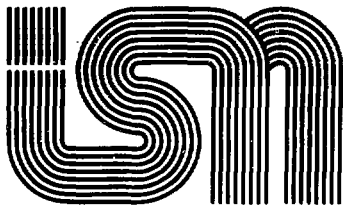


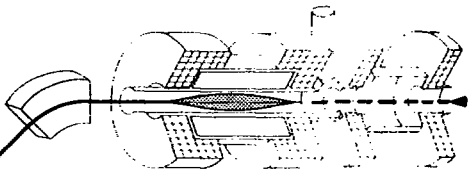
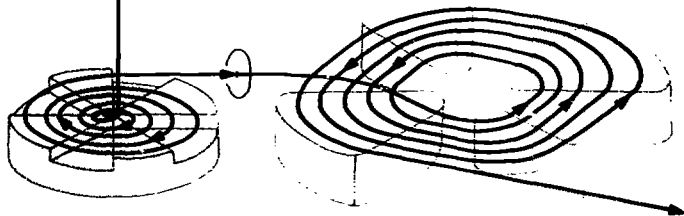
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ON THE NUCLEAR LONGITUDINAL CHARGE RESPONSE IN THE
QUASI-ELASTIC PEAK REGION.

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Abstract

We calculate in a semiclassical RPA approach the nuclear charge response of ^{12}C , ^{40}Ca , and ^{56}Fe in the quasi-elastic peak region for moderate momentum transfers ($q = 1.0 - 2.0 \text{ fm}^{-1}$). Using the Gogny force and taking full account of antisymmetrisation effects we find with no free parameters good agreement with the (e,e') data for ^{12}C . However for ^{40}Ca and ^{56}Fe the missing charge problem persists. Arguments that this may be due to the strong influence of 2p-2h states in the isovector channel are advanced.

+ This work is part of the Ph.D. thesis of U. Stroth

1. INTRODUCTION

Inelastic electron scattering excites in nuclei for momentum transfers $q > 1 \text{ fm}^{-1}$ a broad bump in the cross section, the so-called quasi-elastic peak. The gross properties of the quasi-elastic peak have been explained by one nucleon knock out processes and have been reasonably well described with simple Fermi gas models^{1,2}. Problems, however, showed up when one tried to reproduce the now available longitudinal and transverse responses individually. While the transverse response can be reasonably understood with usual many body theory^{3,4}, the longitudinal response is generally overestimated. Several attempts have been made to improve on this situation: van Giai et al.⁵ find through relativistic corrections a stronger quenching in the charge than in the transverse response, whereas Noble and, later, Shakin proposed that in medium corrections of the nucleon form factor are responsible for additional quenching⁶.

In this work we again attack the problem of the charge response from the conventional many body point of view in adopting our recently developed semiclassical theory for the linear response function³, generalizing it to take fully account of exchange effects in the meanfield as well as in the residual interaction (RPA). In fact at the moderate momentum transfers we will consider ($1 \text{ fm}^{-1} < q < 2 \text{ fm}^{-1}$), the residual particle-hole force still seems to play an important role on the detailed shape and magnitude of the quasi-elastic bump of the charge response. Since on the other hand the residual interactions are quite different in the longitudinal and transverse channels we feel it legitimate to concentrate in this work on the charge response solely.

To that purpose we use in a conventional RPA approach consistently the finite range Gogny force ⁷⁾ whose parameters have been adjusted to low energy phenomena but a comparison ⁷⁾ with more fundamental G-matrix calculations shows that this force should certainly be valid up to momentum transfers of 1.5 fm^{-1} (maybe it can be used up to almost 2 fm^{-1}). This conclusion is backed by several pleasant features of the Gogny force : most importantly it very well fulfills the forward scattering sum rule of Landau's Fermi liquid parameters ⁸⁾ ; this sum rule is a very stringent test on a force and since the Fermi liquid parameters involve through the exchange terms momenta $k = k_F$ the Gogny interaction should be quite accurate at least up to $q = k_F$ but tentatively it could be used beyond (up to $q = 2 \text{ fm}^{-1}$). These satisfying properties are further substantiated by its realistic compressibility coefficient ($K = 228 \text{ MeV}$), by its ability to describe giant resonances ⁹⁾, and by its behavior in describing high lying nuclear excitations as was shown recently ¹⁰⁾. However, we not only will use the Gogny force for the residual ph interaction but also for the construction of the nonlocal mean field potential whose quality has been tested in many Hartree-Fock calculations (it may however be that its nonlocality is somewhat too short ranged ¹¹⁾, a feature of importance only for higher momentum transfers ($q > k_F$)). We want to emphasize that we do not make the often employed effective mass approximation but use the nonlocal mean field in full.

With this in mind we try to solve the RPA equations in the above mentioned range of transferred momenta for the calculation of the nuclear response function in the quasi-elastic peak region. Since in a purely quantum mechanical manner this is a formidable numerical task we have resort to our recently developed semiclassical RPA theory ³⁾ which we generalize to account for antisymmetrisation as well in the

mean field as in the residual interaction. This semiclassical theory has been tested³⁾ and turned out to be extremely accurate for the non interacting response but preliminary studies show that it also works in the interacting case.

Investigations of the longitudinal response have been performed by several authors using various approximation schemes. Cavinato et al.¹²⁾ studied the (e, e') reaction in ^{12}C in a quantal continuous RPA frame using the Skyrme force SK3. This is, however, a contact force whose use may be doubtful since its momentum dependence at higher q -values is not controlled. A more detailed and critical account of the use of this force in the quasi-elastic peak region has been given by Dellafiore et al.¹³⁾ These latter authors investigate the charge response of ^{12}C within the TDA theory using the Kurath interaction. This is a finite range density independent force. This feature makes it less reliable than the density dependent Gogny interaction since it is known⁸⁾ that the rearrangement terms in the ph force coming from a density dependence are very important (for instant to fulfill the forward scattering sum rule). Besides one can ask the question whether one should not use the RPA rather than the TDA approach.

