DYNAMIC CHARACTERISTICS OF HETEROGENEOUS MEDIA IN VIBRATIONAL AND WAVE PROCESSES

Fedotovskij V.S., Sinyavskij V.F., Terenik L.V., Spirov V.S., Kokorev B.V.

ABSTRACT

The complex mechanic systems involving a great quantity of the same type elements, in particular, the rod systems flowed around by the one- or two-phase flow are considered as the two- or three-phase heterogeneous media with certain effective properties. Some recommendations for calculating effective properties and determining those on a base of the dynamic characteristics of various heterogeneous systems are given.

Cores, heat exchangers and steam generators of the nuclear power plants are the complex mechanic systems involving a great quantity of the same type elements flowed around by the one- or two-phase flow. In order to determine the dynamic characteristics of such the systems subjected to vibrational and acoustic influences it is reasonable to consider these ones as the elastic shells or tanks filled with the heterogeneous media with some effective dynamic properties - the vibration density ρ^* and the vibration viscosity μ^* . The first results of this approach have been reported at the previous meeting of specialists in 1977 [1].

This paper presents the main results of developing the methods for predicting the dynamic characteristics of complex systems that have been received in the recent years. The models of systems can be differed depending on the properties of the system elements being a dispersed phase and the properties of a continuous liquid phase. In the tube-and-shell heat exchangers and the heat generating cassettes the continuous phase is a coolant and the dispersed phase is the elastic tubes and rods tightened at the ends. The elementary model of heterogeneous media of such a type is the Newtonian liquid involving ellastically tightened inclusions - the dipole type oscillators.

The two-phase bubble or dispersed media moving in the channels or flowing around some components of the power generating equipment constructions are heterogeneous media with the freely suspended, but in a common case, elastically deformed inclusions - the oscillators of monopole and quadrupole types.

The most important dynamic characteristics of the systems, such as the natural vibration frequences ω_o^* and the dynamic response factors of a resonance Q are expressed by the formulae of the same type as in [2]:

$$\omega_0^* = \omega_0 \left[1 + \frac{\rho^*(\omega_0^*)V}{M} \right]^{-1/2}, \tag{1}$$

$$Q = \frac{M}{\mu^*(\omega_0^*)V} \left[1 + \frac{\rho^*(\omega_0^*)V}{M} \right]^{1/2}, \tag{2}$$

where M, ω_0 are the mass and natural frequency of the empty reservoir having the volume V, respectively.

In heterogeneous media under the vibrational and acoustic influences the processes of dynamic interactions between the continuous phase and the inclusions are occured that are unusual to homogeneous media. The consideration of this interaction results in the effective dynamic properties of heterogeneous media, such as effective condensability, effective density and transmitting viscosity. In a common case these effective propertirs depend on elasticity, density and viscosity of the phases, but the heterogeneous media themselves are the inertial-viscoelastic ones. For the class of heterogeneous media with elastically tightened inclusions the effective density and transmitting viscosity are expressed by the dispersion relations [3]:

$$\rho^* = \rho \left\{ 1 + \left[\frac{\frac{1+\gamma}{\Delta+\gamma}\left(1 - \frac{\omega_0^2}{\omega^2}\right) + \frac{1}{(\omega\tau)^2}}{\left(1 - \frac{\omega_0^2}{\omega}\right)^2 + \frac{1}{(\omega\tau)^2}} \right] \left[(\Delta - 1) + (\Delta+\gamma)\frac{\omega_0^2}{\omega^2} \right] \varphi \right\} \tag{3}$$

$$\mu^* = \frac{\rho(\Delta+\gamma)\varphi}{\tau} \cdot \frac{\left(\frac{\omega_0^2}{\omega^2} + \frac{1-\Delta}{\Delta+\gamma} \right)^2}{\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 + \frac{1}{(\omega\tau)^2}} \tag{4}$$

Here ρ is the density of liquid, $\Delta = \rho_0/\rho$ is the relative density of inclusions, γ is the factor of added masses, $\tau = \frac{\rho a \delta (1-\varphi)^2(\Delta+\gamma)}{4\mu}$ is the relaxation time, ω_0 is the natural vibration frequency of inclusions.

In particular, such the heterogeneous media one can attribute to a system of rods with a radius a and a liquid volume fraction φ (Fig.1).

