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THE PION MASS SPLITTING IN THE STANDARD MODEL

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ABSTRACT

We study the mass difference of pions in the standard model, paying particular attention to the short distance QCD contribution,which has been handled following the prescription of Brodsky, de Téramond and Schmidt, leading to a calculable result.

The computation of mass differences of hadrons within the same isomultiplet shows an interesting link between the standard model interactions. The weak interaction is as important as the electromagnetic one at high energies, i.e. at high momentum the effects of weak gauge bosons and the photon have the same strength. On the other hand, quantum chromodynamics (QCD), as the theory of strong interaction, with its property of asymptotic freedom, enables us to study the convergence or divergence of the mass splitting produced by the electroweak interactions. Under this point of view we have to separate the $\Delta I = 1$ from the $\Delta I = 2$ mass differences, since in the second case, such as $m_{\pi^+} - m_{\pi^0}$, the quark mass contributions cancel. For the $\Delta I = 1$ mass differences, such as $m_p - m_n$, the effect (and origin) of quark masses is important, and, maybe, an extension of the standard model should be considered to reach a complete understanding of these mass splittings.

Recently Machet and the author[1] discussed the mass splitting of pions in the Glashow-Weinberg-Salam model, where the weak neutral boson (Z^0) gives an extra convergence to Δm_π $(= m_{\pi^+} - m_{\pi^0})$. In that paper a straightfoward method to compute mass differences has been settled down, and some points, such as gauge invariance, have been discussed in detail. However, the final result has been obtained only at the chiral limit; for massive pions the theory is as far from a comprehensive result as it was at the time of the pioneer work of Das et al[2] . In this letter we intend to improve our understanding of Δm_π for massive pions, making use of the prescription proposed by Brodsky, de Téramond and Schmidt[3] to deal with the QCD contribution (proportional to quark masses). This prescription is crucial for our result, and it has been fundamented by Craigie et al[4] (see also refs. 5 and 6), leading to a finite value for Δm_π, which is our main result.

Following ref.1 we start with the propagator $(\psi^{(i)}(q^2))$ of the covariant divergences with respect to $SU(2)_L \times U(1)_Y$ of the hadronic currents ($A^{\mu(i)}$, where $i = 1 + i2$, (3) shall stand for the charged (neutral) axial-vector pion current),

$$\psi^{(i)}(q^2) = i \int d^4x \, e^{iq\cdot x} \langle 0 | T \, D_\mu A^{\mu(i)}(x) D_\nu A^{\nu(i)+}(0) | 0 \rangle, \quad (1)$$

which, by PCAC, may also be written as

$$\psi^{(1+i2)}(q^2) \simeq \frac{2 f_{\pi^+}^2 m_{\pi^+}^4}{-q^2 + m_{\pi^+}^2},$$

$$\psi^{(3)}(q^2) \simeq \frac{f_{\pi^0}^2 m_{\pi^0}^4}{-q^2 + m_{\pi^0}^2}. \quad (2)$$

Developing eq.(1) in the standard model, and equalling it to (2) at q=0, we obtain[1]

$$2 f_\pi^2 (m_{\pi^+}^2 - m_{\pi^0}^2) \simeq e^2 \int \frac{d^4q}{(2\pi)^4} \left[\left(D^\gamma_{\mu\nu}(q) - D^Z_{\mu\nu}(q) \right) \left(2 \Pi_V^{(3)\mu\nu}(q) - \Pi_A^{(1+i2)\mu\nu}(q) \right) \right], \quad (3)$$

where we neglected the contributions of Higgs bosons and quark condensates. $D^{\gamma}_{\mu\nu}$ ($D^{Z}_{\mu\nu}$) is the photon (neutral weak boson) propagator and $\Pi^{(i)\mu\nu}_{V(A)}$ are the two-point functions of the vector and axial currents, which can be decomposed as,

$$\Pi^{\mu\nu}(q^2) = -\left(g^{\mu\nu}q^2 - q^{\mu}q^{\nu}\right)\Pi^{1}(q^2) + q^{\mu}q^{\nu}\Pi^{0}(q^2). \quad (4)$$