A preliminary account of the present work is given elsewhere¹⁴⁾. There also exists a recent investigation on the same subject by Alberico et al.¹⁵⁾ using a semiclassical RPA approach very similar to the present one but trying to reproduce, within limits, the experimental data by a best fit procedure on the residual interaction and the mean field properties. Whereas the conclusion of the latter authors supports Noble's idea of a swollen nucleon in a nucleus we here tend to advocate, and in fact will give detailed arguments, that an important coupling of $2p$ - $2h$ states in the isovector part of the response might be at the origin of disagreement with experiment in heavier nuclei.

This paper is organised as follows : in section 2 we give a detailed account of our formalism and in section 3 our results together with a careful discussion and possible interpretations are presented. In section 4 we formulate our conclusions.

2. FORMALISM

As we said in the introduction the aim of this work is to solve the RPA equations using the finite range Gogny force in the quasi-elastic peak region. A fully quantal calculation would be a tremendous numerical task in this energy region ; so we resort here to our semiclassical RPA theory developed recently³⁾ and which turned out to be very precise. We briefly recall the principle. In operator form the free ph Green's function can be written in the following way :

$$G^0(\hat{H}_1, \hat{H}_2) = \frac{\vartheta(\hat{H}_1 - \epsilon_F) \vartheta(\epsilon_F - \hat{H}_2)}{\omega - \hat{H}_1 + \hat{H}_2 + i\eta} - \frac{\vartheta(\epsilon_F - \hat{H}_1) \vartheta(\hat{H}_2 - \epsilon_F)}{\omega + \hat{H}_1 - \hat{H}_2 + i\eta} \quad (1)$$

To lowest order in \hbar we replace the single particle H.F. Hamiltonians \hat{H}_i by their classical counterparts :

$$\hat{H}_i \Rightarrow h_i^c = \frac{p_i^2}{2m} + V(R_i, p_i) \quad (2)$$

where $V(R, p)$ is the Wigner transform¹⁶⁾ of the nonlocal self consistent meanfield potential. In this way we obtain the lowest order semiclassical expression for $G^0(\vec{R}_1, \vec{p}_1, \vec{R}_2, \vec{p}_2)$ expressed in the phase space variables of the particle and the hole. Through inverse Wigner transformation this expression can be obtained in configuration space : $G^0(\vec{r}_1, \vec{r}_2)$ with coordinates as indicated in Fig. 1.

For a local excitation operator such as

$$O(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') e^{i\vec{q}\vec{r}} \quad (3)$$

the free nuclear polarisation propagator is given by

$$\tau^0(q, \omega) = \int d^3r d^3r' e^{i\vec{q}\vec{r}} G^0(\vec{r}\vec{r}', \vec{r}\vec{r}') e^{-i\vec{q}\vec{r}'} \quad (4)$$

While the structure function is defined as the imaginary part of eq.(4) :

$$\begin{aligned} S^0(q, \omega) &= -\frac{1}{\pi} \text{Im} \left\{ \tau^0(q, \omega) \right\} \\ &= -\frac{4}{\pi} \int \frac{d^3R d^3P}{(2\pi)^3} \Theta(|\vec{P} + \vec{q}| - k_F(R)) \Theta(k_F(R) - P) \delta(\omega - \hbar(R) - \hbar(\vec{P} + \vec{q}) + \hbar(R, P)) \end{aligned} \quad (5)$$

Because of the zero range of the excitation operator (3) expression (4) is in semiclassical approximation equivalent to the local momentum approximation, i.e. equal to the nuclear matter expression where k_F is replaced by $k_F(R)$ which in turn is obtained from the implicit equation

$$\varepsilon_F = \frac{k_F^2(R)}{2m} - V(R, k_F(R)) = 0 \quad (6)$$

where ε_F is the global Fermi energy to be determined by the particle number condition

$$4 \int \frac{d^3R d^3P}{(2\pi)^3} \Theta(\varepsilon_F - \frac{P^2}{2m} + V(R, P)) = A \quad (7)$$

This approximation scheme for the non interacting response has been first derived with a somewhat different technique by Rosenfelder²⁾.

We complete and generalize this theory in several respects. First and most importantly we check the accuracy of the approximation. This check

has been published elsewhere³⁾ but for the sake of a self contained presentation we here show our result again. In Fig. 2 is displayed the comparison of a completely quantal calculation of expression (5) using a phenomenological Woods Saxon potential for $q = 2.15 \text{ fm}^{-1}$ together with our semiclassical approximation. The quantal calculation which has been smoothed by a Lorentzian of 3 MeV width for easier representation of the resonating part at low energy has kindly been performed for us by van Gai¹⁷⁾. In Fig. 2 we can appreciate the global accuracy of our approach. Several remarks can however be made : the semiclassical approach is known¹⁶⁾ to give the average part of a quantal calculation : in the continuum region where already the quantal result is smooth both calculations agree and indeed in the low energy part our result passes nicely through the average. Nevertheless a very slight deviation from the quantal result can be observed also in the continuum part which could become more important for lighter nuclei and for surface responses of inelastic hadron scattering. Also our result at the high energy side goes to zero sharply whereas the quantal result vanishes asymptotically. These minor deficiencies are likely to be cured in evaluating \hbar^2 corrections to our lowest order approximation. This is most easily performed in expanding the Wigner transform of (1) around the classical Hamiltonians :

$$G^0(\hat{H}_1, \hat{H}_2) \Big|_W = G^0(h_1^c, h_2^c) + \frac{1}{2} \sum_{i,j} \frac{\hbar^2}{3h_1^c \partial h_j^c} G^0(h_1^c, h_2^c) \left[(\hat{H}_1 - h_1^c) (\hat{H}_j - h_j^c) \right]_{ij} + \dots \quad (8)$$

The evaluation of these corrections is in progress¹⁸⁾.