In the asymptotic cases corresponding to different phase property combinations the formulas (3), (4), when $\omega_0 = 0$, describe the effective dinamic properties of dispersed media with free non-deformed inclusions. So, for example, the heterogeneous medium formed by the liquid with low viscosity and massless inclusions can serve as the model of a two-phase bubble mixture when the frequencies of influencies are essentially smaller as compared to the natural frequencies of bubble oscillations. In this case [4]:

$$\rho^* = \rho \frac{1-\varphi}{1+2\varphi}, \qquad \mu^* = \frac{18\mu\varphi}{a\delta(1+2\varphi)^2}, \tag{5}$$

where a is the bubble radius, $\delta = \sqrt{2\nu/\omega}$ is the thickness of a boundary layer on their surface (it is supposed that $\delta \ll a$).

The other asymptotic case, when the density of inclusions is essentially larger then the density of a continuous phase can correspond to the gas-or-steam dropping media. From the Figs. (3), (4) it follows that, when $\Delta \gg 1$, $\omega\tau \gg 1$, $\omega \ll \omega_0$.

$$\rho^* = \rho \frac{1+\varphi/2}{1-\varphi}, \qquad \mu^* = \frac{9\mu\varphi}{2a\delta(1-\varphi)^2}. \tag{6}$$

It should be noted that the effective dynamic properties of heterogeneous media play a significant role not only in vibrational processes determining dynamical characteristics of elastic structures being in contact with these ones, but also in compression wave distribution processes. It is particularly important to take into consideration the dispersion dependencies of effective density and transmitting viscosity at the frequencies being close to the natural inclusion-oscillator frequencies. Together with the effective compressibility of heterogeneous media the effective density determines phase velosity, and the transmitting viscosity determines the attenuation of compression waves.

Let us consider some concrete equipments with the two phase media and define the dynamic characteristics of these ones. One of the most wide-spread components of power engineering installations is the piping. The problems to secure a vibrating reliability of these devices are closely connected with the defining some dynamic characteristics of piping (natural frequencies and damping factors of vibration) with the purpose of building up these ones from dangerous resonance regimes.

Dynamic characteristics of the elastic piping with a two-phase flow are determined just as by its structural parameters, such as stifness, length mass, structural damping, friction in framings and supports, so by time lag and damping action of a two-phase mixture (by effective mass and hydrodynamic damping).

If the two-phase flow has a bubble structure with the uniform gas content of $\varphi(r)=\mathrm{const}$ through a piping section, then the effective mass of a two-phase mixture ($\mathfrak{M}$), and the hydrodynamic damping factor (Ξ) per unit length are defined by an effective density of ρ^*, an effective transmitting viscosity of μ^* and an area of piping profile of πR_o^2 :

166

$$\mathfrak{M}=\pi\rho^* R_o^2 , \qquad \Xi=\pi\mu^* R_o^2 \tag{7}$$

When substituting the relations of (5) into (1) and (2) we obtain the dynamic characteristics of a piping.

The relations of (5) are checked experimentally [7] by the method of forced vibrations of the tube with the two-phase bubble mixture. From the comparison of the resonance (natural) frequencies of the curved bibration of a tube and the damping coefficient at filling tube with liquid and with the two-phase mixture there are obtained the values of effective density and effective transmitting viscosity at various gas contents. In the Fig.1 and 2 are shown the calculated dependencies of (5) and the experimentally obtained data. Often, the distribution of a gas volume fraction throughout a piping profile is non-uniform. The very significant non-uniformity can take place at the surface boiling of liquid. Another case with extremely non-uniform distribution of a gas-or-steam volume fraction is the annular flow.

In the intermediate case with step distributing a gas content throughout a piping radius

$$\varphi(r),\rho^*(r),\mu^*(r)=\begin{cases}\varphi_1,\ \rho_1^*,\ \mu_1^*, & \text{when } 0<r<R\\ \varphi_2,\ \rho_2^*,\ \mu_2^*, & \text{when } R<r<R_o\end{cases}$$

the effective mass and the hydrodynamic damping are expressed by the relation [5] :

$$\mathfrak{M}=\pi\rho_2^* R_o^2\,\frac{1+\frac{R^2}{R_o^2}\left(\frac{\rho_1^*-\rho_2^*}{\rho_1^*+\rho_2^*}\right)}{1-\frac{R^2}{R_o^2}\left(\frac{\rho_1^*-\rho_2^*}{\rho_1^*+\rho_2^*}\right)} \tag{8}$$

$$\Xi=\frac{4\pi\mu_1^* R^2+\pi\mu_2^*(R_o^2-R^2)\left[\left(\frac{\rho_1^*}{\rho_2^*}+1\right)^2+\frac{R^2}{R_o^2}\left(\frac{\rho_1^*}{\rho_2^*}-1\right)^2\right]}{\left[\left(\frac{\rho_1^*}{\rho_2^*}+1\right)-\frac{R^2}{R_o^2}\left(\frac{\rho_1^*}{\rho_2^*}-1\right)\right]^2} \tag{9}$$

where the ρ_1^*, ρ_2^* and μ_1^*, μ_2^* of bubble media are determined in accordance with a gas volume fraction φ_1 and φ_2 with the equation (5).

