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ABSTRACT

The one loop fermion-fermion-vector three-point functions, the amplitude for the lepton→lepton+photon decays and the anomalous lepton magnetic moments are given in general gauge theories. The results can be used to get restrictions on fermion mixings and the masses of the intermediate bosons. ~~We~~ discuss briefly, ~~the case of~~ some superstring inspired E_6 grand unified theories.

АННОТАЦИЯ

В калибровочной теории общего вида найдены фермион-фермион-векторные трехточечные функции, амплитуды распадов лептон → лептон + фотон и аномальные магнитные моменты лептонов. Эти результаты позволяют получить ограничения на параметры смешивания фермионов и массы промежуточных бозонов. Кратко обсуждаются некоторые E_6-модели великого объединения, связанные с теорией суперструны.

KIVONAT

Egy általános mértékelméletben megadjuk az egyhurok fermion-fermion-vektor hárompontfüggvényt, a lepton→lepton+foton bomlás amplitudóját és a leptonok anomális mágneses momentumát. Ezen eredményeket fölhasználva, lehetőség nyilik arra, hogy megszorításokat tegyünk a fermionkeveredésre és a közvetitő bozonok tömegére. Röviden tárgyaljuk a szuperhur által inspirált E_6 nagy egyesitést is.

I. Introduction

The success of spontaneously broken gauge theories (SBGT) in the form of the standard model [1] has initiated many ambitious attempts at unifying the basic forces of nature. These attempts (e.g. technicolor ideas [2], grand unification [3], family unification, superstring theories [4]) generally lead to hitherto unknown interactions, with possible violation of the baryon and lepton numbers. By estimating the rate of such exotic processes one can get important restrictions on the various models.

Our objective in this paper is to give the amplitude and width of the lepton $\rightarrow$ lepton + photon decays in a general SBGT where the gauge group is an arbitrary compact semisimple group with a possible Abelian factor. We do not consider supersymmetries. The structure of a general gauge theory is described briefly in Chapter II. where we also discuss the one loop correction to the fermion-fermion-vector one-particle irreducible (FFV 1PI) vertex. The detailed expression for the latter is given in the Appendix.

Using our FFV vertex, in Chapter III. we write down the amplitude and the decay width for the $l \rightarrow l' + \gamma$ process. As a by-product of the calculations the anomalous magnetic moments of leptons are also given in Chapter IV. Figures show the dependence of these quantities on the ratio of the masses of the internal leptons and the exchanged boson; the figures can be used to get lower bounds on the mass of the boson, leading to the exotic process, if the decay rate of the lepton is known (Chapter V). (More precisely, one gets bounds for the boson mass divided by the product of its couplings.) Finally, in Chapter VI. a class of E_6 models suggested by superstring theories is discussed.

II. The FFV 1PI Green-function in a general gauge theory

The structure of general spontaneously broken gauge theories and several aspects related to their applications were investigated in [5]. Here we give only a short review. The gauge group is G=K⊗N; K is an arbitrary compact, semisimple group and N is a possible Abelian factor. The generators of G satisfy the commutation relations

$$[e_\alpha, e_\beta] = i g_{\alpha\beta\gamma} e_\gamma \tag{1}$$

with completely antisymmetric structure constants $g_{\alpha\beta\gamma}$. For the adjoint representation $e_\alpha = T_\alpha$ with $(T_\alpha)_{\beta\gamma} = -i g_{\alpha\beta\gamma}$, for fermions $/\psi/$ we use the notation $e_\alpha = t_\alpha$, for scalars $/\varphi/$ $e_\alpha = \vartheta_\alpha$. The Lagrangian has the form

$$L = -\frac{1}{4} F_{\alpha\mu\nu} F_\alpha^{\mu\nu} + \frac{1}{2} (\nabla_\mu \varphi)_i (\nabla^\mu \varphi)_i + i \bar\psi \gamma^\mu \nabla_\mu \psi - \bar\psi m_0 \psi - \bar\psi \Gamma_i \varphi_i \psi - P(\varphi), \tag{2}$$

where

$$F_{\alpha\mu\nu} = \partial_\mu A_{\alpha\nu} - \partial_\nu A_{\alpha\mu} + g_{\alpha\beta\gamma} A_{\beta\mu} A_{\gamma\nu}, \tag{3.a}$$

$$(\nabla_\mu \psi)^a = \partial_\mu \psi^a - i t_{\alpha b}^{a} A_{\alpha\mu} \psi^b, \tag{3.b}$$

$$(\nabla_\mu \varphi)_i = \partial_\mu \varphi_i - i \vartheta_{\alpha ij} A_{\alpha\mu} \varphi_j, \tag{3.c}$$

the Γ_i-s are the Yukawa couplings and $P(\varphi)$ is the most general G-invariant quartic polynomial in the real scalar components φ_i. Spontaneous breaking is introduced by postulating that the vacuum expectation value of the scalar field be different from zero and the exact mass be positive semidefinite. Then writing $\langle \varphi_i \rangle = \Lambda_i$ and $\varphi_i = \Lambda_i + \phi_i$ (where $\langle \phi_i \rangle = 0$) we obtain the Lagrangian of the SBGT [6].

The expression for the one loop contribution to the FFV 1PI Green function was given in [7]. A more explicit form (which is given in the Appendix with the regularization removed and some of the integrals performed) is decomposed as follows:

$$\Gamma_\alpha^\mu(pqk) = \bar{\omega}\,[\,\Gamma_{1\alpha}^\mu(pqk) + \Gamma_{2\alpha}^\mu(pqk)]\,\omega \tag{4}$$

with

$$\Gamma_{i\alpha}^\mu(pqk) = \Gamma_{i\alpha}^{\bar{I}\mu}(pqk) + \Gamma_{i\alpha}^{\bar{II}\mu}(pqk) + \Gamma_{i\alpha}^{\bar{III}\mu}(pqk), \tag{5}$$

for $i=1,2$. They are given in detail in the Appendix.

Γ_1 and Γ_2 are the contributions of the diagrams (1) and (2) respectively, in Fig.1. $\Gamma_{(1,2)}^{\bar{III}}$ vanish if all momenta are on the mass shell and only $\Gamma_{(1,2)}^{\bar{II}}$ contribute to the lepton-lepton transitions to be discussed later on.

In these expressions the fermion gauge and Yukawa coupling constants are in the diagonal, γ_5-free fermion mass representation:

$$\hat{m} = \bar{\omega}^{+} m \omega^{+}, \; (\hat{m} \text{ diagonal}, \; \omega^{+}\omega = 1, \; \bar{\omega} \equiv \gamma^{0} \omega^{+} \gamma^{0}) \tag{6.a}$$

$$\hat{t}_{\alpha} = \omega t_{\alpha} \omega^{+}, \quad \hat{\bar{t}}_{\alpha} \equiv \gamma^{0} \hat{t}_{\alpha}^{+} \gamma^{0} = \bar{\omega}^{+} \bar{t}_{\alpha} \omega^{+}, \tag{6.b}$$

$$\hat{\Gamma}_{i} = \bar{\omega}^{+} \Gamma_{i} \omega^{+} \tag{6.c}$$

The calculation was performed in a "renormalizable" gauge where the gauge fixing term is given by

$$S_{\xi}(A) = \frac{1}{2} (\partial_{\mu} A^{\mu}_{\alpha}) \, \xi^{-1}_{\alpha\beta} (\partial_{\nu} A^{\nu}_{\beta}), \tag{7.a}$$

$$\xi^{T} = \xi \; ; \; [T_{\alpha}, \xi] = 0 \tag{7.b}$$

but the expressions for the 1PI FFV Green function are given only in generalized Landau gauge where $\xi_{\alpha\beta} = 0$.

