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I.W.G.F.R Meeting
on Flow Induced Vibration
21-24 October 1986 - PARIS

VIBRATIONS OF THE SHAFTS OF THE PRIMARY PUMPS :

"ROTOR" : A FINITE ELEMENT PROGRAM FOR COMPUTING THE VIBRATIONS OF ROTATING SHAFTS IMMERSED IN A LIQUID

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AIMS OF "ROTOR" IS TO COMPUTE :

- The complex flexural eigenmodes of the shaft
- The critical angular velocities
- The instability thresholds
- The vibration response to a given unbalance

TAKING INTO ACCOUNT :

- The gyroscopic effect
- The various fluid effects
- The support conditions of fluid bearings

FUTURE EXTENSIONS

- Dynamical coupling of the shaft with the non rotating parts of the pump
- Time histories for transient excitation both in the linear and non linear domains

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GENERAL FEATURES OF THE PROGRAM ROTOR

PHYSICAL PROBLEM

A shaft supported by liquid film bearings and rotating in a liquid is a non conservative mechanical system as Flexural vibrations are concerned :

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Eigen Frequencies ω are complex in nature and depend on the shaft angular velocity Ω

$$\boxed{\omega_c = \omega_R(\Omega) + i\,\omega_I(\Omega)}$$

UNACCEPTABLE VIBRATION CAN OCCUR AT :

- Critical angular velocities : $\omega_R(\Omega) = \Omega$
- Static instability : $\omega_R(\Omega) = 0$
- Dynamic instabilities : $\omega_I(\Omega) = 0$

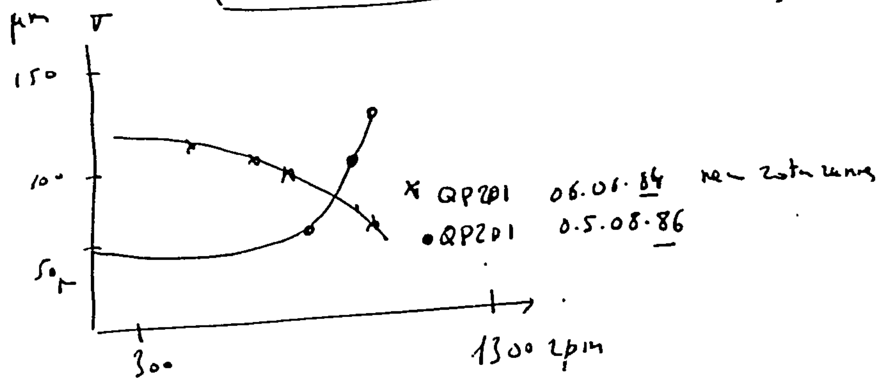
KBG : Company Kanak. reactor Pump.

Continuous increase in vibration from 275 μm

100 μm → 300 μm

2 expl. [illegible] [illegible] [illegible] !

[illegible] [illegible] unbalance



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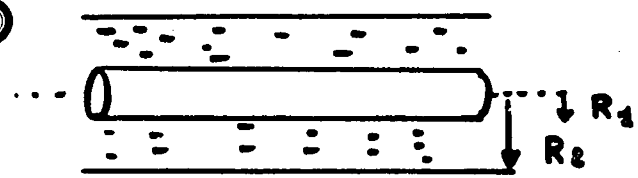
NON ROTATING SHAFT : FLUID EFFECTS



Inertial effect

$$m_a = \rho \pi R_1^2 \frac{R_1^2 + R_2^2}{R_2^2 - R_1^2}$$

For small annular gap $R \simeq R_2 \simeq R_1 \quad R_2 - R_1 = e$

$$m_a \simeq m_f \frac{R}{e}$$

Compressibility effect

negligible as far as $\omega R / c \ll 1$

Viscosity effect

small as far as $\omega e^2 / \nu \gg 1$

leading to a modal damping

$$\zeta = \frac{1}{2} \frac{m_a}{M + m_a} \sqrt{\frac{2\nu}{\omega e^2}}$$

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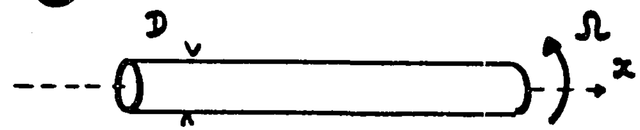
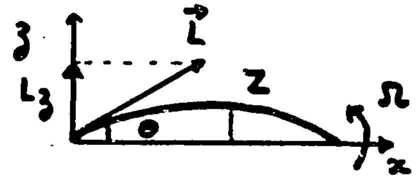
ROTATING SHAFT : GYROSCOPIC EFFECT



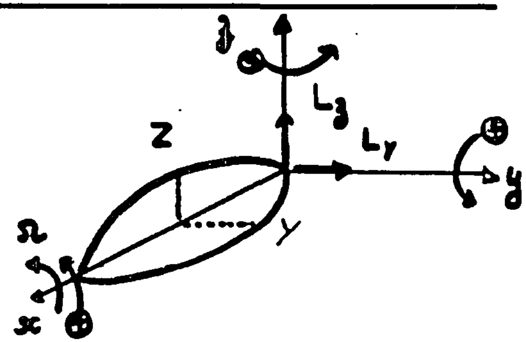
Angular momentum $\vec{L} = J \vec{\Omega}$; $J = MD^2/8$

$$L_z = J \Omega \theta = J \Omega \frac{\partial Z}{\partial x}$$



Equilibrium $\Rightarrow$ $\mu_{(\text{flexion in oxy plane})} = \dot{L}_z$

A.S. COUPLED FLEXURAL EQUATIONS



$$EI \frac{d^4 Z}{dx^4} - \omega^2 \rho S Z + i \omega \Omega \rho J \frac{d^2 Y}{dx^2} = 0$$

$$EI \frac{d^4 Y}{dx^4} - \omega^2 \rho S Y - i \omega \Omega \rho J \frac{d^2 Z}{dx^2} = 0$$

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SHAFT AT REST : $\Omega = 0$

I CLASSICAL F. E. DISCRETISATION PROVIDES

$[K_s]$: Structural stiffness matrix

$[M]$: Structural + incompressible fluid mass matrix

$[A]$: Damping matrix, in particular for viscosity effect of the fluid

Shaft modeled as a beam

Liquid modeled in 2D in a section perpendicular to the shaft axis (3D corrective coefs are possible for variation of section along the shaft)