From the expression (3), (4) there are followed various extreme cases that have practical significance. In particular, at $\varphi_1 = 1$ and $\varphi_2 = 0$ the two-phase flow is an annular one. From (8) it is followed that in this case the effective mass of the two-phase mixture is equal to

$$\mathcal{M} = \pi \rho R_o^2 \frac{1-\frac{R^2}{R_o^2}}{1+\frac{R^2}{R_o^2}} = \pi \rho R_o^2 \frac{1-\varphi}{1+\varphi} \tag{10}$$

It is interesting to note that the effective mass of two-phase mixtures at the bubble structure (see (7)) is smaller than at the annular one at equal φ. On the contrary the hydrodynamic damping of vibrating a piping with the annular two-phase flow is found to be smaller then with a bubble one (with equal φ). In this case from equation (9)is followed

$$\Xi = \frac{4\pi\mu R^4}{R_o^3 \delta \left(1+\frac{R^2}{R_o^2}\right)^2} = \pi\mu R_o^2 \frac{4\varphi^2}{R_o \delta (1+\varphi)^2} \tag{11}$$

Thus, the effective mass of the two-phase mixture detemining along with the mass of a piping and its stiffness the natural frequencies of vibration is differed to some extent from a real one.

Now, consider the bundles of fuel rod assemblies or the tube bundles of heat exchangers in the one-or-two phase flow of a coolant. The dynamic characteristics of such systems depend, to a large extent, on the geomrtric shape of rods, their mutual arrangement, the character of flowing around these bundles by the medium and its inertial and viscous properties.

When the rod bundle is vibrating as a porous half-permeable body in the one-phase liquid the liquid exerts a certain inertial and damping influence upon it, which is taken into consideration when calculating the natural frequencies and amplitude with the help of an added mass and hydrodynamic damping coefficient. For the bundle of a cylindrical shape with a radius R consisting of the cylindrical rods with a radius a and existing in the one-phase liquid confined by a cilindrical shell with a radius R_o, the added mass and the hydrodynamic damping are expressed by the equations [4, 5]:

$$\mathcal{M} = \pi \rho R^2 \varphi \frac{1+R^2\varphi/R_o^2}{1-R^2\varphi/R_o^2} \tag{12}$$

$$\Xi = \frac{4\pi\mu R^2 \varphi}{a\delta(1-R^2\varphi/R_o^2)^2} \tag{13}$$

where ρ and μ are the density and viscosity of the liquid, φ is the volume concentration of rods in the bundle, $\delta = \sqrt{2\nu/\omega}$ is the thickness of a boundary layer when the bundle of rods is vibrating with frewuency ω.

If the vibration of a rod bundle occur in the two-phase bubble mixture with the bubbles whose radius r_n is considerably smaller then the radius of the rods and the gaps between them, the two-phase mixture can be considered as a homogeneous medium with the effective density given by (5):

$$\rho^* = \rho \frac{1-\alpha}{1+2\alpha}$$

where the letter α re-denotes the gas volume fraction.

Thus, the added mass of the two-phase bubble mixture for the rod bundle, as for the other bodies, will be smaller than the added mass of the one-thase liquid by $(1-\alpha)(1+2\alpha)$ times.

This conclusion is confirmed by the experimental data of Carlucci [8].

The hydrodynamic damping of bibrations of the rod bundle in the two-phase bubble mixture conditioned to a large extent by dissipative losses at bubbles is expressed by the formula [4]:

$$\Xi = \frac{4\pi\mu R^2\varphi}{a\delta\left(1+R^2\varphi/R_0^2\right)^2}\left[1+\frac{9a\alpha\left(1-R^4\varphi^2/R_0^4\right)}{2r_n\left(1+2\alpha\right)^2}\right]$$

which is in satisfactory accordance with the known experimental data [8].