III. The lepton $\to$ lepton + photon decay

The decay is described by the following S-matrix element:

$$S(l_1 \to l_2 \gamma) = i(2\pi)^4 \delta(k+p-q) \, e_{\mu}(k,\varsigma) \, \bar{u}(p,\lambda_2) \xi_{A\alpha} \xi_{a}^{l_2} \cdot \\ \cdot \Gamma^{a\mu}_{b\alpha}(p,-q,k) \, \xi^{b}_{l_1} u(q,\lambda_1), \tag{8}$$

where $\Gamma^{a\mu}_{b\alpha}$ is the 1PI FFV three-point function, the ξ factors are finite renormalization constants. They are present because we use the counterterms of the symmetric theory [5]. $\bar{u}(p,\lambda_2)$ and $u(q,\lambda_1)$ are the spinors of the outgoing and incoming particle, respectively, with the mass shell momenta $p^2 = m^2$, $q^2 = M^2$ and $e_\mu(k,\varrho)$ is the photon polarization vector with $k^2 = 0$.

The Lorentz-structure of the physical three-point function is:

$$\tilde{\Gamma}_A^\mu = A_A \gamma^\mu + iB_A \sigma^{\mu\nu} k_\nu + C_A k^\mu + D_A \gamma^\mu \gamma_5 +$$
$$+ iE_A \sigma^{\mu\nu} \gamma_5 k_\nu + F_A k^\mu \gamma_5 . \tag{9}$$

The wavy line refers to the vertex including the finite renormalization constants; the subscript A stands for the photon.

In consequence of the transversality of the photon, the C_A and the F_A formfactors do not give any contribution to the decay amplitude. We can get further restrictions using the Ward identity

$$k_\mu \tilde{\Gamma}^\mu_A(p,-q,k) = 0 \tag{10}$$

which gives $A_A = D_A = 0$. Thus only the formfactors B_A, E_A contribute to the decay. The decay width can be written as follows:

$$\Gamma(l_1 \to l_2 \gamma) = \frac{1}{8\pi M} \frac{(M^2 - m^2)^3}{M^2 + m^2} [|B_A|^2 + |E_A|^2]. \tag{11}$$

Since $\Gamma^{a\mu}_{b\alpha}$ in zeroth order is of the form $\gamma^\mu t_\alpha$, only its one-loop correction gives non-vanishing result. Thus we need the finite renormalization constants only in zeroth order to get the first order contributions.

From the general form (A1-A6) of the 1PI FFV function we get:

$$B_A = B_A^1 + B_A^2 \,, \qquad (12.a)$$

$$16\pi^2 B_A^1 = \tilde{\Gamma}^{\ell}_{+PP} Q_\ell \int_0^1 ds\, \frac{1-s}{s} P(s)\, \frac{m_\ell}{x} -$$
$$- \Gamma^{\ell}_{+PP} Q_\ell \int_0^1 ds\, \frac{1}{x} \{m[-s+P(s)\, \frac{z_{P\ell}(M^2,s)}{xs}] + M[s-P(s)\cdot$$
$$\frac{z_{P\ell}(m^2,s)}{xs}]\} +$$
$$+ \tilde{t}^{\ell}_{+KK} Q_\ell m_\ell \int_0^1 ds\{ \frac{s}{M_K^2} + \frac{P(s)}{s}(3+ \frac{mM}{M_K^2} \frac{(1-s)^2}{s})\} -$$
$$- t^{\ell}_{+KK} Q_\ell \int_0^1 ds\, \frac{1}{x} \{2m[-s+P(s)(1+ \frac{z_{K\ell}(M^2,s)}{xs})] +$$
$$+2M[s+P(s)(1- \frac{z_{K\ell}(m^2,s)}{xs})] + \frac{1}{M_K^2}[m[(M^2-m_\ell^2)s +$$
$$+ \frac{P(s)}{s}[(1-s)(m_\ell^2 - sM^2) + \frac{m_\ell^2-M^2}{x} z_{K\ell}(M^2,s)]] +$$
$$+ M[(m_\ell^2 - m^2)s + \frac{P(s)}{s}[(1-s)(m_\ell^2 - sm^2) + z_{K\ell}(m^2,s)\cdot$$
$$\cdot \frac{m^2-m_\ell^2}{x}]]]\} \,, \qquad (12.b)$$

$$16\pi^2 B_A^2 = -\tilde{\Gamma}^{\ell}_{+PP'} Q_{PP'} \int_0^1 ds\, \frac{m_\ell}{x} R(s) +$$
$$+ \Gamma^{\ell}_{+PP'} Q_{PP'} \int_0^1 ds\, \frac{1}{x} \{(M-m)s + \frac{R(s)}{s}[m\, z_{\ell P}(M^2,s) -$$
$$- M\, z_{\ell P}(m^2,s)]\} +$$
$$+ t^{\ell}_{+KK'} Q_{KK'} \int_0^1 ds\, \frac{1}{x} \{2m[-s+ \frac{R(s)}{s}(1-s+ \frac{z_{\ell K}(M^2,s)}{x})] +$$
$$+2M[s+ \frac{R(s)}{s}(1-s+ \frac{z_{\ell K}(m^2,s)}{x})] +$$
$$+ \frac{1}{M_K^2}[\, m[(M^2-m_\ell^2)s + R(s)(sm_\ell^2 - \frac{1}{xs}(M^2-m_\ell^2) z_{\ell K}(m^2,s))] +$$

$$+ M[(m_\ell^2 - m^2)s + R(s)(sm_\ell^2 + \frac{1}{xs}(m^2 - m_\ell^2)z_{\ell K}(M^2, s))]]] +$$

$$+ \tilde{t}^{\,\ell}_{+KK'} Q_{KK'} m_\ell \int_0^1 ds \{ -\frac{s}{M_K^2} - \frac{R(s)}{2x} [\frac{3(1-s)}{s} + \frac{s m M}{M_K^2}]\}$$

(12.c)