A further important step consists in introducing the residual ph interaction into the semiclassical framework. Let us first consider the Bethe Salpeter equation (see ref. 16) ch. 8) for the interacting

ph Green's function in the case of a direct spin and isospin independent force :

$$\begin{aligned} G(\vec{r}, \vec{r}') &= G^{(0)}(\vec{r}, \vec{r}') + \int d^3 r_1 d^3 r_1' G^0(\vec{r}, \vec{r}_1) v(\vec{r}_1 - \vec{r}_1') G(\vec{r}_1, \vec{r}') \\ G(\vec{r}, \vec{r}') &= G(\vec{r}, \vec{r}', \vec{r}, \vec{r}') \end{aligned} \quad (9)$$

The local Green's function $G(\vec{r}, \vec{r}')$ figuring in (4) can be considered as a nonlocal one body operator and correspondingly eq(8) is effectively a one body equation. We can Wigner transform (9) and remembering that to lowest order in \hbar the Wigner transform of a product of operators is equal to the product of the respective Wigner transforms we obtain from (9) :

$$\begin{aligned} \tau(R, q) &\stackrel{\hbar \rightarrow 0}{=} \tau^0(R, q) + \pi^0(R, q) v(q) \tau(R, q) \\ &= \tau^0(R, q) / (1 - v(q) \tau^0(R, q)) \end{aligned} \quad (10)$$

with

$$\tau(R, q) = \int d^3 S e^{-i\vec{q}\vec{S}} G(\vec{R} + \frac{\vec{S}}{2}, \vec{R} - \frac{\vec{S}}{2}) \quad (11)$$

The condition for (10) to be valid is the same as for the lowest order approximation of G^0 namely gradients of the mean field appearing in the expansion (7) must be negligible. We therefore expect that our approximation for the interacting response is of similar accuracy. Unfortunately the numerical evaluation of the exact interacting response at such high energies is very involved even for the case of a direct or δ -interaction only, so that we do not have presently any definite possibility of checking (10) but preliminary results are available indicating that our theory also works in the interacting case. In order to assure convergence \hbar^2 correction can, however, also be evaluated straightforwardly in the interacting case in going to higher orders in the threefold operator

product of (9) ¹⁶⁾. We again remark that our lowest order solution (10) is of the local density type what is equivalent to take the infinite matter result and replace k_F by $k_F(R)$ everywhere.

For finite range forces to be considered here we however have also to take care of the exchange contribution of the force what is a much harder problem because inclusion of exchange leads even in infinite matter to a genuine integral equation for the response function and therefore no analytical solution exists even in local density approximation. On the other hand it will turn out that direct and exchange contributions are of the same order of magnitude and it is thus very important to treat both on the same footing. Since in total the influence of the interaction (direct plus exchange) is relatively weak ($\sim 30\%$) we here apply a continued fraction expansion of the response function with fully antisymmetrized interaction. This procedure has been proposed by one of the authors some time ago ¹⁹⁾ but independently applied to the response function in the TDA scheme in ref. 13). It has been shown there that the convergence of the continued fraction expansion is very good in the domain of the considered momenta and that usually the first iteration is sufficient. We therefore expand the interacting response to first order in the fully antisymmetrical interaction and to lowest order in \hbar :

$$\pi(R, q, \omega) = \pi^0(R, q, \omega) + \pi^0(R, q, \omega) v_D(R, q) \pi^0(R, q, \omega) + \pi_{ex}^0(R, q, \omega) \quad (12)$$

with

$$\pi_{ex}^0(R, q, \omega) = \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} G^0(R, \vec{k}_1, \vec{k}_1 + \vec{q}) v_{ex}(|\vec{k}_1 - \vec{k}_2|) G^0(R, \vec{k}_2, \vec{k}_2 + \vec{q}) \quad (13)$$

where G^0 is given by :

$$G^0(R, \vec{k}, \vec{k}+\vec{q}) = 4 \left\{ \frac{\theta(|\vec{k}+\vec{q}| - k_F(R))\theta(k_F(R)-k)}{\omega - h(R, |\vec{k}+\vec{q}|) + h(R, k) + i\eta} - \frac{\theta(|\vec{k}-\vec{q}| - k_F(R))\theta(k_F(R)-k)}{\omega + h(R, |\vec{k}-\vec{q}|) - h(R, k) + i\eta} \right\} \quad (14)$$

and π^0 is related to G^0 by the following equation :

$$\pi^0(R, q, \omega) = \int \frac{d^3 k}{(2\pi)^3} G^0(R, \vec{k}, \vec{k}+\vec{q}) \quad (15)$$

The form of the interaction in (12,13) has already been modelled to the Gogny force where we have density independent direct (v_D) and exchange (v_{ex}) parts and a density dependent contact term, which is included in v_D .

We now transform (12) into the lowest order continued fraction expansion¹⁹⁾ and obtain for the interacting polarisation propagator

$$\pi(q, \omega) = \int d^3 R \frac{\pi^0(R, q, \omega)}{1 - \tilde{v}(R, q, \omega)\pi^0(R, q, \omega)} \quad (16)$$

where we introduced the effective interaction

$$\tilde{v}(R, q, \omega) = v_D(R, q) + \frac{v_{ex}(R, q, \omega)}{(\pi^0(R, q, \omega))^2} \quad (17)$$

It is clear from our approximation scheme that direct and exchange term are treated on an equal footing and furthermore eq.(16) has the pleasant feature that for a contact force or in ring approximation (neglecting exchange) it reduces to expression (10). Higher order continued fraction terms can be straightforwardly constructed to the expense of a somewhat greater numerical effort but as we already mentioned \tilde{v} will turn out to be relatively small so that a first order Taylor series almost suffices ; usually however Padé approximations or continued fractions speed up convergency as was also noticed in ref. 13). We thus keep eqs.(16,17) together with (1,2) as our final expression for the response function, remembering that we will use a nonlocal mean field constructed from the Gogny force.

