CTH-IEFT/PP-1986-09 ISSN
SE8700058

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TIME EVOLUTION OF THE VELOCITY DISTRIBUTION OF NEUTRAL BEAM INJECTED IONS HEATED BY ICRH IN A TWO COMPONENT PLASMA

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Abstract

The time evolution of the distribution function of the beam injected particles in the presence of ICRH in a two component plasma is determined. Consideration is restricted to the time development during two completementary time periods: (i) the early time period, i.e. $0 \leq t \ll \tau_s$, and (ii) the quasi-steady-state, i.e. $t > \tau_s$, where τ_s is the slowing-down time for beam ion-electron collisions. Explicit analytical solutions are obtained for anisotropic as well as isotropic beam injection.

1. Introduction

In order to achieve non-negligible fusion reaction rates in a thermonuclear plasma, an average ion temperature of 10 keV must be obtained. In a Tokamak, the inherent ohmic heating is not sufficient to rise the plasma temperature to the required level, and auxiliary heating method must be used. Several heating have been examined both theoretically and experimentally. Two of the leading methods are neutral beam injection (NBI) heating and ion cyclotron resonance heating (ICRH).

NBI has been applied successfully, e.g. a plasma of several keV has been obtained in the PLT device, [1]. An advantage with NBI is that, in addition to providing bulk plasma heating, it supplies a non-Maxwellian high energy ion tail, which may decisively contribute to the fusion rate. This is especially so since the neutral beam injection energy typically falls at the energies at which the fusion reaction rate peaks.

On the other hand, ICRH has recently made remarkable experimental progress, [2-5], and is becoming a promising candidate for efficient heating of plasma ions. The inherent advantage of the high energy tail in NBI heating is parallelled in ICRH by selectively heating a minority of the plasma ions or by heating at the cyclotron harmonic, where preferentially the high energy ions absorb the wave energy, [6].

Recently, there has been considerable interest in the possibility of heating a Tokamak with a combination of ICRH and NBI, [7-10]. This scenario will be of particular importance in the near future, since it is expected that several Tokamaks, including JET, will operate with both heating methods simultaneously. It has been shown theoretically, [8], that the advantages of both heating schemes can be combined by tuning the ICRH to the ion cyclotron frequency of the neutral beam injected ions, thus causing a significant enhancement of the high energy tail.

Much effort has been devoted to analytical as well as numerical investigations of the influence of RF-heating on the stationary distribution function of the beam ions and to the calculation of the resulting fusion power multiplication factor, cf [8].

Since a detailed knowledge of the distribution function is essential to theoretical as well as experimental investigation of the heating process, the purpose of this work is to determine the time evolution of the distribution function of the beam injected particles in the presence of ICRH in a two component plasma. The analysis is based on the time dependent Fokker-Planck equation including a quasilinear RF-diffusion operator. Consideration is restricted to the time development of the beam distribution function during two complementary time periods: (i) the early time period, i.e. $0 \leq t \ll \tau_s$, and (ii) the quasi-steady state, i.e. $t > \tau_s$ where τ_s is the slowing-down time for beam ion-electron collisions. At early times the development of the distribution function is dominated by particle-wave interactions. This stage should be of interest in connection with problems like RF-induced enhanced sawteeth-activity, [11], or excitation of velocity space microinstabilities during the heating process, [12].

In quasi-steady state the evolution of the distribution function is dominated by collisional effects. It is shown that in this stage a group of thermalized beam ions with a Maxwellian distribution appears. The number of particles in this group will increase in time until steady-state is reached. Another group of beam ions, those that have not thermalized forms a time independent non-Maxwellian "tail" in the distribution function. Explicit analytical solutions for the beam distribution function are obtained for two complementary beam injection scenarios: anisotropic and isotropic injection of the beam ions.

