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SCHWINGER MODELS IN ARBITRARY GAUGES AND AT FINITE TEMPERATURE

George Thompson

and

Ruibin Zhang

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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In yielding non-perturbative solutions to the Schwinger-Dyson equations the gauge technique (Delbourgo and West 1977a) has proved invaluable. The method has conventionally made use of the spectral representation of the fermion propagator. While this is very practical in covariant gauges in the non-covariant gauges this situation makes calculations exceedingly difficult. This can be easily appreciated by comparing the original work on four-dimensional qed (Delbourgo and West 1977b) and that done later in the axial gauges (Delbourgo and Phocas-Cosmetatos 1979). The reason for the difficulties is precisely because of the non-covariant nature of the calculation, where one encounters non-trivial spectral representations.

When dealing with the finite temperature Schwinger model (in the Landau gauge) Stam and Visser (1985) circumvented the problem of non-covariant spectral functions (non-covariance arises because of the rather different handling of the time and space co-ordinates). The idea is to use the two Ward identities that are available and to solve the position space Schwinger-Dyson equations directly, in this way it is possible to avoid the introduction of the spectral functions. (This will become clearer in the following).

In this letter, we extend the results of Stam and Visser and so are able to give a solution of the Schwinger models in arbitrary gauges and at finite temperature (Zhang 1985).

Gauge and chiral invariance of the theory (Delbourgo and Thompson 1982; Stam 1983; Thompson 1983) give rise to two identities (for zero temperature) which allow us to express the three-point amplitude (with photon leg amputated) as follows:

$$G_\mu(p, p-k) = \frac{k_\mu}{k^2}\left[S(p-k) - S(p)\right] + i\,\frac{\epsilon_{\mu\nu}k^\nu}{k^2}\left[S(p)\gamma_5 + \gamma_5 S(p-k)\right]. \qquad (1)$$

The relevant Schwinger-Dyson equation is

Let us search for a solution for the fermion propagator that preserves chiral symmetry

$$\{ S_A , \gamma_5 \} = 0 ,$$

consequently Eq.(3) takes the simple form

$$1 = \not p\, S_A(p) + i g^2 \int d^2k \; \gamma_\mu \frac{\not k}{k^2} \gamma_\nu \, D^{\mu\nu}(k) \, S_A(p+k), \tag{4}$$

which in position space becomes

$$\delta(x) = i \not\partial S_A(x) + i g^2 \int d^2k \; \gamma_\mu \frac{\not k}{k^2} \gamma_\nu \, D^{\mu\nu}(k) \, e^{ikx} \; S_A(x). \tag{5}$$

To solve (5) it is expedient to adopt the decomposition

$$S_A(x) = S_0(x) \, \exp(Q(x))$$

where $S_0(x)$ is the free propagator and $Q(x)$ satisfies $[Q,\gamma_5] = 0$ and $Q(0) = 0$. (5) then reduces to

$$\not\partial [\, S_0^{-1}(x) \, Q(x) \, S_0(x)] = - g^2 \int d^2k \; e^{ik\cdot x} \, \gamma_\mu \frac{1}{\not k} \gamma_\nu \, D^{\mu\nu}(k) . \tag{6}$$

The solution to this equation is

$$S_0^{-1}(x) \, Q(x) \, S_0(x) = i g^2 \int d^2k \; \frac{1}{\not k} \gamma_\mu \frac{1}{\not k} \gamma_\nu \, D^{\mu\nu}(k) \left(e^{ik\cdot x} - 1 \right) . \tag{7}$$

So finally we arrive at the desired result

$$S_A(x) = S_0(x) \exp \left[\frac{1}{x\cdot\gamma} \, i g^2 \int d^2k \; \frac{1}{\not k} \gamma_\mu \frac{1}{\not k} \gamma_\nu \, D^{\mu\nu}(k) \left(e^{ik\cdot x} - 1 \right) x\cdot\gamma \right] . \tag{8a}$$

However, (8a) is not the unique solution of (3). In fact other solutions can be obtained by letting

$$S(x) = S_A(x) + S_S(x) \tag{9}$$

with

$$[S_S, \gamma_5] = 0$$

and solving (3) for S_s. Some manipulations lead us to

$$S_S(x) = C \exp\left(ig\int d^2k \frac{1}{k^2} D_\mu{}^\mu(k) e^{ik\cdot x}\right), \tag{8b}$$

where C is an arbitrary constant (and possibly infinite in order to make $S_s(x)$ finite).