3. APPLICATION TO THE CHARGE RESPONSE OF ^{12}C , ^{40}Ca , AND ^{56}Fe AND DISCUSSION OF THE RESULTS.

Before discussing applications and results in detail still the explanation of some more ingredients of our calculus is in order. As we said we use the Gogny force to build up the Wigner transform (2) of the Hartree-Fock field and the effective ph interaction $\hat{V}(R, q, \omega)$ (eq.(17)) ; in doing so we however neglected consistently the spin orbit term which does not have a strong influence on average quantities such as we are considering here ; this has been demonstrated for ground state properties like the detailed r-dependence of the nuclear density but should equally hold for the energy domain we are considering here. The Wigner transform of the non local meanfield

$$V(\vec{r}, \vec{r}') = \frac{1}{4} \int_{\sigma_1 \tau_1} v(\vec{r}, \sigma_1, \tau_1, \vec{r}'_1, \sigma_1, \tau_1, \vec{r}'_1, \sigma_1, \tau_1) \rho(\vec{r}'_1, \sigma_1, \tau_1, \vec{r}'_1, \sigma_1, \tau_1) \quad (18)$$

has then been constructed using for $\rho(\vec{r}, \vec{r}')$ the Thomas Fermi expression¹⁶⁾. In (18) $v(\dots)$ stands for the antisymmetrized matrix element of the Gogny force and together with the Thomas Fermi expression for ρ we arrive at the expression (3.2) given in ref. 11). The Wigner transform is then easily calculated what yields an explicit expression in terms of error functions and $k_F(\vec{R})$. The local Fermi momentum $k_F(R)$ is evaluated everywhere from the expression

$$k_F(R) = \sqrt{2m(\epsilon_F - V(R))} \quad (19)$$

where we took for $V(R)$ the very well tested Woods Saxon like parametrization of ref. 20) ; $k_F(R)$ and thus $V^{H.F.}(R, P)$ should therefore be quite reliable and we can proceed to our final formula which we used for comparison with experiment. The excitation operator \hat{C} for the charge response is of

the form

$$O(q) = \sum_{i=1}^A \frac{1+\tau_3 i}{2} e^{-i\vec{q}\vec{r}_i} \quad (20)$$

and thus the longitudinal charge response is a sum of $S = 0$, $T = 0$ and $S = 0$, $T = 1$ ph responses ; we have to keep that in mind and perform a proper ph spin and isospin coupling for the construction of our effective residual ph interaction (17). Some details of the explicit evaluation of v_{ex} are given in the appendix. The final quantity to be compared with experimental data includes the nucleon form factor and the Darwin Foldy term to partially account for relativistic effects :

$$R_L(q, \omega) = f^2(q_\lambda) \frac{(1+q_\lambda^2/(4m^2))}{(1+q_\lambda^2/(2m^2))} \frac{(-1)}{\pi} \frac{1}{4} \text{Im} \left[\pi^{0,0}(q, \omega) + \pi^{0,1}(q, \omega) \right] \quad (21)$$

The function $f(q_\lambda) = (1+(q_\lambda^2)/(842 \text{ MeV}^2))^{-2}$ is the nucleon form factor and $q_\lambda = q^2 - \omega^2$ the four momentum transfer.

We are now ready for comparison with experiment and as a first example we choose the nucleus ^{12}C for a series of momentum transfers $1 \text{ fm}^{-1} \leq q \leq 2 \text{ fm}^{-1}$. These results together with the experimental points are displayed in Figs. 3a-e where we see besides the individual $T = 0$ and $T = 1$ responses (broken lines) also the free (non interacting) response $R_L^{(0)}$ calculated without the residual interaction \tilde{v} including however the non local Hartree-Fock potential.

A first look on Figs. 3a-e shows that in view of the fact that our theory contains no adjustable parameter the agreement with experiment is globally very satisfying. We see the total influence of the residual interaction is of $\sim 30\%$ as announced earlier ; we also see that individual resonances showing up in the experimental spectrum at low

momentum transfers ($q = 1 \text{ fm}^{-1}$; $q = 1.27 \text{ fm}^{-1}$) are reproduced on the average and that the immersion of these resonances into the quasi-elastic peak at around $q = 1.5 \text{ fm}^{-1}$ is nicely followed by the theoretical curves. In fact the only deficiency which can be seen is that our response somewhat overshoots the experimental data on the high energy side. This feature is quantitatively almost independent of the five momentum transfers considered and it looks difficult to decide from Fig. 3 to which ingredient of our theory this deficiency is due to.

We now come to the heavier elements and we choose to present ^{40}Ca and ^{56}Fe . Unfortunately experimental momentum transfers start only at $q = 1.52 \text{ fm}^{-1}$ so that we display in Figs. 4(a-c) in parallel the charge responses for ^{40}Ca and ^{56}Fe for three momentum transfers between $1.5 \text{ fm}^{-1} < q < 2 \text{ fm}^{-1}$. A dramatic effect can be seen immediately : with exactly the same theory as in the ^{12}C case our results are much worse for the two heavier nuclei. So a strongly mass dependent effect must be missing in our theory and it is precisely this missing charge problem which lead Noble⁶⁾ to the introduction of a mass dependent nucleon size (a swelling of the nucleon with increasing A). Let us now inspect the total of our results more closely and let us try then to draw some conclusions. The first observation which can be made is that it is the strong increase of the overshooting on the high energy side in going from ^{12}C to ^{40}Ca , ^{56}Fe which causes the failure of our results for the ^{40}Ca and ^{56}Fe cases. And in turn this is caused by a strong relative increase of the $T = 1$ response versus the $T = 0$ response for the heavier nuclei. In fact closer inspection shows that the $T = 0$ part by itself follows in its shape very nicely the evolution as a function of momentum transfer and mass number. For instance the plateau like structure of the response observed in the two heavier nuclei