II. Fokker-Planck equation

We shall concentrate on neutral beam injected ions in a two-component plasma which are directly heated by ICRH tuned to the ion cyclotron frequency of the injected ions. The Fokker-Planck equation for the distribution function, f, of these ions can be written as

$$\frac{\partial f}{\partial t} = C(f) + Q(f) - \frac{1}{\tau_{cx}} f + S \qquad (1)$$

where C(f) is the collision operator, Q(f) is the quasi-linear RF-diffusion operator, τ_{cx} is the charge-exchange loss time, and S is the source of the injected particles. We assume that the neutral injection does not affect the equilibrium of the plasma. This requires $n_b \ll n$, where n_b and n are the density of beam ions and plasma ions, respectively. Hence, we consider the plasma distribution to be Maxwellian and disregard collisions between beam particles themselves.

In order to describe the influence of the ICRH on the distribution function we consider only the fundamental ion cyclotron resonance of a small minority ion component in a thermal background plasma and use the quasi-linear diffusion operator derived in Ref. [13]. Assuming the limit of small Larmor-radius and, for simplicity, neglecting particle trapping effects when averaging over toroidal surfaces the explicit forms of the collision and RF-diffusion operators are:

$$C(f) = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{-v^2\alpha(v)f + \frac{1}{2}\frac{\partial}{\partial v}\left[v^2\beta(v)f\right]\right\} +$$

$$+ \frac{\gamma(v)}{4v^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial f}{\partial \mu}\right] \qquad (2)$$

and

$$Q(f) = \frac{3}{2}K\frac{1}{v^2}\left\{(1-\mu^2)\frac{\partial}{\partial v}\left[v\frac{\partial}{\partial v}(vf)\right] +\right.$$

$$+ \frac{\partial}{\partial \mu}\left[\mu(1-\mu^2)\frac{\partial}{\partial \mu}(\mu f)\right] -$$

$$- \frac{\partial}{\partial \mu}\left[\mu(1-\mu^2)\frac{\partial}{\partial v}(vf)\right] -$$

$$\left. - (1-\mu^2)\frac{\partial}{\partial \mu}\left[\mu\frac{\partial}{\partial v}(vf)\right]\right\} \qquad (3)$$

where v is the velocity and $\mu = v_\parallel/v$ is the cosine of the pitch angle. The collision coefficients α, β, and γ describe dynamical friction on the background species, energy diffusion and pitch angle scattering, respectively. The constant K is proportional to the rf power absorbed per unit volume. For further details concerning notations see Refs. [6,14].

The source, S, of injected particles is assumed to supply almost energetic particles with velocity v_o. This means that $S(v,\mu)$ can be written as

$$S(v,\mu) = \frac{S_o}{2\pi v_o^2}\,\delta(v-v_o)K(\mu) \qquad (4)$$

where S_o is the number of particles injected per second and cm^{-3} and the function $K(\mu)$ which represents the angular spread satisfies

$$\int_{-1}^{1} K(\mu)d\mu = 1 \qquad (5)$$

Finally, the charge-exchange time, τ_{cx}, appearing in eq. (1) is taken to be independent of energy (which is a good approximation for energies in the range 1-30 keV).

An approach, which has been widely used in analytic studies of the effects of ICRH on the velocity distribution function, [6,14], is to expand solutions of the Fokker-Planck equation in Lagendre polynomials and to keep only the lowest order term, viz.

$$\langle f\rangle = \frac{1}{2}\int_{-1}^{1} f(v,\mu)d\mu \qquad (6)$$

The approach is justified of the distribution function is only weakly anisotropic. Consequently, this approach is a legitimate one for the low energy part of the distribution function and deteriorates in the high energy tail region, which becomes strongly anisotropic in the presence of significant RF power absorption.

In the present analysis we will apply a modified approach, cf [15] to derive the pitch angle averaged ion distribution in the presence of ICRH. The method provides a consistent description of the gradual transition from the almost isotropic low energy part of the distribution to the anisotropic high energy tail distribution. Furthermore, it does not depend on truncating an expansion and is consequently not restricted to almost isotropic situations.