II RESOLUTION OF THE CONSERVATIVE EIGENVALUE PROBLEM

$$\{[K_s] - \omega^2 [M]\} X(s) = 0$$

$\Rightarrow$ $\omega_n > 0$ and $\varphi_n(s)$ (real quantity)

III MODAL DAMPING BY MODAL PROJECTION

$$A_n = [\varphi_n]_T [A] [\varphi_n]$$

$$\zeta_n = A_n / 2 M_n \omega_n \qquad M_n = [\varphi_n]_T [M] [\varphi_n]$$

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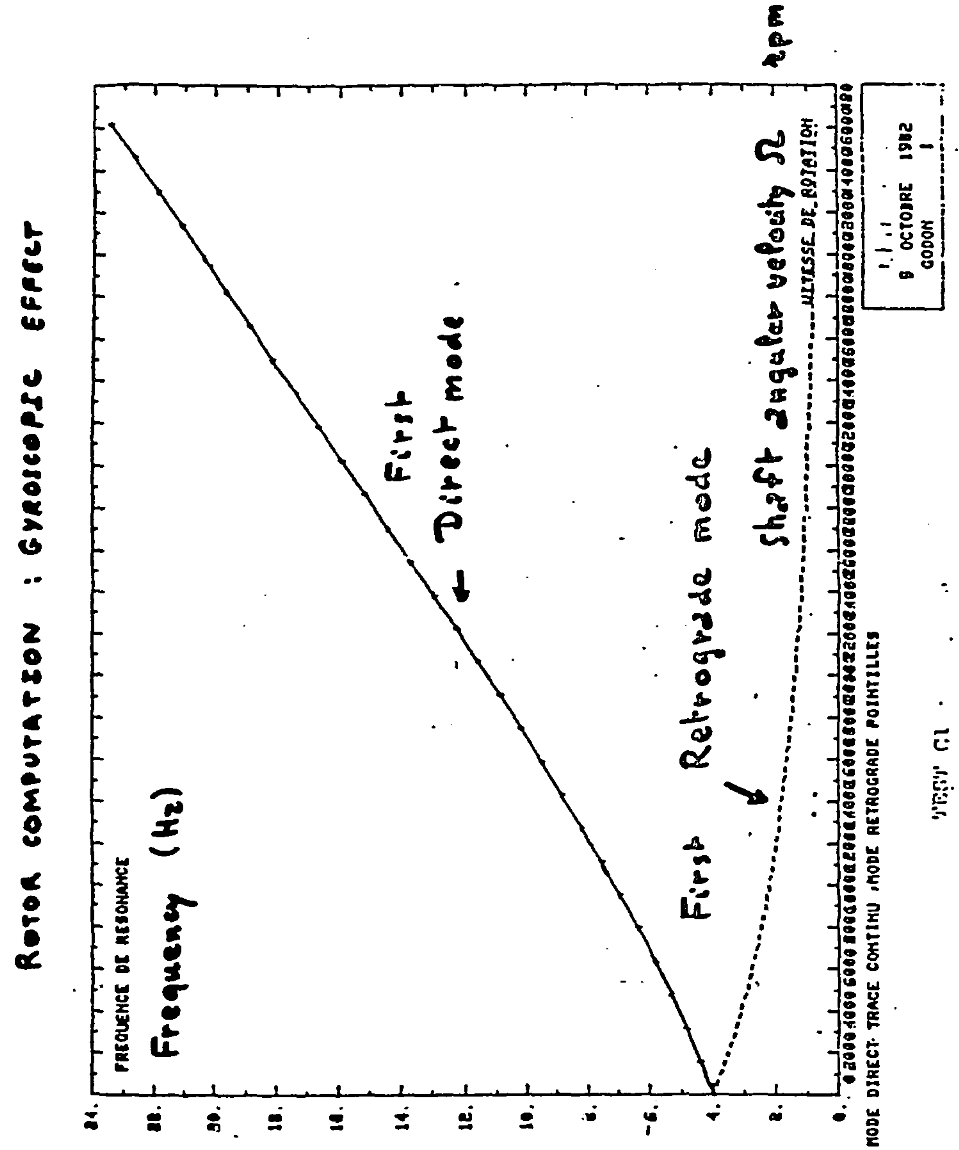

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GYROSCOPIC COUPLING

CHANGE OF VARIABLES

$$\varphi_D = Y + iZ \qquad \varphi_R = Y - iZ$$

For a uniform shaft one obtains the decoupled system

$$\left[EI\frac{d^4}{dx^4} - \omega\Omega J\frac{d^2}{dx^2} - \omega^2\rho S\right]\varphi_D = 0$$

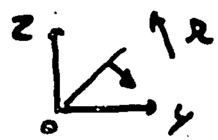
$$\left[EI\frac{d^4}{dx^4} + \omega\Omega J\frac{d^2}{dx^2} - \omega^2\rho S\right]\varphi_R = 0$$

Direct mode $\quad \varphi_D = \Phi(x)\,(1+i)$

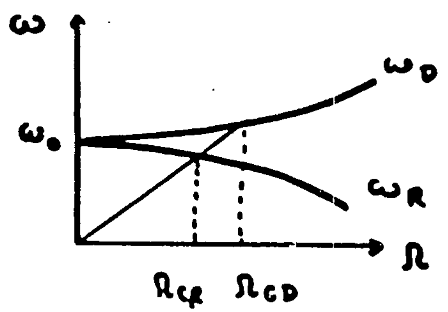
Flexural plane rotates at $+\Omega$

Retrograde mode $\quad \varphi_R = \Phi(x)\,(1-i)$

Flexural plane rotates at $-\Omega$



ω_D et ω_R remains real and positive



$$\frac{\omega_D - \omega_R}{\omega_0} \simeq \frac{1}{2}\,\frac{\rho(\Omega R)^2}{E}$$

Pinned - pinned uniform beam

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ROTATING SHAFT COROTATING FLUID EFFECT

To the 1st order in respect with fluctuating quantities:

$$e\frac{\partial w}{\partial\theta} - \frac{1}{2}\Omega R\frac{\partial\vec{X}\cdot\vec{n}}{\partial\theta} - R(\dot{\vec{X}}\cdot\vec{n}) = 0$$

$$eR\frac{\partial w}{\partial t} + \frac{e}{\rho}\frac{\partial p}{\partial\theta} + \Omega Re\frac{\partial w}{\partial\theta} - \left(\frac{1}{2}\Omega R\right)^2\frac{\partial\vec{X}\cdot\vec{n}}{\partial\theta} - \frac{1}{2}\Omega R(\dot{\vec{X}}\cdot\vec{n}) = 0$$