In the above expressions we distinguished the contributions of the diagrams (1) and (2), the contributions of the vector bosons (with masses M_K) and the physical scalars (with masses μ_p), and the handedness changing and preserving processes, because:

$$\tilde{t}^{\,\ell}_{\pm\alpha\beta} \equiv \frac{1}{2} \{ \hat{t}^{L}_{\alpha,\ell_2\ell} \hat{t}^{R}_{\beta,\ell\ell_1} \pm \hat{t}^{R}_{\alpha,\ell_2\ell} \hat{t}^{L}_{\beta,\ell\ell_1} \}, \qquad (13.a)$$

$$t^{\ell}_{\pm\alpha\beta} \equiv \frac{1}{2} \{ \hat{t}^{L}_{\alpha,\ell_2\ell} \hat{t}^{L}_{\beta,\ell\ell_1} \pm \hat{t}^{R}_{\alpha,\ell_2\ell} \hat{t}^{R}_{\beta,\ell\ell_1} \}, \qquad (13.b)$$

$$\tilde{\Gamma}^{\ell}_{\pm ij} \equiv \frac{1}{2} \{ \hat{\Gamma}^{L}_{i,\ell_2\ell} \hat{\Gamma}^{L}_{j,\ell\ell_1} \pm \hat{\Gamma}^{R}_{i,\ell_2\ell} \hat{\Gamma}^{R}_{j\ell\ell_1} \}, \qquad (13.c)$$

$$\Gamma^{\ell}_{\pm ij} \equiv \frac{1}{2} \{ \hat{\Gamma}^{L}_{i,\ell_2\ell} \hat{\Gamma}^{R}_{j,\ell\ell_1} \pm \hat{\Gamma}^{R}_{i,\ell_2\ell} \hat{\Gamma}^{L}_{j,\ell\ell_1} \}, \qquad (13.d)$$

where $\hat{t}^{L}$, $\hat{t}^{R}$, $\hat{\Gamma}^{L}$, $\hat{\Gamma}^{R}$ are the projections of the fermion gauge and Yukawa couplings by $\frac{1}{2}(1 \pm \gamma_5)$, respectively. We note that the electric charges of the internal particles are:

$$\pi_\ell \hat{t}_A = -Q_\ell \pi_\ell \qquad (14.a)$$

$$T_{A,\alpha\beta} = -Q_{\alpha\beta} = Q_{\beta\alpha} \qquad (14.b)$$

$$\vartheta_{A,ij} = -Q_{ij} = Q_{ji} \qquad (14.c)$$

In the expressions (12.b-c) we used the notations:

$$x = M^2 - m^2 \qquad (15.a)$$

$$P(s) = \ln \frac{z_{Bl}(m^2,s)}{z_{Bl}(M^2,s)} \tag{15.b}$$

$$R(s) = \ln \frac{z_{lB}(m^2,s)}{z_{lB}(M^2,s)} \tag{15.c}$$

where B refers to a boson index and z is the same expression as in (A13).

The expressions for the E formfactor can be given by the substitutions $t_+ \to t_-$, $\Gamma_+ \to \Gamma_-$ and $M \to -M$ in the corresponding expressions of B_A.

If we want to examine the decay of the known charged leptons we can reduce these expressions, because the mass of the internal vector boson is supposed to be much higher ($M_K \geq 80$ GeV) than the masses of the external fermions ($m, M \leq 2$ GeV). We assume that this is true for the masses of the internal Higgs bosons as well [8]. By taking the lowest approximation in $m(M)/M_K(\mu_P)$ for the above expressions and integrating over s we get

$$32\pi^2 B_A^1 = \frac{Q_l}{M_K}\{\tilde{t}^{\,l}_{+KK}\tilde{f}^{\,1}_V(R) - t^{\,l}_{+KK}\frac{m+M}{2M_K}f^{\,1}_V(R)\} + \frac{Q_l}{\mu_P}\{\tilde{\Gamma}^{\,l}_{+PP}\tilde{f}^{\,1}_S(r) + \Gamma^{\,l}_{+PP}\frac{m+M}{2\mu_P}f^{\,1}_S(r)\}, \tag{16.a}$$

$$32\pi^2 B_A^2 = \frac{Q_{KK'}}{M_K}\{-\tilde{t}^{\,l}_{+KK'}\tilde{f}^{\,2}_V(R) + t^{\,l}_{+KK'}\frac{m+M}{2M_K}f^{\,2}_V(R)\} + - \frac{Q_{PP'}}{\mu_P}\{\tilde{\Gamma}^{\,l}_{+PP'}\tilde{f}^{\,2}_S(r) + \Gamma^{\,l}_{+PP'}\frac{m+M}{2\mu_P}f^{\,2}_S(r)\}, \tag{16.b}$$

where

$$\tilde{f}^{\,1}_V(R) = \frac{\sqrt{R}}{(1-R)^2}(4+R+R^2+\frac{6R}{1-R}\ln R) \tag{17.a}$$

$$f^{\,1}_V(R) = \frac{1}{3(1-R)^2}(8-30R+9R^2-5R^3-\frac{18R}{1-R}\ln R) \tag{17.b}$$

$$\tilde{f}_V^2(R) = \frac{\sqrt{R}}{(1-R)^2}\left(-4+11R-R^2+\frac{6R^2}{1-R}\ln R\right) \tag{17.c}$$

$$f_V^2(R) = \frac{1}{3(1-R)^3}\left(10-33R+45R^2-4R^3+\frac{18R^3}{1-R}\ln R\right) \tag{17.d}$$

and

$$\tilde{f}_S^1(r) = \frac{\sqrt{r}}{(1-r)^2}\left(-3+r-\frac{2}{1-r}\ln r\right) \tag{17.e}$$

$$f_S^1(r) = \frac{1}{3(1-r)^3}\left(2+5r-r^2+\frac{6r}{1-r}\ln r\right) \tag{17.f}$$

$$\tilde{f}_S^2(r) = \frac{\sqrt{r}}{(1-r)^2}\left(1+r+\frac{2r}{1-r}\ln r\right) \tag{17.g}$$

$$f_S^2(r) = \frac{1}{3(1-r)^3}\left(1-5r-2r^2-\frac{6r^2}{1-r}\ln r\right) \tag{17.h}$$

with $R = \dfrac{m_\ell^2}{M_K^2}$ and $r = \dfrac{m_\ell^2}{\mu_\rho^2}$.

Our decay amplitude is in agreement with that of [9] where it was given (somewhat less explicitly) in this latter approximation.

IV. Anomalous magnetic moments of leptons

The g-2 factor of a particle of mass m and electric charge Q is defined as

$$g-2 = \frac{4m}{Q} B_A = (g-2)_0 + (g-2)_M \tag{18}$$

where B_A is the corresponding form factor in (9) for $\tilde{\Gamma}^\mu_A(p^2=q^2=m^2, k^2=0)$, diagonal in fermion space. The first and second part of the sum represent the contributions of the massless vectors and the massive bosons, respectively.