In order to derive an equation for $\langle f \rangle$ we integrate the Fokker-Planck equation (1) over μ. In the collision operator, the pitch angle scattering term vanishes, and we obtain

$$\frac{\partial\langle f\rangle}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left\{\left[-\alpha v^2 + \frac{1}{2}\frac{\partial}{\partial v}(\beta v^2)\right] + \right.$$

$$\left. + \frac{1}{2}\beta v^2 \frac{\partial}{\partial v}\right\}\langle f\rangle + \frac{3}{2}\frac{K}{v^2}\left[v^2 \frac{\partial}{\partial v}\langle(1-\mu^2)f\rangle + \right.$$

$$\left. + v\langle(1-3\mu^2)f\rangle\right] - \frac{1}{\tau_{cx}}\langle f\rangle + \frac{S_o}{2\pi v_o}\delta(v-v_o) \qquad (7)$$

Eq. (7) couples the zero (μ^o) and second (μ^2) order moments of f. By successively multiplying the Fokker-Planck equation with μ^{2n} and integrating we can obtain an infinite coupled system of equations determining the moments $\langle\mu^{2n}\rangle$. However, this is not an analytically tractable approach.

Instead, we follow Ref. [15] and decouple the zero order moment from the second order moment by introducing

$$\langle \mu^2 f \rangle \equiv \mu^2_{eff}(v)\langle f \rangle \tag{8}$$

The main role of $\mu^2_{eff}(v)$ is to describe the anisotropy of the distribution function. According to Ref. [15] an appropriate model for $\mu^2_{eff}(v)$ should satisfy the conditions

$$\mu^2_{eff}(0) \equiv \frac{1}{3}$$

$$\mu^2_{eff}(\infty) \rightarrow 0. \tag{9}$$

This means that $\mu^2_{eff}(v)$ should consistently reproduce the low and high energy asymptotic values, i.e. $\mu^2_{eff} = \frac{1}{3}$ for an isotropic distribution and $\mu^2_{eff} = 0$ for a strongly anisotropic distribution.

Considering $\mu^2_{eff}(v)$ as a known function of v and introducing $F \equiv \langle f \rangle$ we can write eq. (7) as

$$\frac{\partial F}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\{v^2[\frac{1}{2}\beta + \frac{3}{2}K(1-\mu^2_{eff})]\frac{\partial F}{\partial v} +$$

$$+ [-\alpha v^2 + \frac{1}{2}\frac{\partial}{\partial v}(\beta v^2) + \frac{3}{2}Kv(1-3\mu^2_{eff} - v\frac{\partial^2_{eff}}{\partial v})]F\}$$

$$-\frac{1}{\tau_{ex}}F + \frac{S_o}{2\pi v_o^2}\delta(v-v_o) \tag{10}$$

Eq. (10) requires a statement of initial condition. If we consider that the beam is injected at t=0, then $F(v,t=0)=0$ and eq. (10) describes the subsequent evolution in time of the beam distribution function. Accordingly the beam density, $n_b = 2\pi \int_0^\infty v^2 F dv$, varies with time as

$$\frac{n_b}{n} = \frac{\tau_{ex}}{\tau_b}\left[1-\exp(-t/\tau_{ex})\right] \ll 1 \tag{11}$$

where $\tau_b = n/S_o$ is the production time of beam ions. It follows from eq. (11) that the steady-state value of the beam density is given by $n_b(\infty)/n=\tau_{ex}/\tau_b$.

III. Initial evolution of the beam distribution function

During the initial stage of the beam injection, when $0 \leq t \ll \tau_s$, where τ_s is the slowing-down time for beam ion-electron collisions, the evolution of the distribution function is determined predominantly by the source term and the RF diffusion. Neglecting the collision terms and the charge exchange loss term, eq. (1) reduces to, cf [16],

$$\frac{\partial}{\partial t} f(v_\perp,v_\parallel,t) = \frac{3}{2} K \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp \frac{\partial}{\partial v_\perp} f(v_\perp,v_\parallel,t)\right]+S(v_\perp,v_\parallel) \tag{12}$$

where $v_\perp$ and $v_\parallel$ are the ion velocity components perpendicular and parallel to the magnetic field, respectively. The solution of eq. (12) is given by

$$f(v_\perp,v_\parallel,t) = \frac{1}{3K}\int_0^t \frac{dz}{z}\int_0^\infty d\xi\xi S(\xi,v_\parallel)\exp\left(-\frac{v_\perp^2+\xi^2}{6KZ}\right)I_o\left(\frac{v_\perp\xi}{3KZ}\right) \tag{13}$$

where $I_o(x)$ is the modified Bessel function of order zero.