is already reproduced by the isoscalar response alone whereas this plateau is absent in ^{12}C as well experimentally as on the theoretical curves. Let us briefly discuss where this qualitative change of behaviour of the isoscalar part comes from. To this purpose we present in Fig. 5 our effective ph force \tilde{V} of eq.(17) (i.e. the real part, the imaginary part being by a factor 10-20 smaller) as a function of momentum transfer once for $k_F = 1.36 \text{ fm}^{-1}$ (bulk) and once for $k_F = 0.75 \text{ fm}^{-1}$ (surface). From this figure we see : the well-known fact that the F^{00} Landau parameter becomes strongly attractive in the surface²¹⁾ persists in \tilde{V} for quite high momentum transfers. On the other hand in the bulk \tilde{V} passes from slightly negative values at low q to appreciable repulsive values at higher q 's. The surface attraction pulls strength to lower energies (compared with the free response) and that is the only effect present in ^{12}C (due to its small size). In ^{40}Ca and ^{56}Fe however the repulsion in the bulk enters into competition with the surface attraction and pushes part of the strength to the high energy side causing the plateau in our isoscalar response in ^{40}Ca and ^{56}Fe . If this feature should be the explanation of the plateau like structure observed in all heavier nuclei measured so far (this is demonstrated in Fig. 6 where the nuclear structure functions at $q = 1.52 \text{ fm}^{-1}$ for the four measured nuclei ^{12}C , ^{40}Ca , ^{48}Ca , ^{56}Fe are displayed showing again the qualitatively different behaviour of ^{12}C from the rest of the nuclei) then we remain with the question of what is happening to the isovector part of our response which for the heavier nuclei destroys agreement with experiment. There could in fact be one explanation which seems quite tempting to us. There have been speculations for some time that the 2p-2h states could be much more effective in spreading the isovector charge response than the isoscalar one²²⁾. That this effect can be dramatic has recently been demonstrated by Drozd et al.²³⁾ where it

is shown that 2p-2h states literally wash out the 1p-1h collective isovector response of ^{40}Ca whereas the spreading width of the isoscalar giant resonances is very small. Should this feature prevail up to momentum transfers considered in this paper, this would indeed permit a quite coherent description of the situation. In fact this would mean that the isoscalar part of our response is only little affected by the coupling to 2p-2h states whereas the isovector part is strongly quenched and broadened. This is exactly what is needed to bring down the overshooting on the high energy side of our total response, since on top of it the coupling of 2p-2h states is strongly mass dependent as we can see on Fig. 7 ; there we display the density of 2p-2h states for a fixed energy as a function of mass number calculated semiclassically with a spherical harmonic oscillator potential²⁴⁾. Last but not least this could also explain the missing charge problem since strength may have been transported to higher energies and thus been missed by the experimental evaluation of the integrated cross section. That such strength exists beyond the measured points of the charge response can, as we think, not be dismissed from the experimental data points.

We are aware of the fact that our explanation is of course tentative and that it cannot rule out other possible explanations such as the increase of the nucleon form factor as long as we have not explicitly performed the coupling to 2p-2h states. We think however that our investigation is sufficiently reliable and the features which emerge of so general character that our proposed explanation of the charge response should be considered as a serious alternative.

4. CONCLUSIONS

In this work we investigated the nuclear charge response of ^{12}C , ^{40}Ca , and ^{56}Fe in the domain $1 \text{ fm}^{-1} \leq q \leq 2 \text{ fm}^{-1}$ of transferred momenta. We used a recently developed semiclassical theory for the nuclear response in the RPA frame. The mean field and the residual interaction were constructed from the Gogny force^{7,8)} and particular emphasis was laid on a correct treatment of exchange effects in both quantities. We gave arguments that the Gogny force should be quite reliable up to momentum transfers of almost 2 fm^{-1} . We found that our theory with no adjustable parameter quite satisfactorily describes the ^{12}C data with a moderate overshooting of the data points on the high energy side as the only drawback. For the nuclei ^{40}Ca and ^{56}Fe , however, suddenly a strong discrepancy with experiment shows up where the overshooting on the high energy side is dramatically increased; this indicates a strongly A-dependent effect which is missing in our theory. Closer inspection of the isoscalar and isovector parts of our response reveals that the former is following the evolution of data points as a function of momentum transfer and mass number quite nicely in shape (of course not in absolute magnitude) and that it is the isovector response which destroys agreement with experiment for instance for the heavier nuclei. Guided by the recent work of Drozd et al.²³⁾ where they show that at low q the coupling of 2p-2h states can have a dramatic broadening effect on the collective isovector oh response and only very little so on the isoscalar giant resonances, we speculate that, if this effect persisted at the momenta considered here, it could give a consistent picture of the charge response. The density of 2p-2h states is strongly increasing with mass number and if essentially only the isovector response is affected (i.e. broadened) this could cure the overshooting of our total response

on the high energy side a feature which is, as we said, also very much mass-dependent. In addition the broadening of the isovector bump could give an explanation of the missing charge problem since strength might be shifted into an experimentally unobserved high energy region. We think that this possibility cannot be ruled out from an analysis of the data points. Though our proposed explanation is only speculative we think that so many pieces would fit together nicely on the one hand and on the other our study seems sufficiently reliable not to put into question the general features which emerge. We feel that our speculation could be a serious candidate among the other explanations of the problem which have been proposed so far^{5,6)}. In any case we are strongly motivated to include $2p-2h$ states into our theory^{19,25)} and studies in this direction are now under way.

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APPENDIX

We here want to specify, how the exchange contribution to the polarisation propagator (eq. 13) has been evaluated. The exchange part of the Gogny force, which has to be inserted into eq. 13 has the following momentum dependence :

$$v_{ex}(|\vec{k}_1 - \vec{k}_2|) = \sum_{i=1}^2 C_i \exp((\vec{k}_1 - \vec{k}_2)^2 \mu_i^2/4) \quad A1$$

The C_i contain the spin-isospin coefficients of the force (see ref. 8).

With eq. A1 we get out of eq. 13 for the imaginary part of π_{ex} :

$$\begin{aligned} \text{Im}\{\pi_{ex}(R, q, \omega)\} &= 2 \sum_{i=1}^2 C_i \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} \text{Im}\{G^0(R, \vec{k}_1 + \vec{q}, \vec{k}_1)\} \text{Re}\{G^0(R, \vec{k}_2 + \vec{q}, \vec{k}_2)\} \\ &\quad \times \exp((k_1^2 + k_2^2 - 2k_1 k_2 \cos \gamma)/\mu_i^2/4) \end{aligned} \quad A2$$

The angle γ between \vec{k}_1 and \vec{k}_2 can be expressed by the angles of \vec{k}_1 and \vec{k}_2 :

$$\cos \gamma = \cos \vartheta_1 \cos \vartheta_2 + \sin \vartheta_1 \sin \vartheta_2 \cos(\varphi_1 - \varphi_2) \quad A3$$

Since the Green's functions do not depend on the angles φ_1 and φ_2 , they can be integrated using the substitution $\phi_- = \varphi_1 - \varphi_2$ and $\phi_+ = \frac{1}{2}(\varphi_1 + \varphi_2)$.