Elimination of fluctuating velocity

$$\boxed{p\frac{e}{\rho} = R^2\ddot{X}\cos\theta - \Omega R^2\dot{X}\sin\theta - \frac{\Omega^2}{4}R^2X\cos\theta}$$

Inertia $\qquad$ gyroscopic coupling $\qquad$ stiffness

Integration of pressure leads to the coupled eigenvalue problem :

$$EI\frac{d^4X}{dx^4} - m_a\frac{\Omega^2}{4}X + i\omega m_a\Omega Y - \omega^2(\rho S + m_a)X = 0$$

$$EI\frac{d^4Y}{dx^4} - m_a\frac{\Omega^2}{4}Y - i\omega m_a\Omega X - \omega^2(\rho S + m_a)Y = 0$$

$$\begin{bmatrix} K(\Omega) - \omega^2 M & +i\omega C \\ -i\omega C & K(\Omega) - \omega^2 M \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = 0$$

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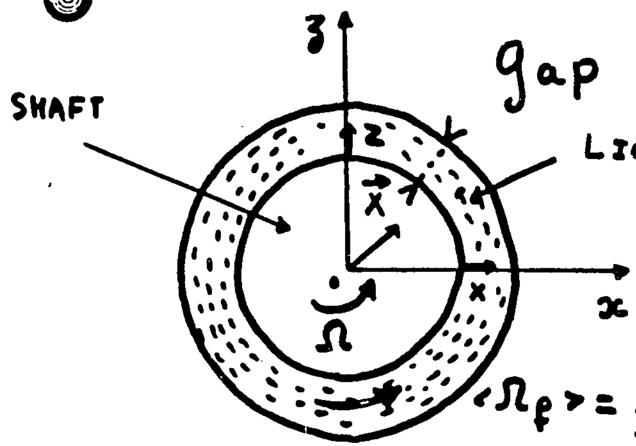
ROTATING SHAFT

COROTATING FLUID EFFECT

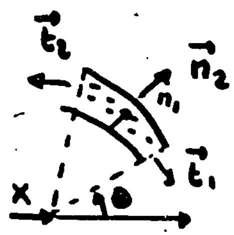


gap e $\quad S = \frac{\omega e^2}{\nu} \gg 1$

$\langle \Omega_f \rangle = \frac{1}{2}\Omega$

Flow velocity in the presence of a lateral vibration of the shaft

$$\vec{V} = \left(\frac{1}{2}\Omega r + w\right)\vec{t} + u\,\vec{n}$$



Pressure Field

$$P = P_0 + p(t, r, \theta)$$

Continuity equation $\displaystyle\iint_\Sigma \vec{V}.\vec{k}\, d\Gamma = 0$

Momentum equation

$$\iiint \frac{\partial \vec{V}}{\partial t} d\tau + \iint_\Sigma \vec{V}(\vec{V}.\vec{k})\, d\sigma + \frac{1}{\rho}\iint_\Sigma P\vec{k}\, d\Gamma = 0$$

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MODAL ALGORITHM IN THE COMPLEX PLANE : DISCRETISED LINEAR OPERATOR

$$\mathcal{L}[\Omega, \omega] =$$

$$\Big\{ [K_s + K_B(\Omega)] - \Omega^2 K_F] + 2i\omega \Big[A_{FS} + A_B(\Omega) + \Omega [G_s + G_F] \Big] - \omega^2 [M_s + M_F] \Big\} \Phi = 0$$

PROBLEM :

COMPUTE THE EIGENMODES Φ_n, ω_n VERSUS Ω

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ROTATING SHAFT : COROTATING FLUID EFFECTS

For uniform shaft and fluid annulus decoupling is readily achieved

Direct mode $\varphi_D = X + iY$

Retrograde mode $\varphi_R = X - iY$

ω_0 natural pulsation of the shaft at rest and in air

M : structural mass, m_a added fluid mass/unit length

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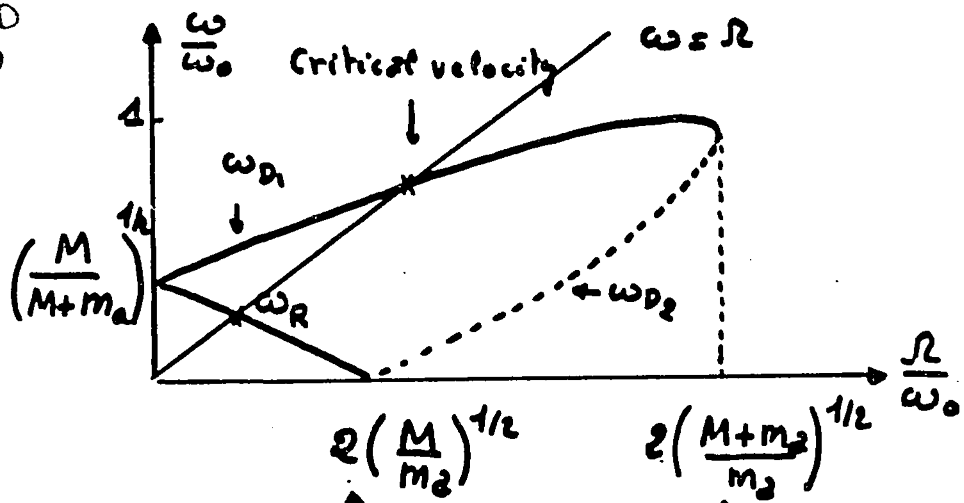
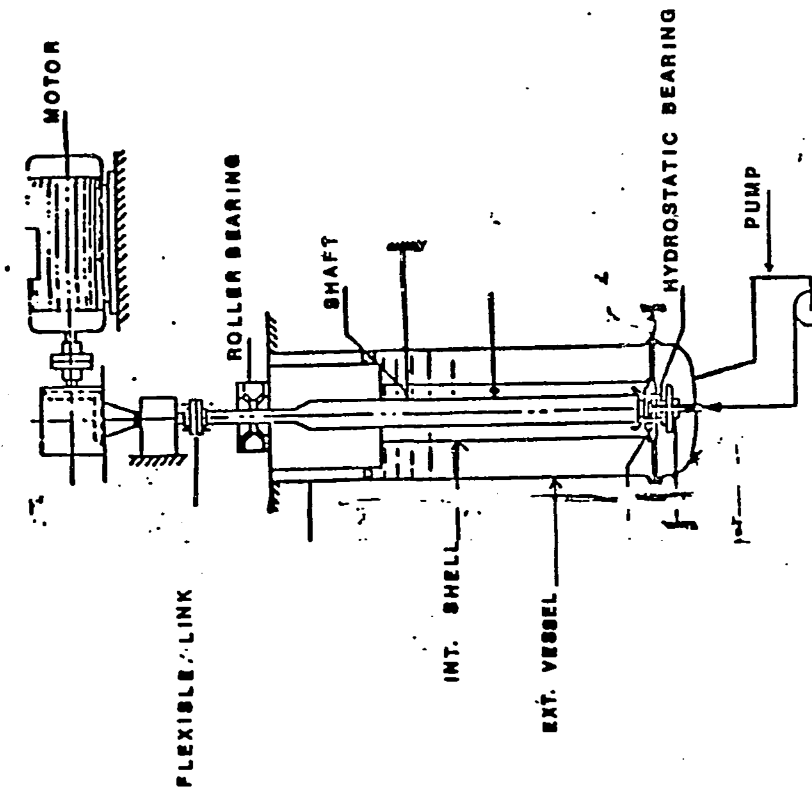


