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DECAY OF FALSE VACUUM IN PRESENCE OF SOFT FERMIONIC AND BOSONIC MODES BOUND ON SOLITON

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Abstract

Decay rate of a false vacuum in (1+1) dimensions is considered in a situation when there are zero or soft fermionic or bosonic modes which are localized on the soliton that converts the false vacuum into a lower one. It is shown that up to exponentially small corrections the decay rate is contributed independently by soliton states, corresponding to different values of the occupation numbers of the soft modes. The contribution of each of the soliton states is given by the formula derived in an earlier work.

Recently there has been some progress in evaluating the rate of quantum decay (at zero temperature) of false vacuum in (1+1) dimensions. For a theory of one scalar field ϕ with potential $V(\phi)$ being a fou.th power polinomial in ϕ, the rate has been calculated[1] in one-loop approximation by an explicit evaluation of the corresponding functional determinant. Subsequently it has been realized[2,3] that in general case the rate of transition from the metastable vacuum ϕ_+ to a lower one ϕ_- is governed by only two renormalized parameters : the difference in the energy density of the phases ϕ_+ and ϕ_-

$$\varepsilon = \mathcal{E}(\phi_+) - \mathcal{E}(\phi_-) \tag{1}$$

and the mass μ of the soliton which converts ϕ_+ into ϕ_-. The probability W of formation of a critical bubble of the phase ϕ_- per unit time and per unit length occupied by the phase ϕ_+ is given by the universal formula[2,3]

$$w = \frac{d^2W}{dx\,dt} = \frac{\varepsilon}{2\pi} \exp\left(-\pi\mu^2/\varepsilon\right) \tag{2}$$

(the system of units $\hbar = c = 1$ is used, where c is the speed of propagation of infinitesimally short-wave small oscillations in the system considered (c is the speed of light in relativistic problems)). Eq.(2) is applicable if there is a gap in the spectrum of small excitations around the soliton, and the value of the gap is much larger than inverse radius 1/R of the critical bubble :

$$R = \mu / \varepsilon \tag{3}$$

When speaking about the gap we do not mean the zero mode which corresponds to displacement of the soliton as a whole. This mode is a Goldstone one and arises due to spontaneous breaking of translational symmetry by soliton /4-6/. It can be recalled that it is path intergation over the Goldstone degrees of freedom which results /2,3/ in eq.(2) for the decay rate. Note also that there are no corrections to eq.(2) which go in powers of ε / μ^2.

In this paper we generalize the consideration /2,3/ to the case when in addition to the leading field ϕ which drives the phase transition there are other fermionic and/or bosonic fields which have low-energy (in particular zero-energy) eigenmodes localized on the soliton. Namely we assume as in Refs.2,3 that the minimal mass m of particles propagating in the bulk of the phases $\phi_\pm$ satisfies the condition

$$m \mu / \varepsilon \gg 1 , \tag{4}$$

however we allow for the possibility that besides the Goldstone zero mode there are other eigenmodes localized on the soliton with eigenvalues $\varkappa$ much smaller than m :

$$\varkappa \ll m . \tag{5}$$

It can be mentioned that in linear structures in condensed matter to which our consideration can be directly applied appearance of fermionic zero or almost zero modes on the soliton is a typical phenomenon/7/.

When a soft mode exists there appear instead of one soliton a spectrum of stable soliton states corresponding to different values of the occupation number of the soft mode, the masses of the states being given by

$$\mu_n = \mu + E_n \tag{6}$$

where E_n is the energy of the mode with the occupation number n.

The main statement of this work is that in the decay rate of the false vacuum contributions of all the soliton states are independently summed up, so that the rate is given by

$$\frac{d^2W}{dx\,dt} = \sum_n w_n = \frac{\varepsilon}{2\pi}\sum_n \exp(-\pi\mu_n^2/\varepsilon) \tag{7}$$

In particular for a fermionic mode the sum contains only two terms corresponding to filled (n=1) and empty (n=0) fermionic level. If this mode is zero one then the two soliton states with different values of the fermionic number are degenerate in mass. According to eq.(7) the fermionic zero mode gives a factor of two in the probability.

It is straightforward to derive eq.(7) within the effective action method/2,3/. According to this approach the Euclidean path integral Z which determines the decay rate[8,9] ($W = 2\,\mathrm{Im}\ln Z$) is reduced under the condition (4) to an integral over closed curves γ on a plane with an effective action S, which depends on the length $L(\gamma)$ and the area $A(\gamma)$ inside the curve:

$$S(\gamma) = \mu L(\gamma) - \varepsilon A(\gamma). \qquad (8)$$

The meaning of the curve γ is that it describes the closed Euclidean world line of the soliton, so that inside the curve there is the vacuum state ϕ_-, while outside is the false vacuum ϕ_+. The parameters μ and ε are determined by the underlying microscopic theory and are renormalized by high energy degrees of freedom. Within the effective low-energy theory of Goldstone degrees of freedom with the action (8) these parameters are fixed and recieve no renormalization.

When in addition to the Goldstone modes there are other soft ones, these should be included in the effective low-energy theory. Let χ generically denote the fields of the soft modes. After integration over the high-energy sector of the theory (i.e. over modes with eigenvalues larger than m) the path integral for the false vacuum state ϕ_+ is given by $Z = \exp Z_1$, where

$$Z_1 = \int \mathcal{D}\chi\, \mathcal{D}\gamma \exp\left(-S(\gamma) - S(\chi,\gamma)\right). \qquad (9)$$

Here $S(\gamma)$ is the geometric action (8), $S(\chi,\gamma)$ is the action for the soft moes which live on the curve γ and $D\gamma$ ($D\chi$) is the measure in the space of closed curves (fields of soft modes).

Consider first integration over $D\chi$ with a fixed curve γ (the curve is assumed to be non-self-intersecting since otherwise its contribution contains an additional exponential damping factor /3/). The fields χ depend on the length parameter l of the curve ($\oint_\gamma dl = L(\gamma)$), which can be viewed as an Euclidean time in the sense of evolution of the fields χ . Since the curve γ is closed l is periodic with the period equal to L . Correspondingly the bosonic (fermionic) fields $\chi(l)$ should be periodic (antiperiodic) functions of l. Under these circumstances the path integral over $D\chi$ is nothing else than the partition function for the soft modes at the temperature $T = L^{-1}$, and one can write the result of the integration in terms of the energy spectrum E_n :

$$Z_1 = \sum_n \int \mathcal{D}\gamma \exp\left(-S(\gamma) - E_n L(\gamma)\right). \qquad (10)$$

Each term of the sum in this expression is an integral with the action (8) in which μ is replaced by μ_n (see eq.(6)). Therefore one can directly apply the results of previous calculations /2,3/, thus obtaining eq.(7).

I acknowledge a remark by V.G.Kiselev and K.G.Selivanov that a direct computation of the fermionic determinant in a specific version of the theory with a zero mode gives the factor of two in the probability in agreement with the general formula (7).

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