By using the one-loop expressions (A1-A6) for $\tilde{\Gamma}^{\mu}_{A}$, B_A can be evaluated in the same manner as in the previous paragraph. For the first part of (18) we simply get

$$(g-2)_0 = \frac{1}{4\pi^2} \sum_{\alpha} (\hat{t}_{\alpha})^2_{l_0 l_0} \tag{19}$$

where the sum extends over the conserved generators; for $\alpha \equiv A$ one has the usual electromagnetic expression $Q^2/4\pi^2$. The contribution of the massive vectors and physical scalars

$$(g-2)_{\mu} = \frac{4m}{Q} B_A \tag{20}$$

can actually be obtained from (12.a-c) by putting $l_1 = l_2 = l_0$ and taking the limit $q \to p$, $M \to m$. Thus, using the same notations (with the trivial modification that in (13) both the initial and the final lepton become l_0), we have

$$B_A = B^1 + B^2 \tag{21.a}$$

$$\begin{aligned}
16\pi^2 B^1 = {} & m_l Q_l \tilde{t}^{\,l}_{+KK} \int_0^1 ds\,(1-s)\ \{4s + \frac{1-s}{M_K^2}[m_l^2 + (1-2s)m^2]\} \cdot \\
& \cdot z^{-1}_{Kl}(m^2, s) - \\
& - m\, Q_l t^{l}_{+KK} \int_0^1 ds\,(1-s)\ \{2s(1+s) + \frac{1-s}{M_K^2}[(2-s)m_l^2 - sm^2]\} \cdot \\
& \cdot z^{-1}_{Kl}(m^2, s) + \\
& + Q_l \int_0^1 ds\,(1-s)\ [m_l \tilde{\Gamma}^{l}_{+PP} + sm\Gamma^{l}_{+PP}]\, z^{-1}_{Pl}(m^2, s)
\end{aligned} \tag{21.b}$$

$$\begin{aligned}
16\pi^2 B^2 = {} & -m_l Q_{KK'} \tilde{t}^{\,l}_{+KK'} \int_0^1 ds\ s\{4s + \frac{1-s}{M_K^2}[m_l^2 + (1-2s)m^2]\} \cdot \\
& \cdot z^{-1}_{Kl}(m^2, s) +
\end{aligned}$$

$$+ m\, Q_{KK'}\, t^{l}_{+KK'} \int_0^1 ds\; s\{2s(1+s) + \frac{1-s}{M_K^2}[(2-s)m_l^2 - sm^2]\} \cdot$$

$$\cdot\, z^{-1}_{Kl}(m^2, s) + \tag{21.c}$$

$$+ Q_{PP'} \int_0^1 ds\; s(s-1)[sm\, \Gamma^{l}_{+PP'} + m_l\, \tilde{\Gamma}^{l}_{+PP'}]\; z^{-1}_{Pl}(m^2, s).$$

Similarly, the lowest order approximation in $m/M_K(\mu_P)$ can be given by the expressions (16.a-b) with the substitutions $B^i_A \to B^i$, $(m+M)/2M_K \to m/M_K$.

V. Mass bounds

In sections III. and IV. we gave the amplitude of the lepton $\to$ lepton + photon decay and the anomalous magnetic moment of a lepton in one-loop order. When the masses of the external leptons are much lighter than the mass of the internal boson we got the simple expressions (16.a-b). Thus we can easily investigate - in a model independent way - the contributions of one concrete process to the amplitude we are interested in. If there are no significant cancellations between the different processes the total decay width or the g-2 factor correction can be estimated by the main concrete contribution. By comparing these computed values with the experimental limits we can obtain some lower bounds for the masses of the theory. The present experimental limits of the processes are [10]:

$$\Gamma(\mu \to e\gamma) \leq 1.5 \cdot 10^{-29}\ \mathrm{GeV} \tag{22.a}$$

$$\Gamma(\tau \to \mu\gamma) \leq 1.5 \cdot 10^{-15}\ \mathrm{GeV} \tag{22.b}$$

$$\Gamma_{(\tau \to e\gamma)} \le 1{,}3 \cdot 10^{-15}\,\mathrm{GeV} \tag{22.c}$$

$$\Delta(g-2)_e \le 1{,}24 \cdot 10^{-5} \tag{23.a}$$

$$\Delta(g-2)_\mu \le 1{,}69 \cdot 10^{-7} \tag{23.b}$$

The first three quantities are the corresponding partial decay widths. They are given in 90% confidence limit. The quantity $\Delta(g-2)$ is the absolute value of the difference between the experimental and QED $g-2$ values including the errors.

Applying these inequalities to a concrete process we get the bounds for the internal boson mass divided by its couplings [11]:

$$M_B\left[\frac{\alpha^2}{Q^2\{|\alpha^{\ell}_{+B}|^2+|\alpha^{\ell}_{-B}|^2\}}\right]^{\frac{1}{k}} \ge \left[\frac{M^{k+1}}{\Gamma(\ell_1\to\ell_2\gamma)}\cdot\frac{\alpha^3}{32\pi^2\cdot 2^k}\right]^{\frac{1}{k}}\cdot\{f(\xi)\}^{\frac{2}{k}} =$$

$$\equiv N_1(l_1, l_2; K)\cdot\{f(\xi)\}^{\frac{2}{k}}, \tag{24.a}$$

$$M_B\left[\frac{\alpha}{|Q\cdot\alpha^{\ell}_{+B}|}\right]^{\frac{2}{k}} \ge m\cdot\left[\frac{\alpha}{2\pi\cdot\Delta(g-2)_{\ell_0}}\right]^{\frac{2}{k}}\cdot\{f(\xi)\}^{\frac{2}{k}} =$$

$$\equiv N_2(l_0; K)\cdot\{f(\xi)\}^{\frac{2}{k}}. \tag{24.b}$$

Here M_B is the internal boson mass, Q is equal to the electric charge (in elementary units) of the particle coupled to the photon, and $4\pi\cdot\alpha^{\ell}_{\pm B}$ is one of the combinations of the couplings defined in equations (13.a-d): $t^1{}_{\pm KK'}$, $\tilde{t}^1{}_{\pm KK'}$, $\Gamma^1{}_{\pm PP'}$, $\tilde{\Gamma}^1{}_{\pm PP'}$. α is the fine structure constant. For handedness changing processes K=2, for the preserving ones k=4. The function f is given by one of the expressions of (17.a-h) so that ς stands for R or r.

Since the ς dependence of the right hand sides of (24.a-b) are the same we plot only the functions $f^{2/2}$ in Fig. 2.-5.. The normalization factors $N_1(l_1, l_2; k)$ and $N_2(l_0; k)$ are given in Table I., and by using them one can easily read off from the Fig. 2.-5. the allowed region for the left hand sides of (24.a-b).

In the case of the processes with internal vector boson (17.a-d) it appears that the Appelquist-Carazzone decoupling theorem is violated: we get nonzero or divergent contribution as $R = \left(\frac{m_\ell}{M_K}\right)^2 \to \infty$. Since the masses come from the spontaneous breaking ($m = g\lambda$), we have to distinguish [12] two possible situations: i) $g \to \infty$, ii) $\lambda \to \infty$. Our perturbative calculation can be valid only in the second case. It is easy to see that for this latter possibility the aforementioned theorem is valid. Indeed, if there is a mass hierarchy: $M \lesssim M_B$, $m_\ell >> M_B$, we know [12] that the gauge coupling of a vector boson with mass M_K between fermions with masses M, m_ℓ is of the order $\frac{M_K}{m_\ell} = \varsigma^{-\frac{1}{2}}$. Thus from $t^l{}_{\pm KK}$, we get an extra ς^{-1} factor and the contributions (17.a-d) will be zero in the limit $\varsigma \to \infty$.