With the transformation $\chi_1 = \cos \vartheta_1$ and $\chi_2 = \cos \vartheta_2$ this yields for A2 :

$$\begin{aligned} \text{Im}\{\pi_{ex}(R, q, \omega)\} &= \sum_{i=1}^2 \frac{8\pi^2 C_i}{(2\pi)^6} \int_{-1}^1 dk_1 k_1^2 dk_2 k_2^2 \int_{-1}^1 d\chi_1 d\chi_2 \\ &\quad \text{Im}\{G^{(a)}(R, \vec{k}_1 + \vec{q}, \vec{k}_1)\} \text{Re}\{G^0(R, \vec{k}_2 + \vec{q}, \vec{k}_2)\} \\ &\quad \times I_0(\mu_i^2 k_1 k_2 \sqrt{1-\chi_1^2} \sqrt{1-\chi_2^2}/2) \\ &\quad \times \exp\left\{-\mu_i^2(k_1^2 + k_2^2 - 2k_1 k_2 \chi_1 \chi_2)/4\right\} \end{aligned} \quad A5$$

I_0 is the Bessel function.

We calculate eq. A5 numerically and get the real part out of the dispersion relation.

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FIGURE CAPTIONS

- Fig. 1 : Coordinates of the free ph propagator : arrow to the right (left) representing a particle (hole).
- Fig. 2 : Structure function of a noninteracting nucleon gas in a Woods-Saxon potential with the parameters (in usual notation) $V_0 = -50$ MeV, $R_0 = 1.2 A^{1/3}$ fm, $a = 0.5$ fm. Continuous line exact calculation¹⁷⁾ and "+" indicates the semiclassical results.
- Fig. 3 : The charge response (eq.21) for ^{12}C at five momentum transfers : the free response ($R_L^{(0)}$) and the interacting one (full lines) are compared to data from Barreau et al.²⁶⁾. The decomposition of the interacting response into isoscalar and isovector contributions is given by the dashed lines.
- Fig. 4a: The charge response (eq. 21) for ^{40}Ca and ^{56}Fe at $q = 1.52 \text{ fm}^{-1}$: the free response ($R_C^{(0)}$) and the interacting one (full lines) are compared to data from Meziani et al.²⁷⁾. The decomposition of the interacting response into isoscalar and isovector contributions is given by the dashed lines.
- Fig. 4b: Same as Fig. 4a but $q = 1.67$.
- Fig. 4c: Same as Fig. 4a but $q = 1.87$.
- Fig. 5 : Real part of the effective ph force (eq.17) for the isoscalar (left) and isovector (right) channel at two different densities. The contributions from the exchange part (v_{ex}) and the direct plus contact term (v_D) are given by the dashed lines.

Fig. 6 : The charge response (eq. 21) divided by Z for ^{12}C (ϕ),
 ^{40}Ca (ϕ), ^{48}Ca (ϕ) and ^{56}Fe (ϕ) at $q = 1.52 \text{ fm}^{-1}$.

This compilation has been taken from Ref. 15.

Fig. 7 : $2p$ - $2h$ level densities at fixed excitation energy ($\omega = 50 \text{ MeV}$)
as a function of mass number for a harmonic oscillator potential²⁴.