Often the bodies of heat exchangers or the cans of cassettes with fuel assemblies are rather subjected to direct vibration influences than the rod bundles and tubes. In this case for the dynamic characteristics prediction it is advantageous to consider those ones as the elastic cans or reservoirs containing the homogeneous media as the one- or two-phase bubble mixture with elastic cylindrical inclusios, i.e. as the two- or three-phase inertial-elastic media. The effective density of such a three-phase medium is expressed by the relation:

168

$$\rho^{**} = \frac{\rho_n^*\rho_{cm}^*}{\rho} = \rho\,\frac{1-\alpha}{1+2\alpha}\cdot\frac{1+\varphi}{1-\varphi}$$

where ρ_n^* is the effective density of a bubble mixture, ρ_{cm}^* is the effective density of liquid with cylindrical inclusions (rods).

Ot should be noted that if the effective densities of the two-phase media ρ^* and ρ_{cm}^* are described by some dispersion relations having special features at the appropriate inclusion resonances (the bubbles and the rods), then the effective density of the three-phase media ρ^{**} will have more complex dependence on vibration influence frequencies. It follows from this that the vibratory system (the reservoir or the shell) having one natural

frequency, for example, when filled with the three-phase homogeneous medium is become a three frequecy one and its aplitude-frequency characteristics will have three resonances. The amplitudes of these three resonances are determined both by the applied vibrational load and the hydrodynamic damping or the effective transmitting viscosity of the tree-phase heterogeneous medium. The latter, along with the effective density, can be expressed through the effective properties of the two-phase media:

$$\mu^{**} = \mu_n^*\frac{\rho_{cm}^*}{\rho} = \frac{18\mu\alpha}{r_n\delta(1+2\alpha)^2}\cdot\frac{1+\varphi}{1-\varphi}\,.$$

REFERENCES

1. Sinyavskij V.F. et al. The Investigation of Added Masses and Damping Factors for Vibrations of Tube Bundles in Fluid. - Specialists Meeting on LMFBR Flow Induced Vibration. Argonne, Ilinois, USA, 20-23 Sept., 1977. Summary Report, p.224.
2. Fedotovskij V.S. Continuum Approach to Problems of Reservoir Dynamics Containing Rod Bundles or Particular Clusters and Liquid at Vibrating Influences: Preprint FEI-1528, Obninsk, 1984.
3. Fedotovskij V.S. Effective Properties of Heterogeneous Media with Elastically Tightened Inclusions at Vibrational Influences: Preprint FEI-1636, Obninsk, 1984.
4. Fedotovskij V.S. Effective Properties of Heterogeneous Media with Free Non-Deformable Inclusions at Vibratory Influences: Preprint FEI-1507, Obninsk, 1984.
5. Fedotovskij V.S. Dynamics of Pipelines and Rod Bundles with Two-

Phase Flow at Bibratory Influences: Preprint FEI-1660, Obninsk, 1985.

6. Fedotovskij V.S., Terenik L.V., Spirov V.S. Experimental Investigation of Dynamic Characteristics of Reservoir with Elastic Rod Assembly at Vibratory Influences by Liquid: Preprint FEI-1659, Obninsk, 1985.

7. Fedotovskij V.S., Spirov V.S., Kukhtin A.B. Inertial and Damping Effect of Two-Phase Flow upon Vibratory Piping: Preprint FEI-1178, Obninsk, 1980.

8. Carlucci L.N. Flow Vibration in Vertical Two-Phase Flow Fuel Channels. Part A: Hydrodynamics Mass and Fluid Damping of Fuel Bundles Vibration in Confined Water and Air-Water Mixtures. AECL, Oct., 1976.