Static instability of retrograde mode

Dynamic instability flutter of the two direct modes



MALVINA TEST RIG

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MODAL ALGORITHM IN THE COMPLEX PLANE :

STEP BY STEP IN Ω ITERATIVE PROCEDURE

Velocity step n $\Omega = n\,\Delta\Omega$

(starting from $n = 1$ $\Delta\Omega \ll \Omega$ final

Iteration k provides ω_k, φ_k

Approximation $k+1$ of φ satisfies:

$$\mathcal{L}[\Omega, \omega_k]\, \varphi_{k+1} = \varphi_k$$

then approximation $k+1$ of ω satisfies:

$$< \varphi^*_{k+1}, \mathcal{L}[\Omega, \omega_{k+1}]\, \varphi_{k+1} > = 0$$

next step

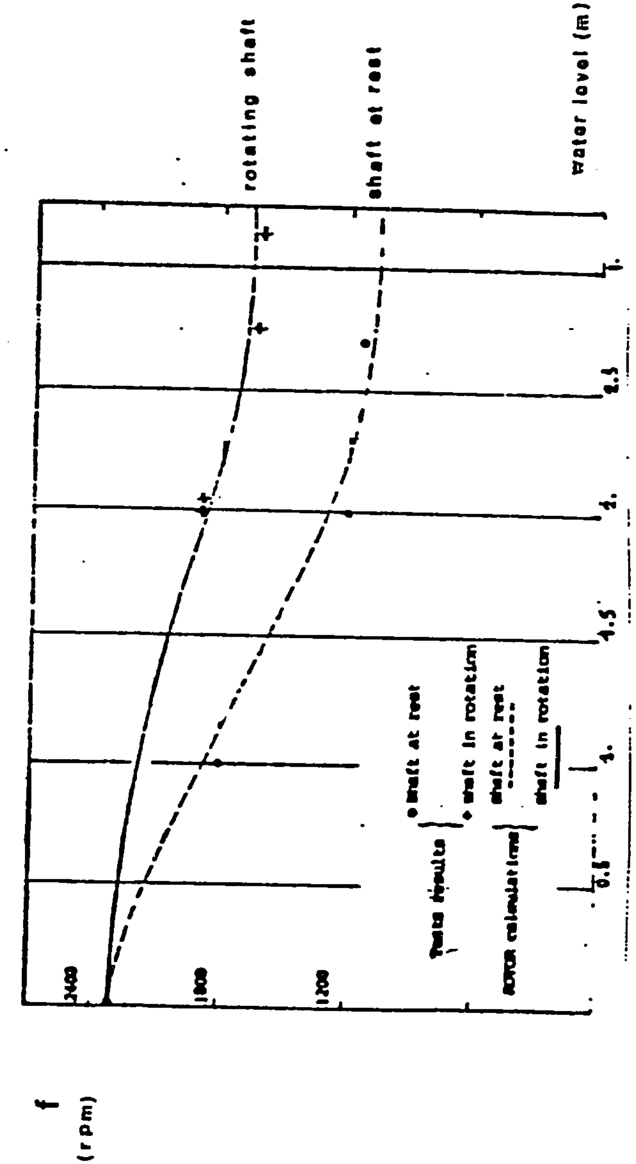
CONVERGENCE :

$$|\omega_{k+1} - \omega_k| \to 0 \quad \text{and} \quad \|\varphi_k\| \to \infty$$

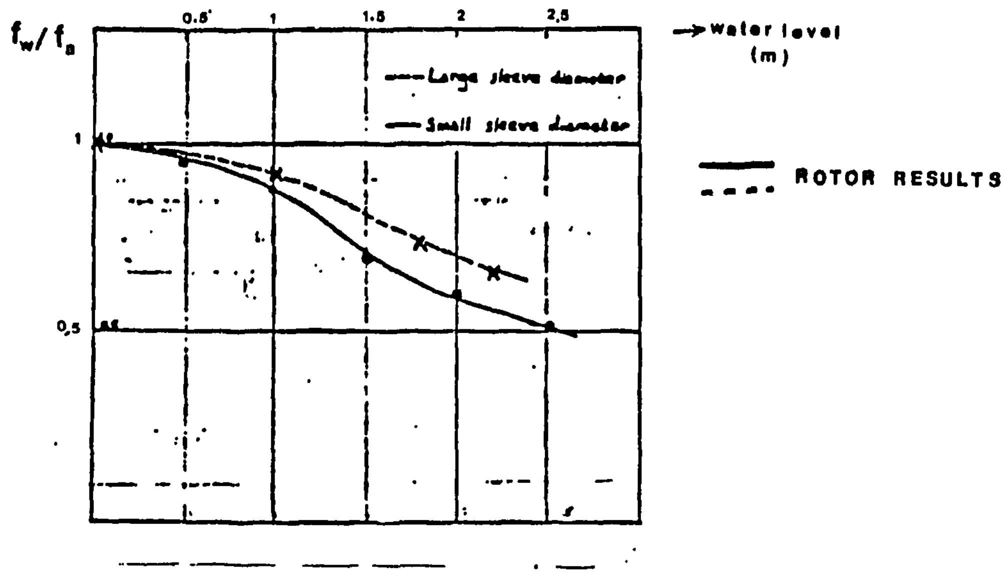
INITIALISATION :

$$\Omega = 0 \Rightarrow \varphi_0, \omega_0 \Rightarrow \begin{array}{l} \varphi_D = \varphi_0 (1+i) \\ \varphi_K = \varphi_0 (1-i) \\ \omega_1 = \omega_0 (1 + i\zeta) \end{array}$$