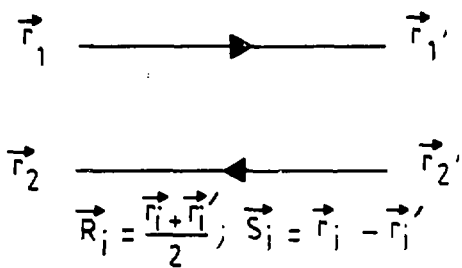


Fig. 1

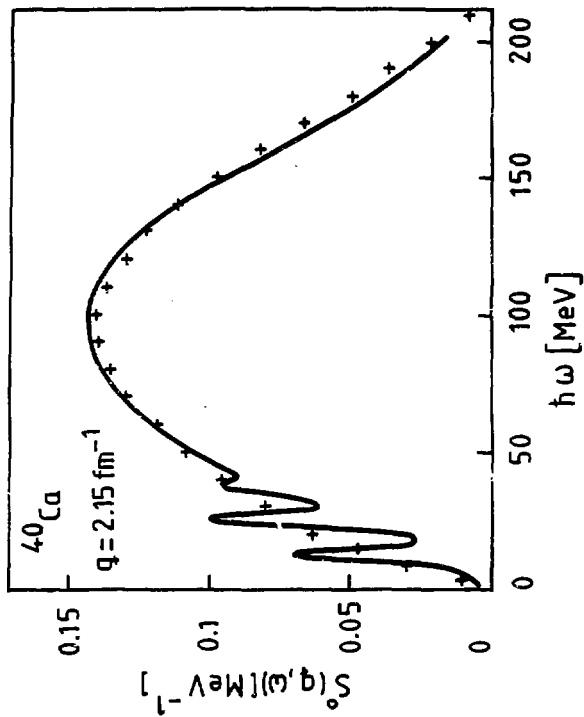


Fig. 2

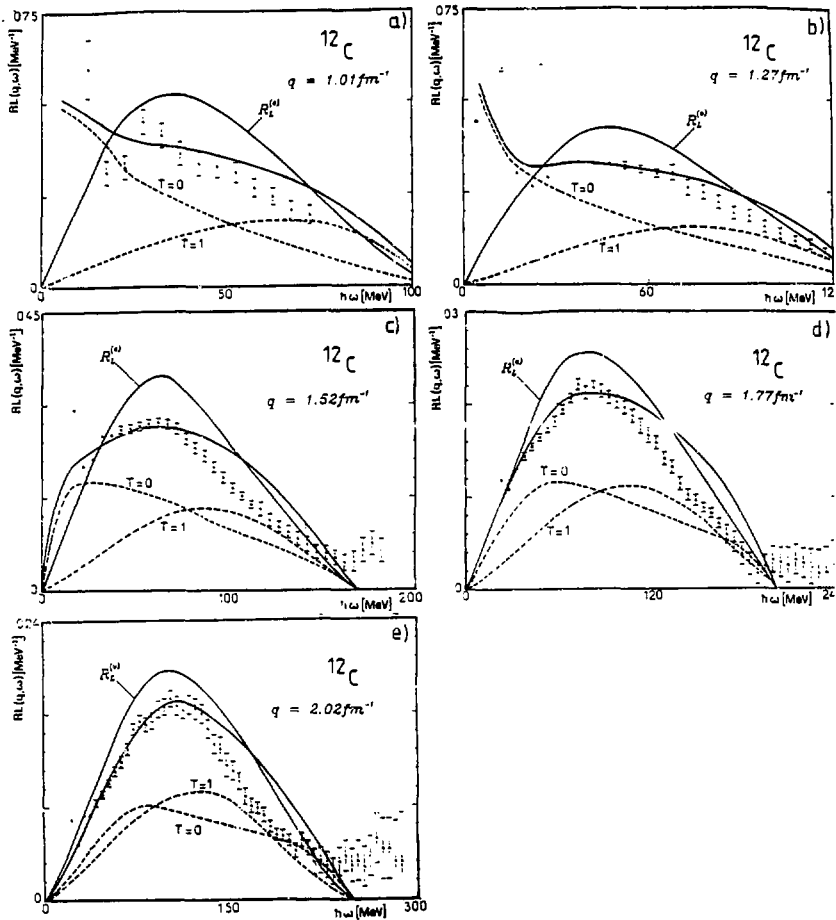


Fig. 3

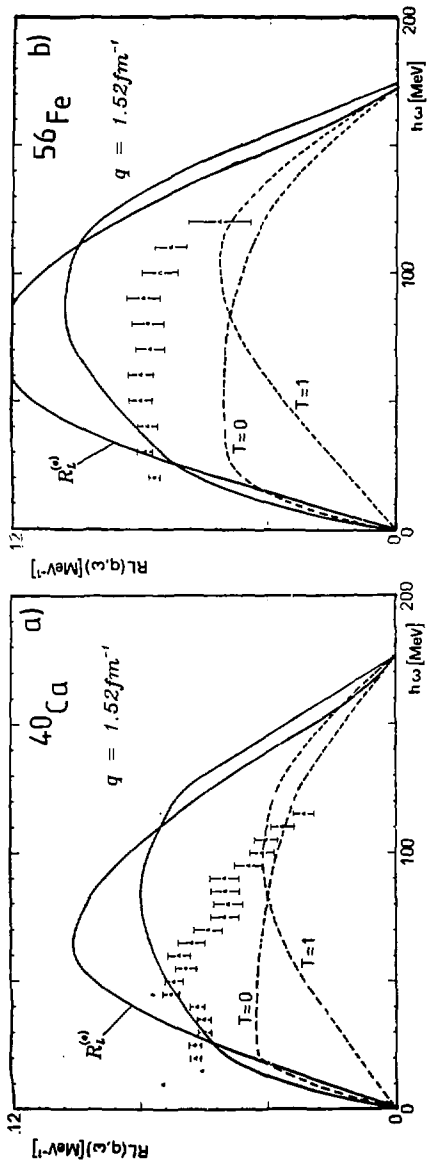


Fig. 4a

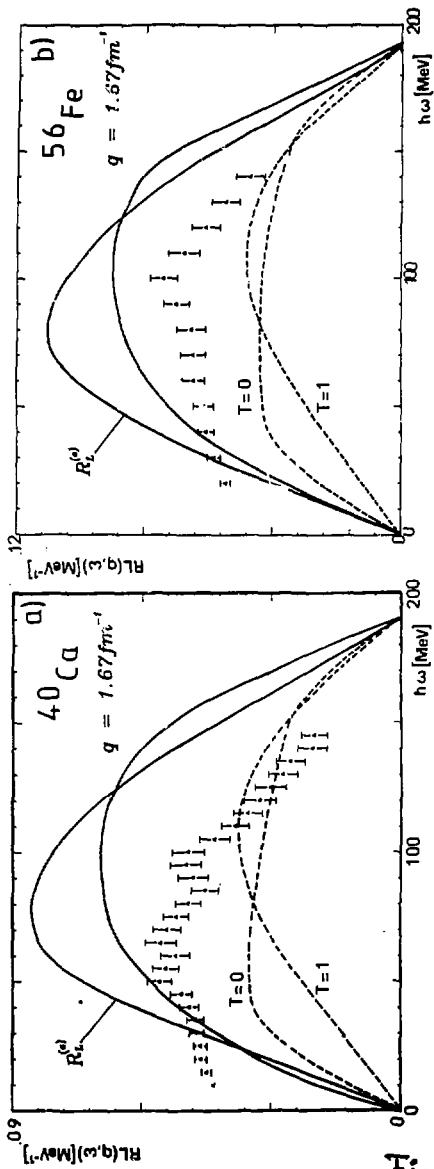


Fig. 4b

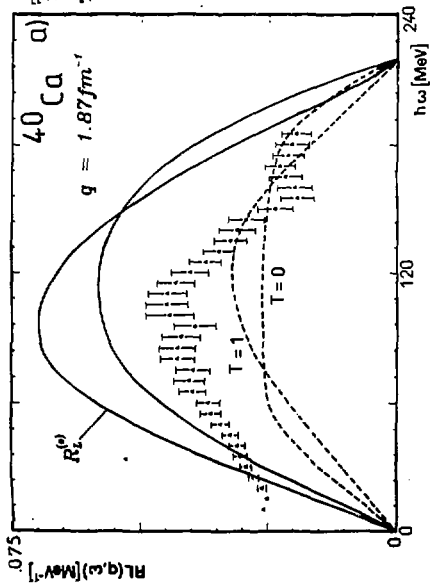
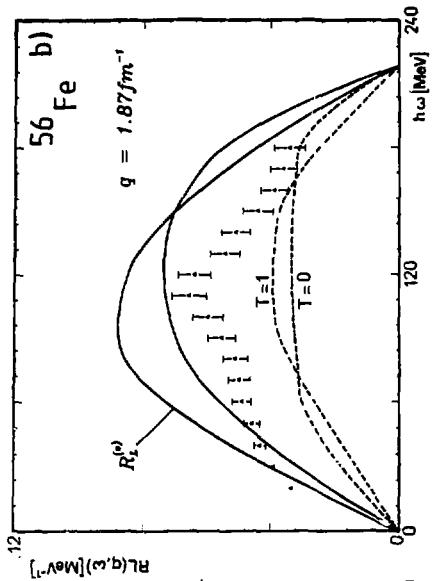