169

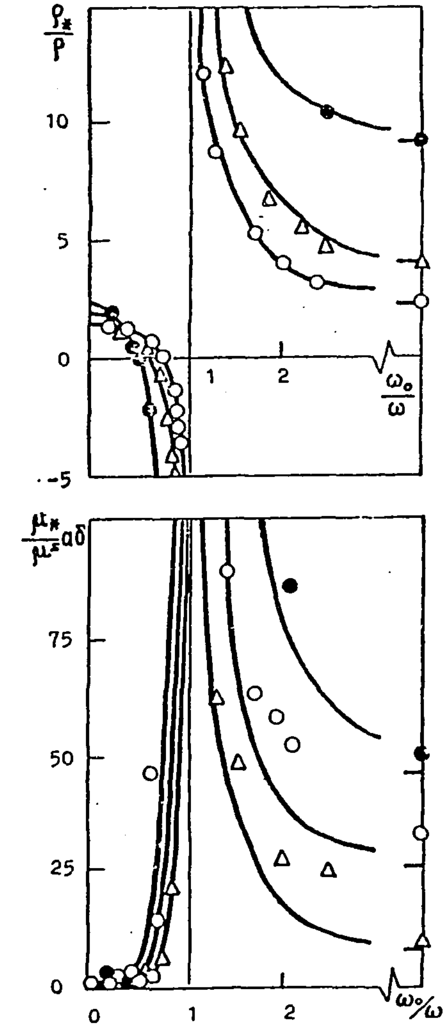

Fig.1 The calculated and experimental [6] dependences of the effective density and the effective transmitting viscosity for the system of elastic rods in liquid at the volume fractions of φ=0.536;0.63 and 0.749 or at the relative bundle pitches of ○-X=1.3; △-X=1.2; ●-X=1.1.

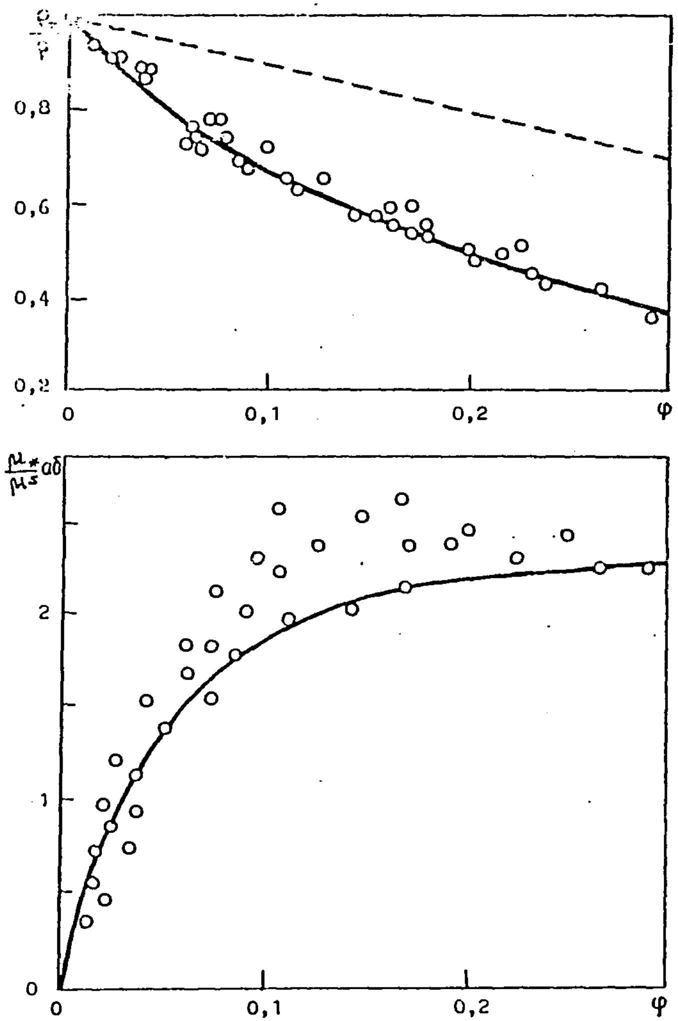

Fig.2 The dependence of the effective properties of the buble mixture on the gas content.
—— theory; o -experiment.

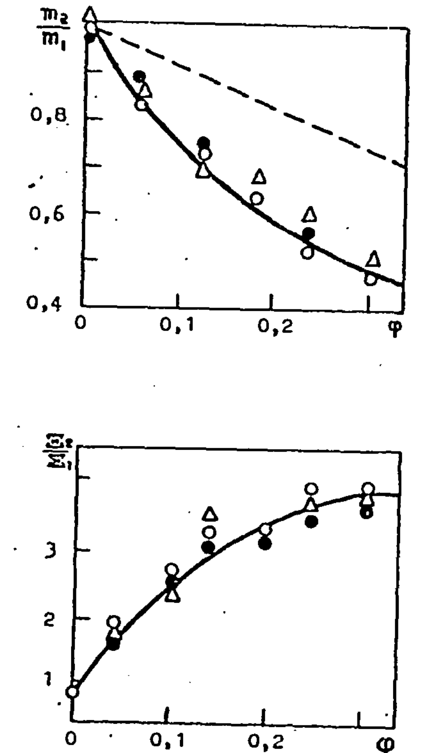

Fig.3. The dynamic characteristics of the rod bundle in the bubble mixture. The indices of 1 and 2 correspond to the one-and-two phase medium.
—— theory;
●,○,△ experimental data of the AECL [8]