VI. Discussion of E_6 models

The possibility that superstring theory [4] could lead to unifying models of the electroweak, strong and gravitational interactions has revived the interest in GUTs based on the group E_6. The reason for this is that the anomaly-free $E_8 x E'_8$ heterotic string theory [13] reduces to a four dimensional $E_6 x E'_8$ model with N=1 supersymmetry after compactification on a manifold with SU(3) holonomy [14]. The group E'_8 describes a "shadow world", which interacts with ordinary matter only gravitationally; it may generate the supersymmetry breaking. Here we shall be concerned only with the E_6 gauge group considered as an effective GUT.

Investigations into the symmetry breakdown of such theories [14], [15] in the context of superstrings, imply that the low energy gauge group will be necassarily larger than the standard one at least by an extra U(1). This $U(1)_X$ can be chosen in different ways [16] but from the view of the exotic leptons in the 27-plets of E_6 there are only two possibilities: either they have $U(1)_X$ invariant masses or they become massive when the $U(1)_X$ symmetry is broken, in the latter case their masses will presumably be considerably lower.

A possible mechanism for the $\mu \to e\gamma$ decay is the following: one can have a mixing betveen the exotic charged leptons (E,M) and the usual ones (e, μ) and the process goes through the diagram (i) given in Fig. 1. with the mediation of the Z and X bosons. The generation mixing is uncontrollable within the E_6 theory while the E-e, M-μ mixings come from the breaking of the (extended) low energy group. Their order of magnitude will partly be determined by the ratio of two breaking scales.

The flavour changing part of the Z-boson exchange is chiral [17] therefore only the handedness preserving amplitude is different from zero; this however is suppressed by a factor of $\frac{M}{M_E}$ (see the term in (16.a.) which is proportional to $t^{l}{}_{+KK}$). The handedness changing process, proportional to the internal lepton mass, is possible only with the mediation of the X boson, whose FC couplings are generally not chiral. From the equations (17. a-b) (as well as from the Fig. 2-3.) one can see that the approximations

$$\tilde{f}^{1}_{V}(R) \approx \sqrt{R} \equiv \frac{m_l}{M_K} \qquad (25.a)$$

$$f^{1}_{V}(R) \approx 1 \qquad (25.b)$$

are roughly valid. Thus the main contribution to the decay comes from the handedness changing process with internal exotic leptons, mediated by the X-boson.

In order to get some mass bounds we have to estimate the mixings between the ordinary and the exotic leptons. Because the left handed e, μ, E, M have the same standard group quantum numbers they can mix at the level of the $U(1)_X$

breaking λ_X, their right partners mix only at the level of the breaking of the standard weak group, λ_W. Thus when the exotic leptons have $U(1)_X$ invariant masses ($\sim \Lambda$), the left and the right handed mixings are of the order of $\frac{\lambda_X}{\Lambda}$ and $\frac{\lambda_W}{\Lambda}$, respectively. If the exotic leptons get their masses from the $U(1)_X$ breaking the mixings are in the order of 1 and $\frac{\lambda_W}{\lambda_X}$.

When the mixings are given by the ratio of two different breaking scales, the unitary matrices, which diagonalize the left and the right handed mass matices, have unitary submatrices in the subspaces of the ordinary leptons up to first order in $\frac{\lambda_X}{\Lambda}$, $\frac{\lambda_W}{\Lambda}$ and $\frac{\lambda_W}{\lambda_X}$, respectively. Because in these subspaces the $U(1)_X$ charges are the same we have the following estimates:

$$\tilde{t}^{E(M)}_{\pm XX} \approx \left[\left(\frac{\lambda_X}{\Lambda}\right)^2 + \left(\frac{\lambda_W}{\Lambda}\right)^2 \right] g_X^2 \quad ; m_E, m_E \sim \Lambda \qquad (26.a)$$

$$\tilde{t}^{E(M)}_{\pm XX} \approx \left[1 + \left(\frac{\lambda_W}{\lambda_X}\right)^2 \right] g_X^2 \quad ; m_E, m_M \sim \lambda_X \qquad (26.b)$$

where g_X is the gauge compling of the X-boson. Supposing that $\Lambda > \lambda_X > \lambda_W$ is valid and the possible mixings are not prohibited, from the equation (24.a) we get:

$$M_X \frac{\alpha}{\alpha_X} \left(\frac{\Lambda}{\lambda_X}\right)^2 \geq N_1(\mu, e \; ; k=2) \frac{m_E}{M_X} \qquad (27.a)$$

$$M_X \frac{\alpha}{\alpha_X} \geq N_1(\mu, e; k=2) \frac{m_E}{M_X} \tag{27.b}$$

Using $M_X \approx g_X \lambda_X$ and $m_E \approx \Gamma\Lambda$ in the first and $m_E \approx \Gamma\lambda_X$ in the second inequality we get:

$$\Lambda \geq \frac{\Gamma}{4\pi\alpha} 1{,}6 \cdot 10^{8}\ \mathrm{GeV}, \tag{28.a}$$

$$M_X \geq \frac{\alpha_X}{\alpha} \frac{\Gamma}{g_X} 1{,}6 \cdot 10^{8}\ \mathrm{GeV}. \tag{28.b}$$

Appendix

In this paragraph we give the 1PI FFV three point function in detail. We use the decomposition indicated in equations (4-5).

$$
\begin{aligned}
32\pi^2\,\Gamma^{I\mu}_{1\alpha}(pqk) = {} & -4\bar{\hat{t}}_b\gamma^\mu\hat{t}_\alpha\hat{t}_b - \hat{\Gamma}_i(1+\ln\pi+\gamma_E)\hat{t}_\alpha\gamma^\mu\hat{\Gamma}_i + \\
& + \int_0^1 ds\,\bar{\hat{t}}_b\pi_a\{\,2[\ln z_{ab}(p^2,s) + \ln z_{cb}(q^2,s) - \\
& - \ln z_{b\Lambda}(0,s)]\gamma^\mu\hat{t}_\alpha + \\
& + \frac{2s}{\Lambda^2-m_b^2}[\ln z_{ab}(p^2,s) - \ln z_{a\Lambda}(p^2,s)](\hat{p}+\hat{m})\hat{p}\gamma^\mu\hat{t}_\alpha + \\
& + \frac{2s}{\Lambda^2-m_b^2}[\ln z_{cb}(q^2,s) - \ln z_{c\Lambda}(q^2,s)]\gamma^\mu\hat{t}_\alpha\hat{q}(\hat{q}-\hat{m})\}\cdot \\
& \cdot\pi_c\hat{t}_b\,, \qquad\qquad \text{(A1)}
\end{aligned}
$$

$$
\begin{aligned}
32\pi^2\,\Gamma^{\overline{II}\mu}_{1\alpha}(pqk) = {} & \bar{\hat{t}}_b\pi_a[2\tilde{U}_{abc} - \frac{1}{\Lambda^2-m_b^2}(\hat{m}-\hat{p})U_{abc}(\hat{m}+\hat{q})]\pi_c\hat{t}_b + \\
& + \delta_{bp}\hat{\Gamma}_b\pi_a U_{abc}\pi_c\hat{\Gamma}_b\,, \qquad\qquad \text{(A2)}
\end{aligned}
$$