MALVINA / ROTOR RESULTS



MALVINA TESTS



Eigenfrequencies variation for the shaft at rest

Table I : Experimental results

Water level (m)	Critical speed (rpm)	Mean Displacements (mm)		Mean Damping	
		at mid-shaft	at lower bearing	at mid-shaft	at bearing
1.7	>1950	>7.6	> 0.50	/	/
2	1875	3.6	0.38	3.9	4.1
2.15	1800	2.0	0.36	/	/
2.30	1740	1.6	0.28	6.2	8.3
2.70	1630	1.3	0.14	7.3	10.5

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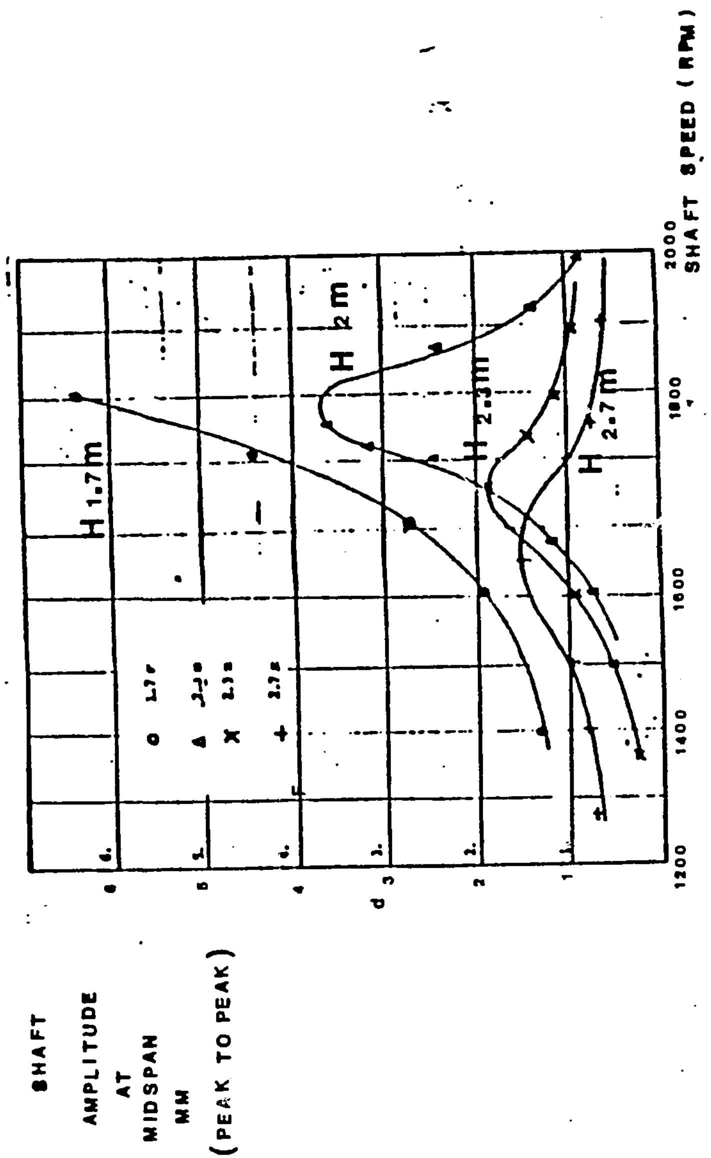
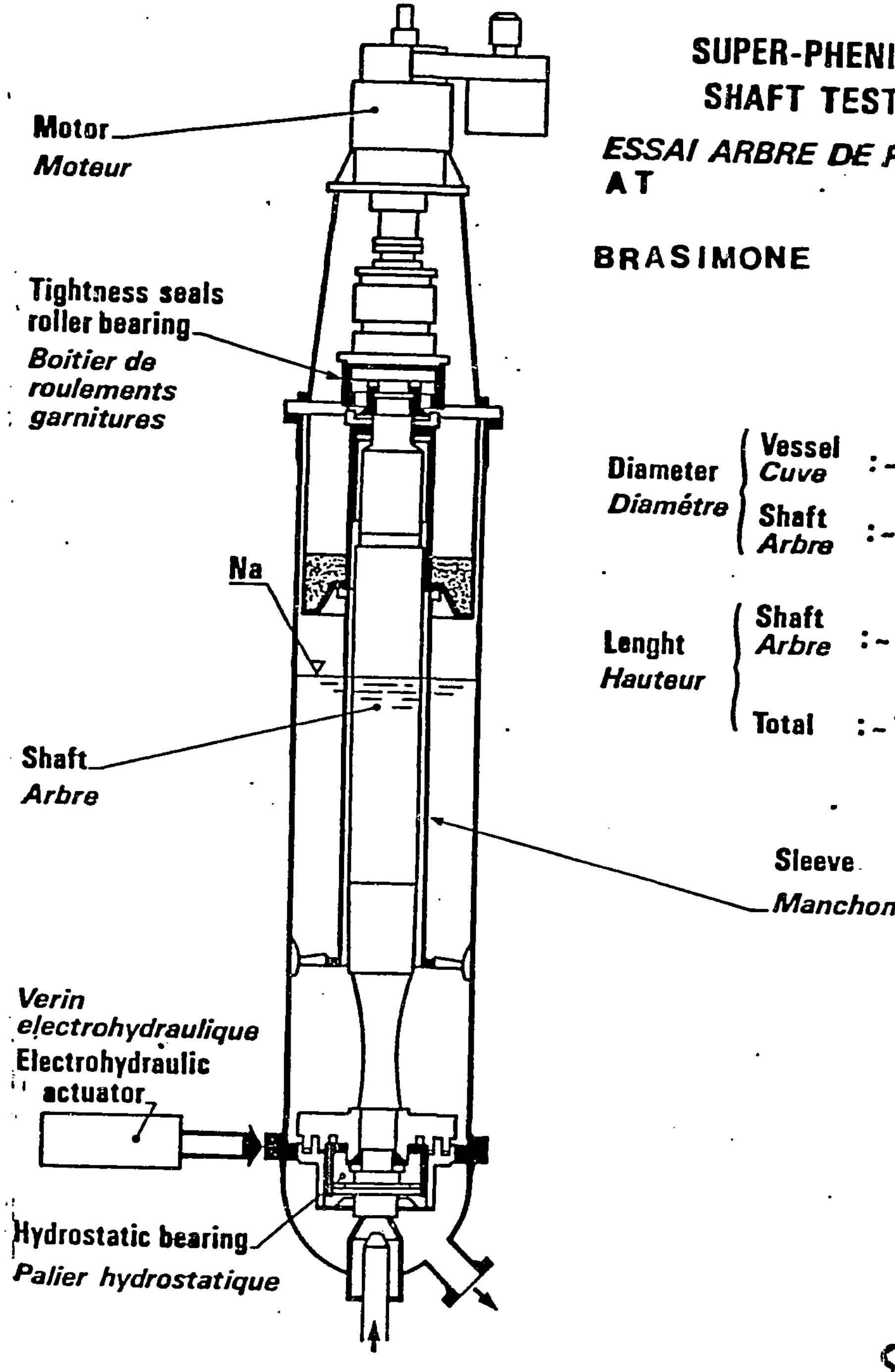