Fig. 4c

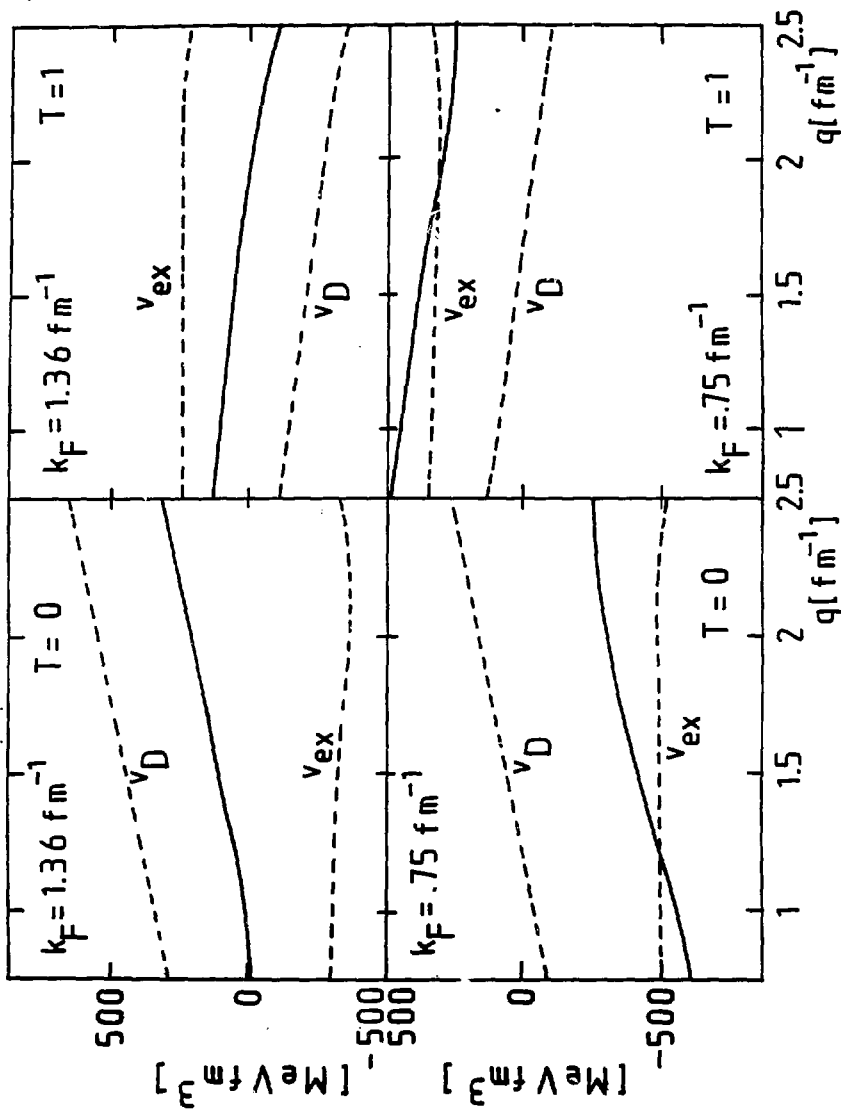


Fig. 5

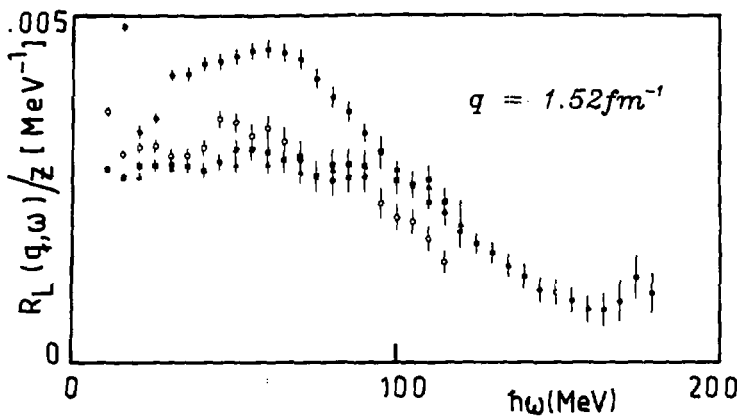


Fig. 6

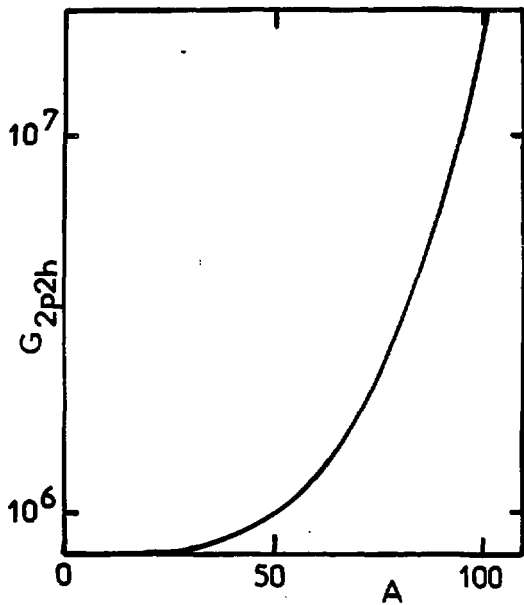


Fig. 7