$$
\begin{aligned}
32\pi^2\,\Gamma^{\overline{III}\mu}_{1\alpha}(pqk) = {} & \frac{1}{\Lambda^2-m_b^2}[\,\hat{t}_b\pi_a(\hat{m}-\hat{p})\,U_{a\Lambda c}\pi_c\bar{\hat{t}}_b(\hat{m}+\hat{q}) + \\
& + (\hat{p}-\hat{m})\hat{t}_b\pi_a\,U_{a\Lambda c}\pi_c(\bar{\hat{t}}_b\hat{m}-\hat{m}\hat{t}_b)] \qquad\qquad \text{(A3)}
\end{aligned}
$$

$$32\pi^2\,\Gamma^{\overline{\mathrm{I}}\,\mu}_{2\alpha}(pqk) = \bar{\hat{t}}_b\,[2+3(\ln\pi+\gamma_E)]\gamma^\mu\,\hat{t}_c\,T_{\alpha cb} + \hat{\Gamma}_i\,(\ln\pi+\gamma_E)\cdot$$

$$\cdot\gamma^\mu\hat{\Gamma}_j\,\vartheta_\alpha\,{}_{ji} \qquad +$$

$$+\;\bar{\hat{t}}_a\pi_b\int_0^1 ds\{\gamma^\mu\ln z_{ac}(k^2,s)+\frac{1}{\Lambda^2-m_a^2}[(k^\mu\hat{k}-k^2\gamma^\mu)(2s^2-5s+1)\cdot$$

$$\cdot\ln\frac{z_{c\Lambda}(k^2,s)}{z_{ca}(k^2,s)}+((k^2-m_c^2)\gamma^\mu-2k^\mu(\hat{p}-\hat{m}))s\,\ln\frac{z_{c\Lambda}(k^2,s)}{z_{ca}(k^2,s)}-$$

$$-(2(\hat{p}-\hat{m})\hat{p}\gamma^\mu s-2(p^2-\hat{m}^2)\gamma^\mu)\,\ln\frac{z_{b\Lambda}(p^2,s)}{z_{ba}(p^2,s)}]\;+$$

$$+\frac{1}{\Lambda^2-m_c^2}[(k^\mu\hat{k}-k^2\gamma^\mu)(2s^2-5s)\,\ln\frac{z_{a\Lambda}(k^2,s)}{z_{ac}(k^2,s)}+((k^2-m_a^2)\gamma^\mu-$$

$$-2k^\mu(\hat{m}+\hat{q}))s\,\ln\frac{z_{a\Lambda}(k^2,s)}{z_{ac}(k^2,s)}-2(\gamma^\mu\hat{q}(\hat{m}+\hat{q})s+\gamma^\mu(\hat{m}^2-q^2))\cdot$$

$$\cdot\ln\frac{z_{b\Lambda}(q^2,s)}{z_{bc}(q^2,s)}]+\frac{k^2}{(\Lambda^2-m_a^2)(\Lambda^2-m_c^2)}(k^2\gamma^\mu-k^\mu\hat{k})(2s^2-s)\cdot$$

$$\cdot\ln\frac{z_{ac}(k^2,s)\,z_{\Lambda\Lambda}(k^2,s)}{z_{a\Lambda}(k^2,s)\,z_{\Lambda c}(k^2,s)}\quad\}\;\hat{t}_c\,T_{\alpha ca}\;+$$

$$+2k^\mu\int_0^1 ds\{\frac{1}{\Lambda^2-m_a^2}\,\bar{\hat{t}}_a\pi_b\hat{\Gamma}_c\,\bar{\beta}_{\alpha ca}\,\delta_{cp}\,s\,\ln\frac{z_{c\Lambda}(k^2,s)}{z_{ca}(k^2,s)}+$$

$$+\frac{1}{\Lambda^2-m_c^2}\,\delta_{ap}\hat{\Gamma}_a\pi_b\hat{t}_c\beta_{\alpha ca}\,s\,\ln\frac{z_{a\Lambda}(k^2,s)}{z_{ac}(k^2,s)}\}\;+$$

$$+\frac{2k^\mu T_{\alpha ca}}{(\Lambda^2-m_a^2)(\Lambda^2-m_c^2)}\int_0^1 ds\{(m_a^2-m_\alpha^2)\bar{\hat{t}}_a\pi_b(\hat{m}\hat{t}_c-\bar{\hat{t}}_c\hat{m})s\,\ln\frac{z_{\Lambda a}(k^2,s)}{z_{\Lambda\Lambda}(k^2,s)}\;+$$

$$+(m_c^2-m_\alpha^2)(\hat{m}\hat{t}_a-\bar{\hat{t}}_a\hat{m})\pi_b\hat{t}_c\,s\,\ln\frac{z_{\Lambda c}(k^2,s)}{z_{\Lambda\Lambda}(k^2,s)}\}\,, \qquad \text{(A4)}$$

$$32\pi^2\,\Gamma^{\overline{\mathrm{II}}\,\mu}_{2\alpha}(pqk) = \bar{\hat{t}}_a\pi_b\{4\,\tilde{Y}_{abc}+2\gamma^\mu X_{abc}(\hat{k}-\hat{p}+\hat{m})+2(\hat{q}-\hat{k}+\hat{m})\cdot$$

$$\cdot X_{abc}\gamma^\mu-2k^\mu\hat{k}\;I^{00}_{abc}\;+$$

$$+\;\frac{1}{\Lambda^2-m_a^2}\;[2(k^2-m_c^2)(\hat{p}-\hat{m})\;X_{abc}\gamma^\mu-2(p^2-\hat{m}^2)k^\mu(X_{abc}-X_{\Lambda bc}-$$

$$-(\hat{p}+\hat{m})(I^{00}_{abc}-I^{00}_{\Lambda bc}))]\;+$$

$$+\frac{1}{\Lambda^2-m_a^2}\left[-2(k^2-m_a^2)\gamma^\mu X_{abc}(\hat m+\hat q)+2(q^2-\hat m^2)k^\mu\left(X_{abc}-X_{ab\Lambda}-\right.\right.$$

$$\left.\left.-(\hat m-\hat q)(I^{00}_{abc}-I^{00}_{ab\Lambda})\right)\right]\ +$$

$$+\frac{1}{(\Lambda^2-m_a^2)(\Lambda^2-m_c^2)}(\hat p-\hat m)\left[(m_\alpha^2-m_a^2-m_c^2)\,Y_{abc}+k^\mu(m_a^2-k^2)(X_{\Lambda b\Lambda}-X_{ab\Lambda})+\right.$$

$$\left.+(m_c^2-k^2)k^\mu(X_{\Lambda bc}-X_{\Lambda b\Lambda})\right](\hat m+\hat q)\ \}\ \hat t_c T_{\alpha ca}\ +$$

$$+\ \delta_{ap}\hat\Gamma_a\pi_b 2\,Y_{abc}\hat\Gamma_c\,\delta_{cp}\,\vartheta_{\alpha,ca}\ +$$

$$+\ \delta_{ap}\hat\Gamma_a\pi_b\{2\,X_{abc}\gamma^\mu+\frac{1}{\Lambda^2-m_c^2}[\,Y_{abc}+k^\mu(X_{ab\Lambda}-X_{abc})](\hat m+\hat q)\}\cdot$$