Finally, inserting (4) into (3) and working in Landau gauge we obtain,

$$2f_\pi^2\left(m^2_{\pi^+} - m^2_{\pi^0}\right) \approx$$
$$\approx 3ie\int\frac{d^4q}{(2\pi)^4}\left(\frac{1}{q^2} - \frac{1}{q^2 - M_Z^2}\right)\left(2\Pi^{(3)\,1}_{V}(q^2) - \Pi^{(1+i2)\,1}_{A}(q^2)\right). \quad (5)$$

In ref.1 we have worked at the chiral limit, i.e. neglected the short distance contribution proportional to quark masses, and saturated the Π's by the low-energy resonances ρ and A_1, what has led us to[1],

$$f_\pi^2\left(m^2_{\pi^+} - m^2_{\pi^0}\right) \approx 3\alpha\left(\frac{m_\rho^4}{g_\rho^2}\ln\frac{m_\rho^2/g_\rho^2}{m_\rho^2/g_\rho^2 - f_\pi^2}\right) \quad , \quad (6)$$

and

$$\Delta m_\pi \approx 4.95\ \mathrm{MeV} \quad . \quad (7)$$

As claimed in ref.1 when we compare (1) to (2) at $q = 0$, we are forced to compute Δm_π at the chiral limit, otherwise the computation is not gauge invariant. Actually, we have already made another approximation when we neglected the continuum contribution to (2). For massive pions any calculation not performed at pion mass-shell will inevitably have a gauge dependence of $O(\alpha m_\pi)$

(the gauge term in (5) has disappeared with the choice of Landau gauge, it also does not contribute at the chiral limit). In conjunction with these aspects we could recognize in eq.(5) a peculiar combination of quark self-energies[1,7].

We intend to take into account the short distance QCD contribution, therefore we introduce a scale $q^2 = Q_o^2$ in the integral (5), separating the large and short distance contribution to Δm_π ; the first is dominated by the ρ and A_1 resonances (denoted by Δ_{RES})and the second described by perturbative QCD (and indicated by Δ_{SD}) entailing

$$\Delta m_\pi = \Delta_{RES} + \Delta_{SD} \quad . \qquad (8)$$

It is easy to verify that cutting the integral (5) at $q^2 = Q_o^2$, the low energy term will be given by

$$\Delta_{RES} \approx \frac{3\alpha}{2 f_\pi^2 m_\pi} \left(\frac{m_\rho^4}{g_\rho^2} \ln \frac{m_\rho^2/g_\rho^2}{m_\rho^2/g_\rho^2 - f_\pi^2} \right) \qquad (9)$$

$$+ \; O\left(\frac{\alpha}{f_\pi^2 m_\pi} \frac{m_{RES}^4}{g_{RES}^2} \ln \frac{m_{RES}^2/M_Z^2 + m_{RES}^2 Q_o^2}{1 + m_{RES}^2 Q_o^2} \right),$$

where m_{RES} is a resonance mass (ρ or A_1).Obviously, for moderately large values of Q_o^2 we get $\Delta_{RES} \approx 4.9$ MeV (= eq. (7)). The resonance region is weakly dependent on the choice of Q_o^2 .

To determine Δ_{SD} we recall that in QCD we have[8]

$$2 \Pi_V^{(3)\,1}(q^2) - \Pi_A^{(ii2)\,1}(q^2) \approx \frac{3}{8\pi} \frac{(m_u + m_d)^2}{q^2} , \quad (10)$$

and assuming

$$m_u \approx m_d \approx \hat{m} \approx 10 \text{ MeV} \quad ,$$

we get

$$\Delta_{SD} \approx \frac{9\alpha}{32\pi^2} \frac{M_Z^2}{f_\pi^2 m_\pi} \int_{Q_0^2}^{\infty} dq^2 \frac{m^2(q^2)}{(q^2 + M_Z^2)} \quad . \qquad (11)$$

As will be justified in the following, the quark mass ($m(q^2)$) appearing in (11) is the running mass

$$m(q^2) \approx \hat{m}(q_0^2) \left[\alpha_s(q^2) / \alpha_s(q_0^2) \right]^{\gamma} , \qquad (12)$$

where $\alpha_s(q^2)$ is the QCD running coupling constant, $\gamma = 12/(33 - 2m_f)$ and m_f is the number of flavors, leading us to

$$\Delta_{SD} \approx \frac{9\alpha}{32\pi^2} \frac{M_Z^2 \hat{m}^2}{f_\pi^2 m_\pi} \int_{\hat{Q}_0^2}^{\infty} dq^2 \frac{(\ln q^2/\Lambda_{QCD}^2)^{-2\gamma}}{(q^2 + M_Z^2)} \cdot (13)$$