Assuming now an almost perpendicular beam injection, the source function can be written in the form

$$S(v_\perp,v_\parallel) = \frac{S_o}{2\pi v_{\perp o}}\,\delta(v_\perp-v_{\perp o})\delta(v_\parallel) \tag{14}$$

which substituted into (13) gives

$$f(v_\perp,v_\parallel,t) = \frac{S_o}{6\pi K}\,\delta(v_\parallel)\int_0^t \frac{dz}{z}\exp\left(-\frac{v_\perp^2+v_{\perp o}^2}{6KZ}\right)I_o\left(\frac{v_\perp v_{\perp o}}{3KZ}\right) \tag{15}$$

Averaging the distribution (15) over the pitch angle μ we obtain for $v > 0$

$$F(v,t) = \langle f\rangle = \frac{S_o}{6\pi Kv}\int_0^t \frac{dz}{z}\exp\left(-\frac{v^2+v_o^2}{6KZ}\right)I_o\left(\frac{vv_o}{3KZ}\right) \tag{16}$$

which is exactly the result if we solve eq. (10) in the absence of collisions and charge exchange loss and by assuming $\mu_{eff}^2=0$. Since we are interested in the short time limit we assume

$$t \ll \min\left(vv_o(3K;\ (v^2+v_o^2)/6K\right) \tag{17}$$

and use the asymptotic expansion of $I_o(x)$ to obtain

$$F(v,t) = S_o\left(\frac{t}{6\pi^3 K v_o v^3}\right)^{1/2} \left\{\exp\left[-\frac{(v_o-v)^2}{6Kt}\right) \pm\right.$$

$$\left. \pm (v_o-v)\left(\frac{\pi}{6Kt}\right)^{1/2}\left[1\pm\phi\left(\frac{v_o-v}{\sqrt{6Kt}}\right)\right]\right\} \qquad (18)$$

where $\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt$ and the signs "+" and "-" correspond to the solutions in regions $v \geq v_o$ and $0 < v \leq v_o$, respectively. It can be seen from eq. (18) that the ions injected at the velocity $v = v_o$ are, due to the RF-induced velocity space diffusion, spread toward lower as well as higher energies. We note that particles will appear at thermal energies on the time scale $\tau_{RF} = v_o^2/6K$, which implies an anomalous "slowing down" in situations when $\tau_{RF} < \tau_s$.

IV. Quasi-steady state of the beam distribution function

When $t \lesssim \tau_s$, the collision terms in eq. (10) will become important and when $t > \tau_s$, it will be possible to distinguish a group of beam ions with a Maxwellian distribution. The number of particles in this group will increase in time until steady-state is reached due to the charge-exchange losses. On the other hand, the plasma will contain non-thermalized beam ions whose distribution function at $t > \tau_s$ does not depend on time but is determined by the balance between the systematic arrival of beam ions at the point $v = v_o$ and the diffusion flow towards low energy as a result of the RF diffusion and the collisions between the beam ions and the plasma electrons and ions. Thus, it is natural to call this state at $t > \tau_s$ a quasi-steady state and the solution of eq. (10) can be written in the following form, cf. [17],

$$\frac{F(v,t)}{n} = h(v)\left[\frac{\tau_{ex}}{\tau_b}\left(1-e^{-t/\tau_{ex}}\right)-\Lambda\right]+g(v) \tag{19}$$