MALVINA
crossing of critical speed

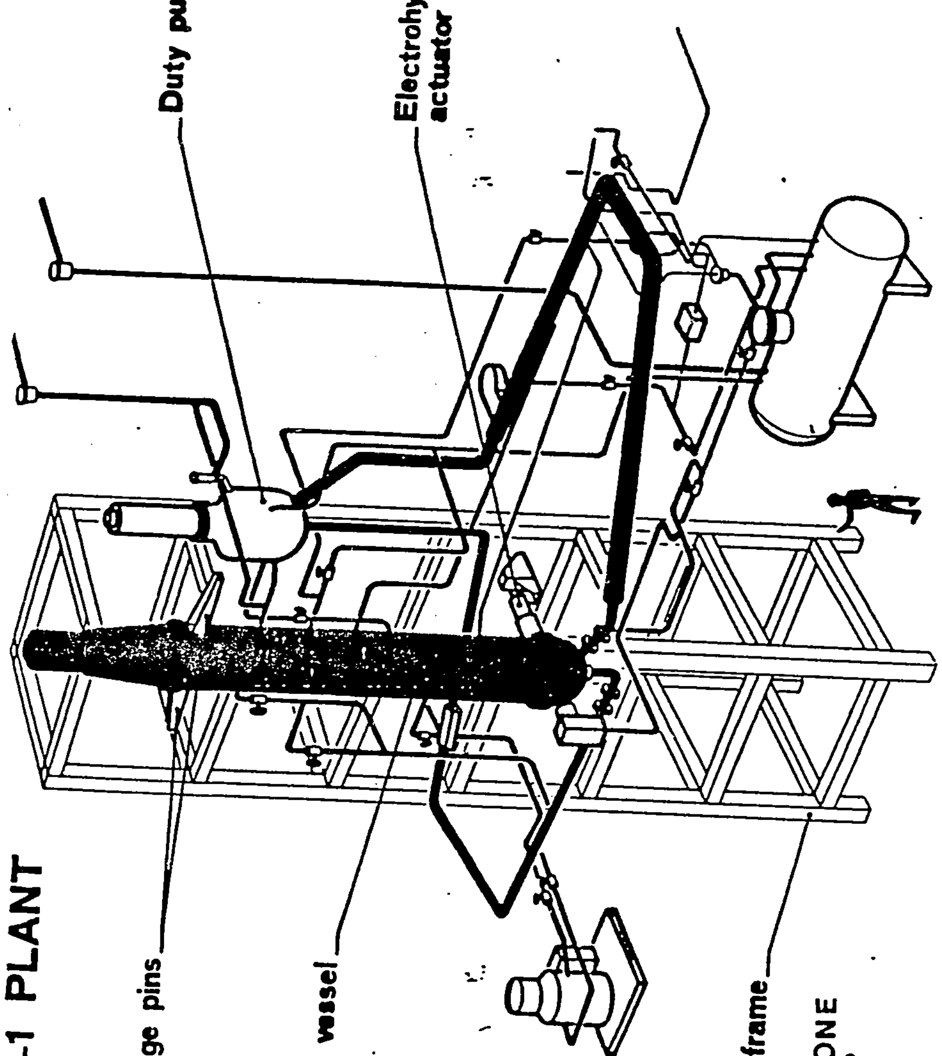
SUPER-PHENIX SHAFT TEST

ESSAI ARBRE DE POMPE
AT

BRASIMONE

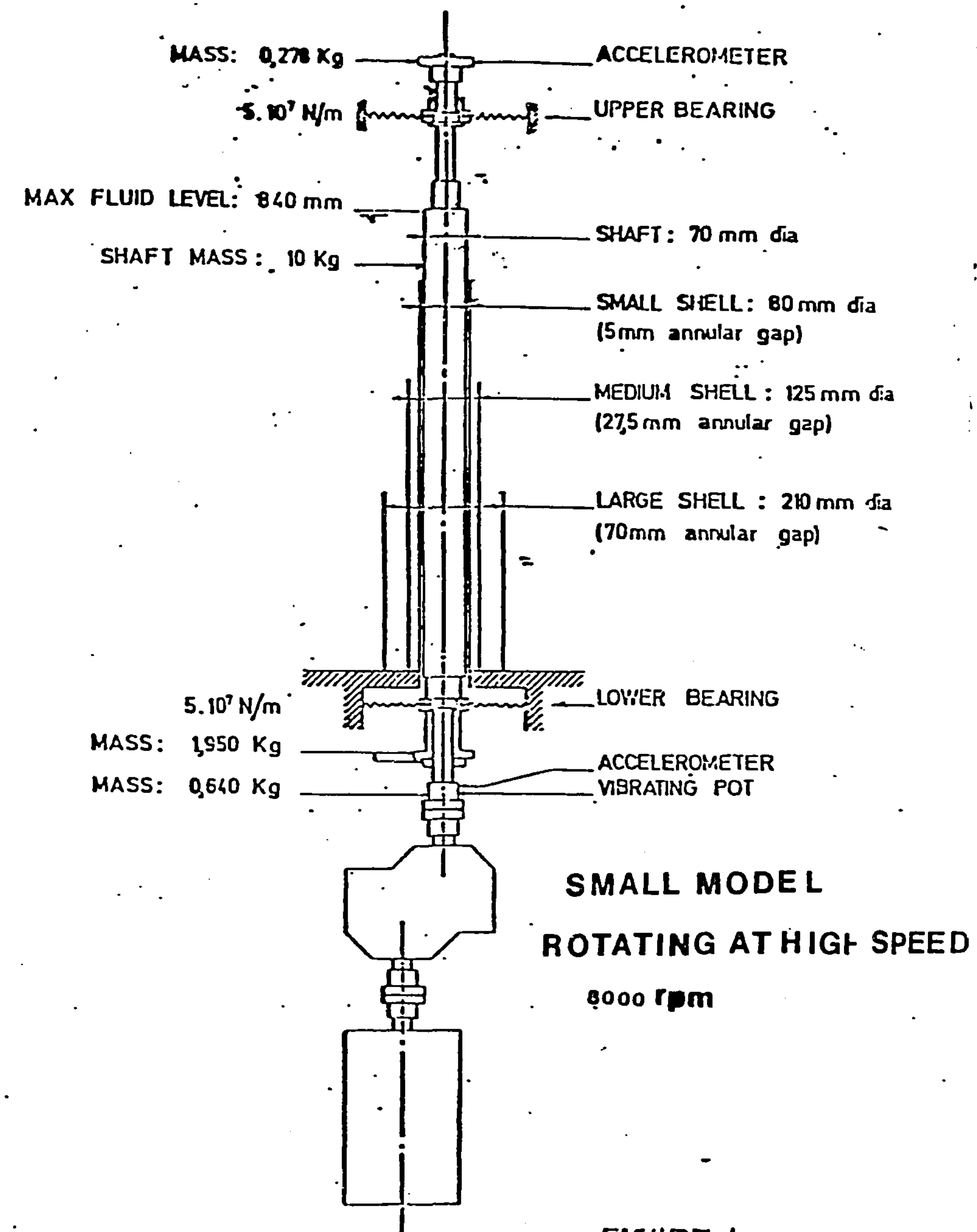


Diameter / *Diamétre*	Vessel / *Cuve*	: ~ 1.5 m	
	Shaft / *Arbre*	: ~ 0.6 m	
Lenght / *Hauteur*	Shaft / *Arbre*	: ~ 10 m	
	Total	: ~ 13.8 m	



