambda c_{g\lambda}\, \varphi_{g\lambda}(x)\,dx = c_{g\lambda}$$

since $\varphi_{g\lambda}$ and $\varphi_{g\lambda'}$, $\lambda \neq \lambda'$, are orthogonal and assumed normalised.

2) Second term:

$$s_{gg} \int_D \varphi_{g\lambda}(x)\,dx \int_D T(t|x-x'|) \sum_\lambda c_{g\lambda}\,\varphi_{g\lambda}(x')\,dx'$$
$$= s_{gg} \int_D \varphi_{g\lambda}(x)\,dx \sum_\lambda c_{g\lambda}\,\lambda_g\,\varphi_{g\lambda}(x)$$
$$= s_{gg}\,\lambda_g\,c_{g\lambda}$$

since $\varphi_{g\lambda}$ is an eigenflux relative to the transport operator T, and $\varphi_{g\lambda}$ and $\varphi_{g\lambda'}$ are orthogonal if $\lambda \neq \lambda'$.

3) Third term:

$$\int_D \varphi_{g\lambda}(x)\,dx \sum_{g'} s_{g'g} \sum_{\lambda'} c_{g'\lambda'}\,\varphi_{g'\lambda'}(x)$$
$$= \sum_{g'} \sum_{\lambda'} s_{g'g}\,c_{g'\lambda'}\,\mathscr{S}_{g\lambda,g'\lambda'}$$

with $\mathscr{S}_{g\lambda,g'\lambda'} = \int_D \varphi_{g\lambda}(x)\,\varphi_{g'\lambda'}(x)\,dx$

a scalar product of eigenflux, which will be calculated from their probability table.

4) Fourth term:

$$\int_D \varphi_{\lambda g}(x)\,dx \sum_k s_{kg} \int_{D'} T(|x-y|) \sum_{\lambda'} d_{k\lambda}\,\Psi_{k\lambda}\,dy$$

$$= \sum_h s_{hg} \sum_{\lambda'} d_{k\lambda'} \int_D \varphi_{\lambda g}(x)\,du \int_{D'} T(|x-y|)\,\Psi_{k\lambda'}\,dy$$

$$= \sum_h s_{hg} \sum_{\lambda'} d_{h\lambda'} \sum_j q_j \int_D \varphi_{\lambda g}(x)\,T(|x-y_j|)\,du$$

$$= \sum_k s_{hg} \sum_{\lambda'} d_{h\lambda'} \sum_{i,j} \rho_i\,q_j\,T(|x_i-y_j|)$$

$$= \sum_k s_{hg} \sum_{\lambda'} d_{h\lambda'}\quad g\lambda,h\lambda'$$

For these transformations we performed two space quadratures over D'(coefficient q_j) and over D (coefficient p_i).

The equation can then be written as:

$$c_{g\lambda} = s_{gg}\,\lambda_g\,c_{g\lambda} + \sum_{g'} s_{g'g} \sum_x c_{g'\lambda'}\,\mathscr{S}_{g\lambda,g'\lambda'} + \sum_h s_{hg} \sum_{\lambda'} d_{h\lambda'}\,\mathscr{C}_{g\lambda,h\lambda'}$$

$$\phi_g(x) = \sum_x c_{g\lambda}\,g\lambda(x)$$

where $\mathscr{S}$ and $\mathscr{C}$ characterize the coupling of eigenfluxes belonging to different cells (different group or subgroup, or different geometry) and are calculated only just once for given geometry and total cross-sections. The problem will be to treat a linear system coupling the $c_{g\lambda}$ intensities of the various eigenfluxes.

7 - COMPARISON WITH OTHER EXPANSION METHOD

Several attempts have been made to deal with the transport equation by flux expansion on a complete basis: mainly those by ASAOKA [8] and CARLVIK [9]. These have been shown to be basically the same: the basis is geometrical, i.e. LEGENDRE polynomials in the case of infinite slabs.

The basis we propose is different: the eigenflux depends of course on the geometry, but also on the total cross section. Their analytical formula is unknown: they are defined by their probability table (which defines the series of orthogonal polynomials from which the eigenflux can be reconstructed).

CONCLUSION - Our first problem was to establish the completeness of the sets of eigenfluxes generated and to give an analytical expression of the matrix coefficient a_{nm} allowing computation of the eigenflux moments of, then of the probability table describing every eigenflux. Up to now we have failed to establish these elements in the case of cylinders.

We now plan to give some practical example of the application of this new approach to neutronics.

We feel that the main difficulty will be the extension of this method to space, i.e. the problem of establishing a 2 (or 3) dimensional probability table, a difficulty linked with the problem of multivariate PADE approximants.

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