where the term proportional to h(v) describes the distribution of the thermalized particles and g(v) is the stationary non-thermalized part of the beam distribution. Integrating eq. (19) over velocity space and using the relation (11) the constant Λ is found to be

$$\Lambda = \frac{\int_0^\infty g(v)v^2 dv}{\int_0^\infty h(v)v^2 dv} \tag{20}$$

We substitute (19) into eq. (10) and separate with respect to the time variation. This yields the following two equations for g(v) and h(v):

$$\left[\frac{1}{2}\beta+\frac{3}{2}K(1-\mu_{eff}^2)\right]v^2\frac{dh}{dv}+\left[-\alpha v^2+\frac{1}{2}\frac{d}{dv}(v^2\beta)+\right.$$

$$\left.+\frac{3}{2}Kv\left(1-\mu_{eff}^2-v\frac{d\mu_{eff}^2}{dv}\right)\right]h = 0 \tag{21}$$

and

$$\frac{1}{v^2}\frac{d}{dv}\{[\frac{1}{2}\beta + \frac{3}{2}K(1-\mu^2_{eff})]v^2\frac{dg}{dv} + [-\alpha v^2 + \frac{1}{2}\frac{d}{dv}(v^2\beta) +$$

$$+\frac{3}{2}Kv(1-3\mu^2_{eff} - v\frac{d\mu^2_{eff}}{dv})]g\} = h(v)(\frac{1}{\tau_b} - \frac{\Lambda}{\tau_{cx}}) +$$

$$+\frac{1}{\tau_{cx}}g - \frac{1}{2\pi v_o^2\tau_b}\delta(v-v_o) \qquad (22)$$

In order to solve eqs. (21) and (22) it is necessary to define an approximate model for $\mu^2_{eff}(v\rangle)$ which allows us to treat the problem analytically. It has previously been mentioned that the anisotropy of the RF-distorted distribution vanishes in the low energy limit where collisional effects, in particular pitch angle scattering, is strong enough to keep the distribution isotropic. Since the function h(v) represents the low energy part of the distribution, i.e. for $v \leq v_o$, we assume $\mu^2_{eff} = 1/3$ when determining h(v). On the other hand the degree of anisotropy of the non-Maxwellian high-energy part of the distribution function, is strongly dependent on the beam injection angle relative to the toroidal axis. In particular, two complementary situations may be considered: (i) perpendicular beam injection, i.e. the cosine of the injection angle is $\mu_o = 0$, and (ii) isotropic beam injection, i.e. the beam source function is effectively isotropic. Correspondingly, we assume the following models for $\mu^2_{eff}(v)$, which determine the behaviour of g(v):

(i) anisotropic injection

$$\mu^2_{eff} = 0 \qquad (23)$$

(ii) isotropic injection

$$\mu_{eff}^2 = \begin{cases} 1/3 & ; \quad v \leq v_o \\ 0 & ; \quad v > v \end{cases} \tag{24}$$

A formal integration of eq. (21) yields

$$h(v) = h(0)\exp\left[-\int_0^v u\, R(u)du\right] \tag{25}$$

where h(0) is an integration constant determined by the condition

$$4\pi \int_0^\infty v^2 h(v)dv = 1 \tag{26}$$

and

$$R = \frac{-\alpha v^2 + \frac{1}{2}\frac{d}{dv}(\beta v^2) + \frac{3}{2}Kv(1-3\mu_{eff}^2 - v\frac{d\mu_{eff}^2}{dv}}{[\frac{1}{2}\beta + \frac{3}{2}K(1-\mu_{eff}^2)]v^3} \tag{27}$$

Using the low energy expansions for the collision coefficients α and β, and taking $\mu_{eff}^2 = 1/3$ we obtain

$$h(v) \simeq h(0)\exp\left[-\frac{m_b v^2}{2kT_*(1+2K/D)}\right] \equiv h(0)\exp\left(-\frac{v^2}{v_1^2}\right) \tag{28}$$

where D represents the low energy limit of β and T_* is the "collective" temperature of the background particles, cf [14]. From eq. (26) together with (11) and (28) we also find that

$$h(0) = (\pi^{1/2}v_1)^{-3} \tag{29}$$

We note that the expression (28) is the correct low energy limit according to weakly anisotropic theory, cf. [14].