The use of the running mass in (11) is a consequence of applying the Schwinger-Dyson procedure for quark self-energies as prescribed by Brodsky et al[3]. As shown in ref. 1, Δm_π can be related to the self-energy of quarks forming chiral currents with pion quantum numbers, therefore the translation of their results[3] to our case is imediate(we neglected corrections of $O(\alpha_s \Delta m_\pi)$). Performing the integral in (13) we obtain

$$\Delta_{SD} \simeq -\frac{9\alpha}{32\pi^2}\,\frac{\hat{m}^2 Q_o^2}{f_\pi^2\, m_\pi}\left(\ln\frac{Q_o^2}{\Lambda_{QCD}^2}\right)^{-2\gamma}, \qquad (14)$$

or

$$\Delta_{SD} \simeq -\frac{9\alpha}{32\pi^2}\,\frac{Q_o^2\, m^2(Q_o^2)}{f_\pi^2\, m_\pi}\,. \qquad (15)$$

The integral leading to (15) is finite if $2\gamma > 1$,

or $m_f \geqslant 5$.[F1]

The conciliation of the Brodsky et al[3] procedure with earlier results stating that the bare mass should appear in (11), has been well explained by Craigie et al[4] and Dine[5]. The main point is that in earlier works, the Cottingham formula[9] has been used to compute electro(weak) mass shifts, and this one assumes that the photon (weak boson) integration is performed last (i.e. after the summation of strong interactions), and this may not be valid! Imposing that the physical result should not depend on which order strong or electroweak interactions are taken into account, Craigie et al[4] confirmed the hypothesis of ref.3. For pratical purposes, following the work of refs. 4 and 5, we may use the running quark mass when computing electroweak mass shifts, no matter this leads to a finite result or not, because even in this last case (i.e.,when the integrai is not convergent), the renormalized result is the analytic continuation of the integral under consideration[4,6].

Unfortunately the short distance behavior of Δm_π is highly dependent on the cut-off Q_0^2, and we must have $\partial \Delta m_\pi / \partial Q_0^2 = 0$ reflecting the independence of the physical result on the choice of subtraction scales. Actually, in a complete determination of Δm_π we should expect only one mass scale, m_π, and ultimately this one would be written in terms of Λ_{QCD}; obviously we continue to be far from this point. We could expect that with a better knowledge of the transition between the nonperturbative resonance region and the perturbative one, the abrupt change in the behavior of $(2\Pi_V^{(3)} - \Pi_A^{(1+i2)})$ may be softened. Anyhow, we can estimate Δ_{SD} assuming $Q_0^2 \approx O(1\,GeV^2)$ as a good value above which we can apply perturbative QCD, doing so we get

$$\Delta_{SD} \approx -0.02 \ MeV \quad .$$

Compared to Δ_{RES} this is a quite small value, and the negative signal is welcomed, because Δ_{RES} is slightly larger than the experimental value of Δm_π $(\sim 4.6\,MeV)$, and the short distance contribution tends to compensate this shift, bringing the theoretical value of Δm_π near to the experimental one.

We recall that we continue to be far from a complete calculation of Δm_π for massive pions. The steps followed from eq.(1) to (3) involve uncertainties of $O(\alpha\, m_\pi)$, and the short distance behavior is highly dependent of an arbitrary scale (Q_0^2). The fact that the short distance QCD contribution is small (considering a Q_o^2 value of O (1 GeV^2)) , may be a justificative for the good result obtained at the chiral limit by Das et al[2]. Finally, despite the above problems, the possibility to have a finite result may be enough to stimulate deeper studies about this quite old subject.

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Footnote

F_1 - The above analysis follows closely the work of ref.4. Notice that the dependence on m_q^2 in eq. (10) (proper of a $\Delta I = 2$ mass difference) is fundamental to the convergence of eq. (11) for $m_f \geqslant 5$.

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