In deriving (8a) and (8b) we have not specified the gauge, nor have we elaborated on the complete solution to the photon propagator. The gauge is clearly arbitrary, and now we establish that the complete photon polarization is determined by the one loop bubble diagram. Making use of (8), (1) may be written as

$$\begin{aligned} G_\mu(p, p-k) &= \int d^2q \frac{1}{\not p - \not q} \gamma_\mu \frac{1}{\not p - \not q - \not k} F(q) \\ &+ \frac{1}{k^2}\left\{ k_\mu [S_S(p-k) - S_S(p)] + i\epsilon_{\mu\nu}k^\nu\gamma_5[S_S(p-k) + S_S(p)]\right\}, \end{aligned} \tag{10}$$

where $F(q) = \int d^2x\, e^{-iq\cdot x}\, e^{Q(x)}$. Substituting (10) into the vacuum polarization tensor and noting that the second term does not contribute, we obtain

$$\Pi_{\mu\nu}(k) = ig^2\int d^2p\, \mathrm{tr}\left[\int d^2q \frac{1}{\not p - \not q}\gamma_\mu \frac{1}{\not p - \not q - \not k} F(q)\gamma_\nu\right].$$

Translating variables (which is allowed in this integral) and using Q(0) = 0 one arrives at

$$\begin{aligned} \Pi_{\mu\nu}(k) &= ig^2\int d^2p\, \mathrm{tr}\left[\frac{1}{\not p}\gamma_\mu \frac{1}{\not p - \not k}\gamma_\nu\right] \\ &= \mu^2(\xi\eta_{\mu\nu} - k_\mu k_\nu/k^2). \end{aligned} \tag{11}$$

$\mu = \frac{g}{\sqrt{\pi}}$ and the extra parameter ξ arises through the various choices of regularization schemes (Hagen 1967 ; Thompson 1983). With $\xi = 0$ in the covariant gauges (parameterized by a) and the light cone gauge particularly elegant solutions arise. These are, respectively,

$$S(x) = a^{-\frac{1}{4}} S_0(x) \exp\left\{\frac{1}{2}\left[K_0(\mu\sqrt{-x^2}) - K_0(\mu\sqrt{-ax^2})\right\}\right.$$
$$- \frac{C}{x^2} \exp\left\{-\frac{1}{2}\left[K_0(\mu\sqrt{-x^2}) + K_0(\mu\sqrt{-ax^2})\right\}\right.$$

and

$$S(x) = S_0(x) \exp\left\{\frac{n\cdot x\, x\cdot\gamma}{n\cdot x}\left[K_0(\mu\sqrt{-x^2}) + \ln(\mu\sqrt{-x^2})\right]\right\} + \text{Const.}$$

Clearly, the above analysis (till (10)) may be carried through for the finite temperature version of the theory, one arrives at a simple generalization of the result of Stam and Visser (1985)

$$S_A(x) = S_0(x) \exp\left\{-\frac{g^2}{\beta\, x\cdot\gamma}\sum_{k_0}\int dk_1 \frac{1}{k\!\!\!/}\gamma_\mu \frac{1}{k\!\!\!/}\gamma_\nu D^{\mu\nu}(k)\left(e^{ik\cdot x} - 1\right) x\cdot\gamma\right\}$$

and

$$S_s(x) = C \cdot \exp\left\{-\frac{g^2}{\beta}\sum_{k_0}\int dk_1 \frac{1}{k^2} D^\mu_\mu(k)\, e^{ik\cdot x}\right\}.$$

We would like to stress that in the gauge where $D^{\mu\nu}(k)$ is simply proportional to $\eta^{\mu\nu}$, $Q(x)$ vanishes and one finds that the complete fermion propagator coincides with the bare one, except for the symmetric contributions. The fact that by solving the Schwinger-Dyson equation, symmetric contributions to the fermionic propagator can arise was first noted by Stam (1983) in the context of the gauge technique spectral representation. The fact that this solution emerges at all is rather surprising, as in path integral and perturbative solutions of the propagator, the symmetric term is absent. This may mean that some boundary condition has been overlooked.

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