In order to determine the non-Maxwellian part of the distribution function, g(v) we neglect the charge-exchange loss term and the contribution from h(v) to eq. (22) for velocities $v > v_o$. Then eq. (22) can be integrated once to yield

$$\begin{aligned}&[\tfrac{1}{2}\beta + \tfrac{3}{2}K(1-\mu_{eff}^2)]v^2\frac{dg}{dv} + [-\alpha v^2 + \tfrac{1}{2}\frac{d}{dv}(\beta v^2) + \\ &+ \tfrac{3}{2}Kv(1-3\mu_{eff}^2 - v\frac{d\mu_{eff}^2}{dv})]g = \\ &= \frac{H(v_o-v)}{2\pi\tau_b}[\tfrac{1}{2}(1-\frac{\tau_b}{\tau_{cx}}\Lambda)\mu(v/v_1)+1]\end{aligned} \tag{30}$$

where H(x) is the step function and $\mu(x) = \frac{4}{\sqrt{\pi}}\int_0^x t^2\exp(-t^2)dt$. The approximate solution of eq. (30) is given by

$$g(v) = g_1(v) \simeq \frac{1}{2\pi\tau_b}\left[G(v) - \frac{1}{vR(v)}\frac{dG(v)}{dv}\right]\left[1-\exp\left(-\int_0^v uR(u)du\right)\right] \quad (31)$$

for $v \leq v_o$, and

$$g(v) = g_2(v) = g_1(v_o)\exp\left[-\int_{v_o}^{v} uR(u)du\right] \quad (32)$$

for $v \geq v_o$, where

$$G(v) = \frac{\frac{1}{2}\left(1-\frac{\tau_b}{\tau_{cx}}\Lambda\right)\mu\left(\frac{v}{v_1}\right)+1}{\left[-\alpha v^2+\frac{d}{dv}(\beta v^2)+\frac{3}{2}Kv\left(1-3\mu^2_{eff}-v\frac{d\mu^2_{eff}}{dv}\right)\right]} \quad (33)$$

and $R(v)$ is given by eq. (27). Since $g(v)$ represents the stationary high-energy part of the beam distribution function the expressions (31) and (32) have to be evaluated by using the high-energy expansions of α and β and by assuming the appropriate models for $\mu^2_{eff}(v)$ as given by eqs. (23) and (24). Thus, the solutions may be written in the following form:

(i) Anisotropic injection

$$g_1^{(a)}(v) \simeq \frac{\tau_s}{2\pi\tau_b}\frac{\left[\frac{1}{2}\left(1-\frac{\tau_b}{\tau_{cx}}\Lambda^{(A)}\right)+1\right]}{(v^3+v_\alpha^3+\kappa_e\xi v)}\cdot\left[1+\frac{v^2\kappa_e\left(1+\xi+\frac{v_\beta^3}{v^3}\right)(3v^2+\kappa_e\xi)}{(v^3+v_\alpha^3+\kappa_e\xi v)^2}\right] \;;v \leq v_o \;, \quad (34)$$

aud

$$g_2^{(a)}(v) = g_1^{(a)}(v_o)\exp[-\frac{m_b}{2K}(\frac{v^2}{T_h(v)} - \frac{v_o^2}{T_h(v_o)})] \quad ; \quad v \geq v_o \tag{35}$$

(ii) Isotropic injection

$$g_1^{(i)}(v) = \frac{\tau_s}{2\pi\tau_b} \frac{\frac{1}{2}(1-\frac{\tau_b}{\tau_{cx}}\Lambda^{(i)})+1]}{(v^3+v_\alpha^3)}]1 + \frac{3\kappa_e v^4(1+\frac{2}{3}\xi + \frac{v_\beta^3}{v^3})}{(v^3+v_\alpha^3)^2}], \quad v \leq v_o , \tag{36}$$

and

$$g_2^{(i)}(v) = g_1^{(i)}(v_o)\exp[-\frac{m_b}{2K}(\frac{v^2}{T_h(v)} - \frac{v_o^2}{T_h(v_o)})] , \quad v \geq v_o , \tag{37}$$