$$\cdot\,\hat t_c\,\beta_{\alpha ca}\ +$$

$$+\ \delta_{cp}\,\bar{\hat t}_a\pi_b\{-2\gamma^\mu X_{abc}+\frac{1}{\Lambda^2-m_a^2}(\hat p-\hat m)[\,Y_{abc}+k^\mu(X_{abc}-X_{\Lambda bc})]\}\cdot$$

$$\cdot\,\hat\Gamma_c\bar\beta_{\alpha ca}\,, \tag{A5}$$

$$32\pi^2\,\Gamma_2^{\overline{III}\,\mu}(pqk)=-\frac{1}{\Lambda^2-m_c^2}\,\delta_{ap}\hat\Gamma_a\,\pi_b\,Y_{ab\Lambda}\,\bar{\hat t}_c(\hat m+\hat q)\beta_{\alpha ca}-$$

$$-\frac{1}{\Lambda^2-m_a^2}\delta_{cp}(\hat p-\hat m)\hat t_a\pi_b\,Y_{\Lambda bc}\hat\Gamma_c\,\bar\beta_{\alpha ca}\ +$$

$$+(k^2-m_\alpha^2)\bar{\hat t}_\alpha\pi_b\,\{-\frac{2}{\Lambda^2-m_a^2}(\hat p-\hat m)\ X_{\Lambda bc}\gamma^\mu+\frac{2}{\Lambda^2-m_c^2}\gamma^\mu X_{ab\Lambda}(\hat m+\hat q)+$$

$$+\frac{1}{(\Lambda^2-m_a^2)(\Lambda^2-m_c^2)}(\hat p-\hat m)[\,Y_{abc}-Y_{\Lambda bc}-Y_{ab\Lambda}+Y_{\Lambda b\Lambda}+k^\mu(X_{\Lambda bc}-$$

$$-X_{ab\Lambda})](\hat m+\hat q)\}\ \hat t_c\,T_{\alpha ca}\ +$$

$$+\frac{m_a^2-m_\alpha^2}{\Lambda^2-m_c^2}\,\bar{\hat t}_a\,\pi_b\ \{-2\gamma^\mu X_{ab\Lambda}-\frac{1}{\Lambda^2-m_a^2}(\hat p-\hat m)[Y_{\Lambda b\Lambda}-Y_{ab\Lambda}+k^\mu(X_{\Lambda b\Lambda}-$$

$$-X_{ab\Lambda})]\}\ \bar{\hat t}_c(\hat m+\hat q)\ T_{\alpha ca}\ +$$

$$+\frac{m_c^2-m_d^2}{\Lambda^2-m_a^2}(\hat{p}-\hat{m})\ \hat{t}_a\pi_b\ [\ 2X_{\Lambda bc}\gamma^\mu-\frac{1}{\Lambda^2-m_c^2}[Y_{\Lambda b\Lambda}-Y_{\Lambda bc}+k^\mu(X_{\Lambda bc}-$$

$$-X_{\Lambda b\Lambda})]\ (\hat{m}+\hat{q})]\ \hat{t}_c T_{\alpha ca}+$$

$$+\frac{m_d^2-m_a^2-m_c^2}{(\Lambda^2-m_a^2)(\Lambda^2-m_c^2)}[\ \bar{\hat{t}}_a\pi_b(\hat{m}-\hat{p})\ Y_{\Lambda b\Lambda}\bar{\hat{t}}_c(\hat{m}+\hat{q})+(\hat{p}-\hat{m})\ \hat{t}_a\pi_b Y_{\Lambda b\Lambda}\cdot$$

$$\cdot(\bar{\hat{t}}_c\hat{m}-\hat{m}\hat{t}_c)]\ T_{\alpha ca}\ . \qquad (A6)$$

Here π_a is a projection into the fermion subspace with mass m_a. δ_{bP} means that the scalar index runs over physical scalars only. $\beta_{\alpha\gamma i}$ is a vector-vector-scalar coupling generated by spontaneous breaking:

$$\beta_{\alpha\gamma i}\equiv T_{\alpha,\gamma\delta}\ \vartheta_{\delta,ij}\lambda_j-2\,\vartheta_{\alpha,ki}\,\vartheta_{\gamma,kj}\lambda_j\qquad ,$$

where the λ_j-s are the tree level scalar vacuum expectation values. Λ^2 is an infrared cut-off to avoid infrared divergencies at an intermediate level of the calculations. The further notations are:

$$U_{abc}=2[\ \hat{q}\gamma^\mu\hat{t}_\alpha\hat{q}\ I^{20}_{abc}+\hat{p}\gamma^\mu\hat{t}_\alpha\ \hat{p}\ I^{02}_{abc}+(\hat{q}\gamma^\mu\hat{t}_\alpha\ \hat{p}+$$

$$+\hat{p}\gamma^\mu\hat{t}_\alpha\ \hat{q})\ I^{11}_{abc}+[(\hat{k}-\hat{m})\gamma^\mu\hat{t}_\alpha\ \hat{q}-\hat{q}\gamma^\mu\hat{t}_\alpha\ \hat{m}]\ I^{10}_{abc}+$$

$$+[(\hat{k}-\hat{m})\gamma^\mu\hat{t}_\alpha\ \hat{p}-\hat{p}\gamma^\mu\hat{t}_\alpha\ \hat{m}]\ I^{01}_{abc}+$$

$$+(\hat{m}-\hat{k})\gamma^\mu\hat{t}_\alpha\ \hat{m}\ I^{00}_{abc}-\hat{t}_\alpha\gamma^\mu L_{abc}\ ], \qquad (A7)$$

and

$$\tilde{U}_{abc}=\bar{\hat{t}}_\alpha\ u_1-\hat{m}\hat{t}_\alpha\ (u_2+\bar{u}_2)-\bar{\hat{t}}_\alpha\ \hat{m}\ (u_3+\bar{u}_3)+$$

$$+\hat{m}\hat{t}_\alpha\ \hat{m}\ \bar{u}_4\ , \qquad (A8)$$

if

$$U_{abc} = \hat{t}_\alpha u_1 + \hat{m}\tilde{\hat{t}}_\alpha u_2 + \hat{t}_\alpha \hat{m} u_3 + \hat{m}\hat{\hat{t}}_\alpha \hat{m} u_4 \; ; \tag{A9}$$

$$\begin{aligned} Y_{abc} = {} & 2p^\mu \hat{p}\, I^{02}_{abc} + 2(p^\mu \hat{q} + q^\mu \hat{p})\, I^{11}_{abc} + 2q^\mu \hat{q}\, I^{20}_{abc} + \\ & + [k^\mu \hat{p} + 2p^\mu (\hat{m}-\hat{q})]\, I^{01}_{abc} + [k^\mu \hat{q} + 2q^\mu (\hat{m}-\hat{q})]\, I^{10}_{abc} + \\ & + k^\mu (\hat{m}-\hat{q})\, I^{00}_{abc} + \gamma^\mu L_{abc} , \end{aligned} \tag{A10}$$