CPV-1 PLANT

BRASIMONE NA LOOP

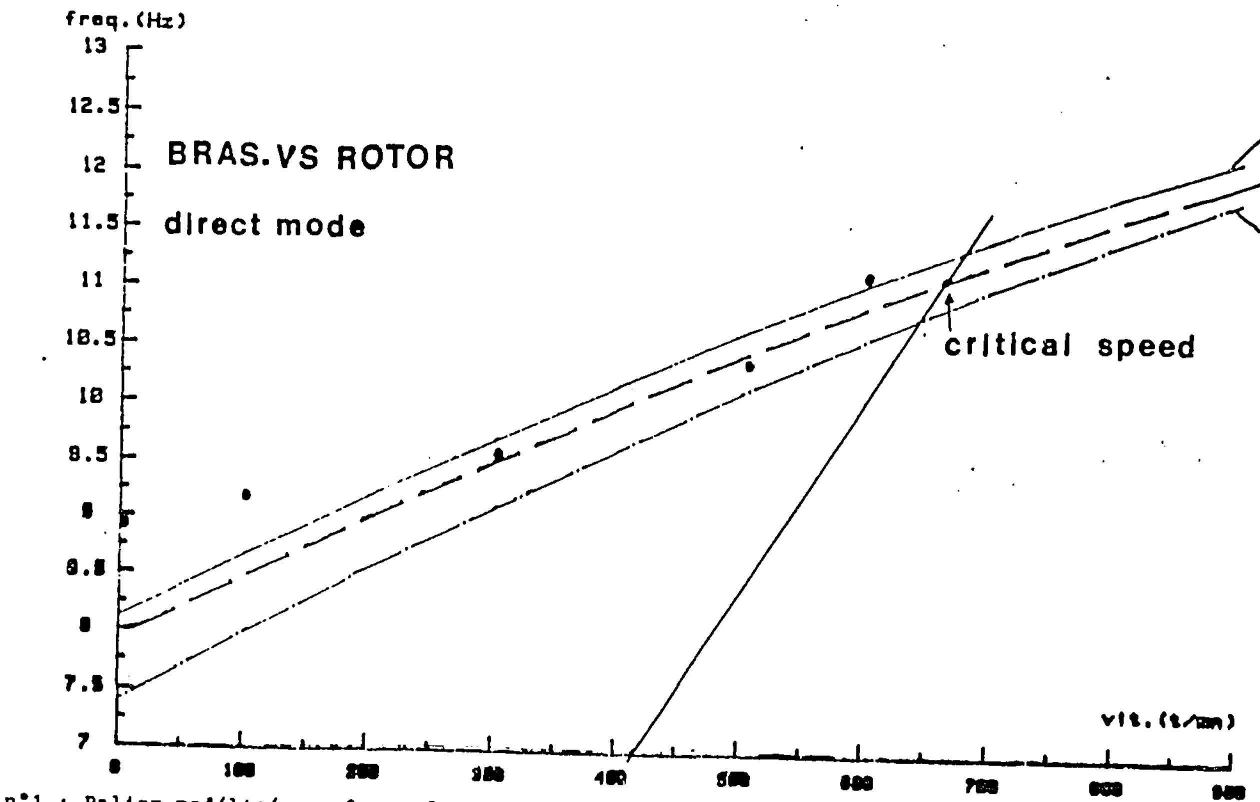


SMALL MODEL

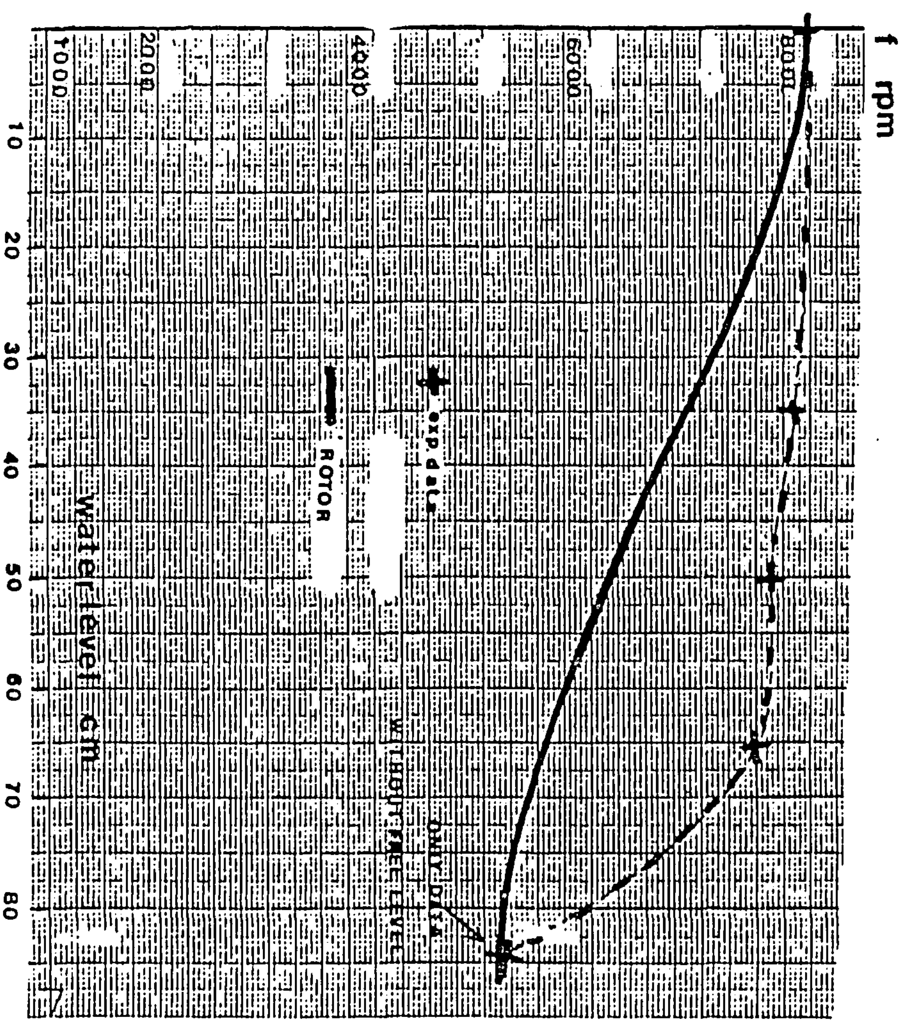
ROTATING AT HIGH SPEED

8000 rpm

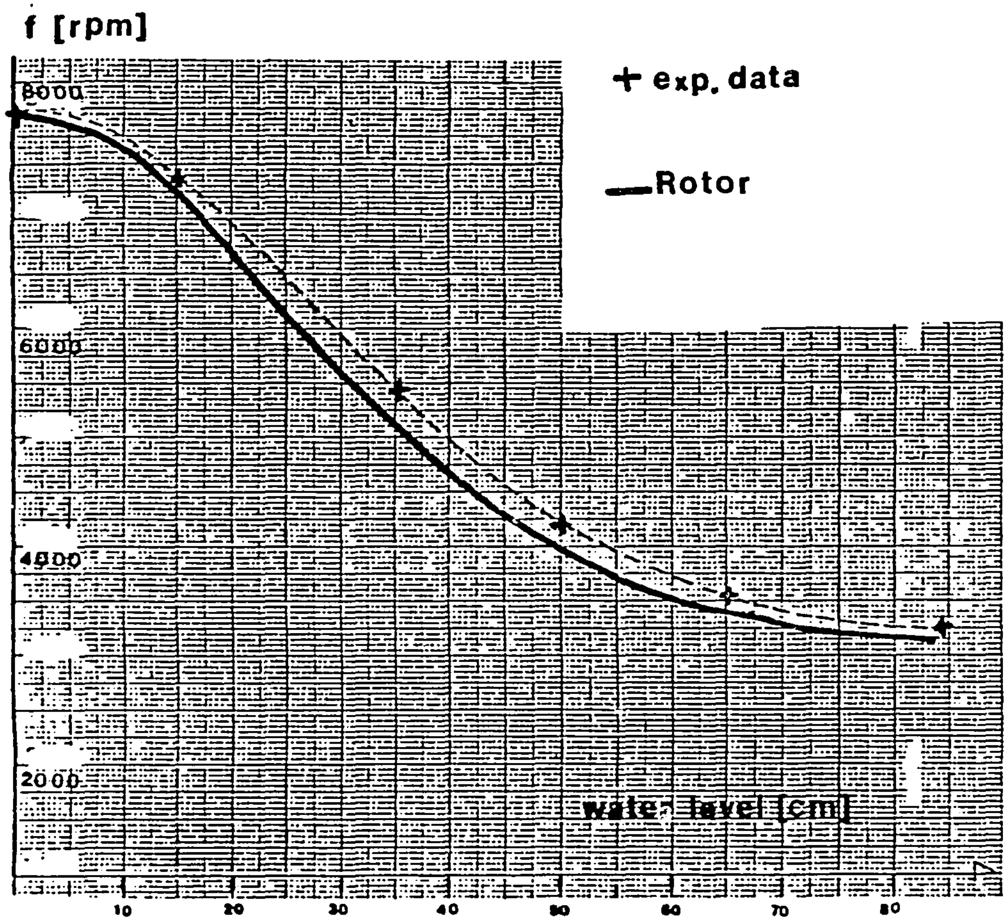
FIGURE 1



n°1 : Palier modélisé sur 2 noeuds + labyrinthe représenté par son confinement (§III.6.B)
n°2 : Palier modélisé sur 2 noeuds (§III.5)
n°3 : Palier modélisé sur 2 noeuds + labyrinthe modélisé comme palier (§III.6.a)
• : Points de mesure.



ROTATING SHAFT: free level effect



SHAFT AT REST

small shell , gap/radius:0.14

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