Here, $T_h(v)$ is defined by

$$\frac{m_b}{kT_h(v)} = \frac{1}{\kappa_e(1+\xi)}[1+\frac{(1+\xi)(v_\alpha^3+\kappa_e\xi v)-v_\beta^3}{(1+\xi)v^3+v_\beta^3}] , \tag{38}$$

and further notations are as follows: $\kappa_e = kT_e/m_b$, $\xi = 3m_b K\tau_s/(2kT_e)$, T_e is the electron temperature, τ_s is the slowing-down time given by

$$\tau_s = \frac{3}{16\pi^{1/2}}(\frac{2kT_e}{m_e})^{3/2}\frac{m_e\, m_b}{Z_b^2\, n_e\, e^4 L} \tag{39}$$

and the characteristic velocities v_α and v_β are defined by, cf [6],

$$v_\alpha^2 = \frac{3\pi^{1/2}}{4} \left(\frac{2kT_e}{m_e}\right)^{3/2} \sum_i \frac{n_i}{n_e} Z_i^2 \frac{m_e}{m_i} ,$$

$$v_\beta^2 = \frac{3\pi^{1/2}}{4} \left(\frac{2kT_e}{m_e}\right)^{1/2} \sum_i \frac{n_i}{n_e} Z_i^2 \frac{2kT_i}{m_i} \tag{40}$$

Using now eqs. (20), (28) and (29) we find that the number of particles contained in the non-Maxwellian part of the distribution function is equal to

$$4\pi \int_0^\infty g(v) v^2 dv = \Lambda \tag{41}$$

which by means of the relations (34)-(38) can be approximated as

$$\Lambda \approx \frac{\tau_s}{\tau_b} \frac{\gamma}{\left(1 + \frac{1}{3} \frac{\tau_s}{\tau_{cx}} \gamma\right)}$$

where

$$\gamma = \gamma^{(a)} = \ln\left(1 + \frac{v_\alpha^2}{v_o^3} + \frac{\kappa_e \xi}{v_o^2}\right) + \frac{3\kappa_e^2(1+\xi^2)}{v_o(v_o^3 + v_\alpha^3 + \kappa_e \xi v_o)} \tag{42}$$

for the anisotropic injection, and

$$\gamma = \gamma^{(i)} = \ln\left(1+\frac{v_\alpha^3}{v_o^3}\right) + \frac{3\kappa_e^2(1+\xi^2)}{v_o(v_o^3+v_\alpha^3)} \tag{43}$$

for isotropic injection. The solutions (34)-(35) and (36)-(37) describe the deviation of F(v,t)/n from the Maxwellian distribution and are important for velocities $v^2 \gg 2kT_*(1+2K/D)/m_b$, when the Maxwellian part of F(v,t)/n is exponentially small. Note from eqs. (34)-(35) and (36)-(37) that a stationary non-Maxwellian tail in the distribution function may be formed if the charge-exchange loss time, τ_{cx}, is such that $0 < \tau_s/\tau_{cx} < 3$.

V. Conclusions

Analytical solutions have been obtained for the time evolution of the velocity distribution of neutral injected ions heated by ICRH in a two component plasma. In particular, two time periods of the development have been considered: early times ($0 \leq t \ll \tau_s$) when the development is dominated by RF-induced velocity diffusion, and the quasi-steady-state ($t > \tau_s$) when the

beam ion distribution consiste of a time dependent Maxwellian part and a time independent non-Maxwellian "tail". It should be noted that the toroidal effects on the beam ion distribution function have been neglected. However, in a device such as JET these effects will be of importance, since they scale as $\varepsilon^{1/2}$, where ε is the inverse aspect-ratio. Thus the analysis in this paper needs to be extended.

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