$$\tilde{Y}_{abc} = Y_{abc} \; (\hat{m} \to -2\hat{m}) , \tag{A11}$$

$$X_{abc} = \hat{p}\, I^{01}_{abc} + \hat{q}\, I^{10}_{abc} + (\hat{m}-\hat{q})\, I^{00}_{abc} , \tag{A12}$$

$$z_{ab}\,(p^2, s) = m_b^2 + s(m_a^2 - m_b^2 - p^2) + s^2 p^2 . \tag{A13}$$

Finally the integrals in (A7-A12) are

$$I^{kl}_{abc} = \int_0^1 ds \int_0^1 dt \; z^{-1}_{abc}\,(pqk; st)\; s^k t^l , \tag{A14}$$

$$L_{abc} = \int_0^1 ds \int_0^s dt \; \ln(z_{abc}\,(pqk; st)) \tag{A15}$$

with

$$\begin{aligned} z_{abc}(pqk; st) = {} & m_c^2 + s(m_b^2 - m_c^2 - q^2) + t(m_a^2 - m_b^2 + \\ & + q^2 - k^2) + (sq+tp)^2 . \end{aligned} \tag{A16}$$

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	k=2	k=4
$N_1(\mu, e; k)$	$1.6 \cdot 10^8$ GeV	$2.9 \cdot 10^3$ GeV
$N_1(\tau, \mu; k)$	$1.2 \cdot 10^3$ GeV	33 GeV
$N_1(\tau, e; k)$	$1.2 \cdot 10^3$ GeV	32 GeV
$N_2(e; k)$	48 MeV	5.0 MeV
$N_2(\mu; k)$	$7.3 \cdot 10^2$ GeV	8.9 GeV

Table I. The values of the N_1 and N_2 quantities which are the normalization factors in the inequalities (24.a-b). $N_1(l_1, l_2; k)$ comes from the $l_1 \to l_2 \gamma$ decay; $N_2(l_0; k)$ from the (g-2) factor of the lepton l_0. k=2 and k=4 mean the handedness changing and preserving processes, respectively.

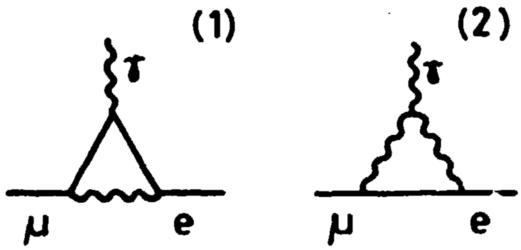

Fig. 1

The two types of diagrams which contribute to the 1PI FFV three point functions. The straight and the wavy lines denote fermions and bosons, respectively

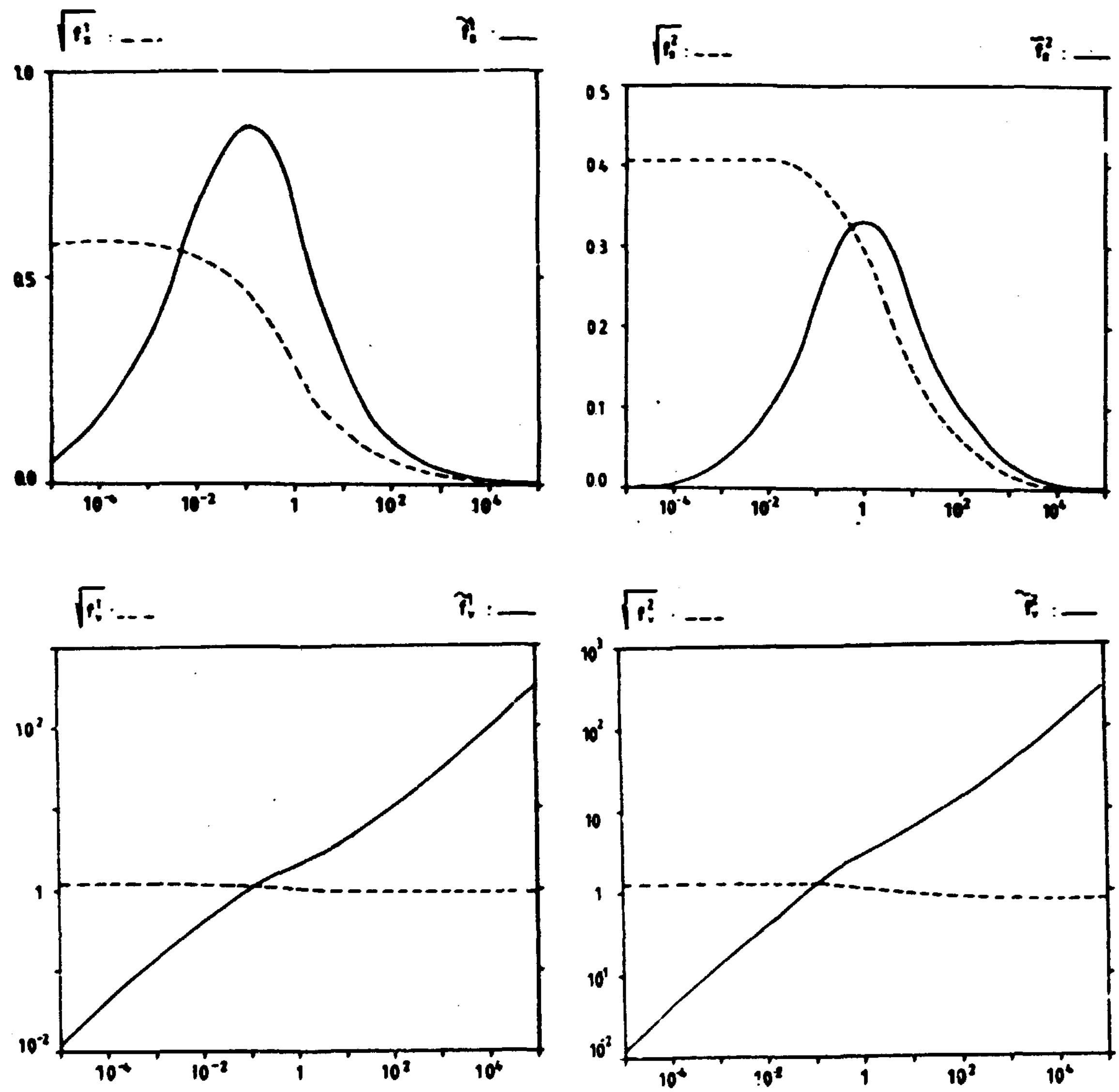

Fig. 2-5

We plotted here the functions in pairs: $\tilde{f}^1_S$, $\sqrt{f^1_S}$; $\tilde{f}^2_S$, $\sqrt{f^2_S}$; $\tilde{f}^1_V$, $\sqrt{f^1_V}$; $\tilde{f}^2_V$, $\sqrt{f^2_V}$. *The superscripts 1 or 2 denote which diagram in Fig.1 they come from, the subscript letters S, V refer to the internal scalar and vector particle, respectively. There is a tilde for the functions which describe the mass dependence of a handedness changing process. For convenience we have a square root of the handedness preserving functions because these are the quantities which occur in the inequalities (24.a-b)*

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