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AN ELEMENTARY PRESENTATION OF THE PS "BEAM CONTROL" SYSTEM

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INTRODUCTION

Here is a fresh attempt, following a few others, to demystify the PS beam control system. The writer is making this attempt on his own account, hoping thereby to convince himself that he has indeed understood how "his" system works, and he hopes it may also prove helpful to others. So the point here will not be to do original work, but rather to give a rundown of what has been written on the subject from time to time, and I am thinking especially of the course written by H.G. Hereward[1] for PS operators, which has been the direct inspiration for this description.

The engineering specifications used to build the equipment have been omitted in this paper; they would fill a bulky file, perusal of which would be too laborious for the non-specialist.

Actual talks on the subject at the PS have already served as a proving ground for this presentation, and the writer has benefitted as much as anyone from the interest shown by many on the occasion of those gatherings.

1. THE FREQUENCY PROGRAM

The simplest RF system imaginable consists of an oscillator whose frequency can be controlled by means of an outside voltage. The oscillator acts on the RF cavities, and a function generator connected to the magnetic field (frequency program) controls the frequency of the oscillator (Fig. 1). For, this is connected with the guiding field by the relationship

$$f_{RF} = h \frac{c}{2\pi R} \frac{B}{\sqrt{B^2 + (E_0/c\rho)^2}} \qquad (1)$$

in which:

- h is the harmonic number ($h = 20$)
- c is the velocity of light ($3 \cdot 10^8$ m/sec)
- R is the mean radius of the machine ($R = 100$ m)
- B is the magnetic field (in teslas)
- E_0 is the proton rest energy, $E_0 = 938\ 10^6$ eV
- ρ is the radius of curvature of the magnets (70 m)

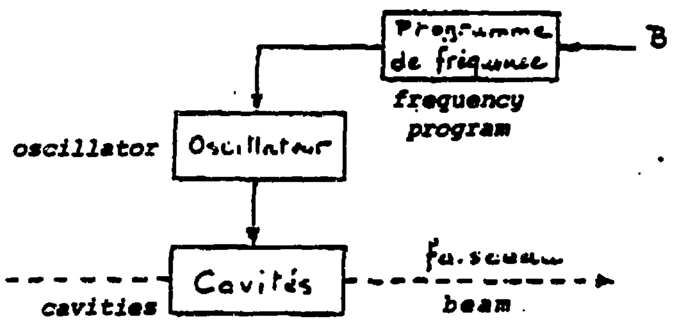

Fig. 1. The simplest of all RF systems.

In the case of the PS, the frequency program is obtained by a digital function generator using a particular train B derived from the voltage induced in a coil placed in the reference magnet. The function is approximated by nine segments whose termini are defined by the coincidences between the train B and preregulated values.

Note that for a given machine, the function (1) depends only on the field; the coincidences are regulated once for all.

What is the precision required for the accelerator frequency? For a given magnetic field, an error in the accelerator frequency, the frequency of revolution, that is, manifests itself by a deviation in radial position given by the relationship

$$\frac{\Delta R}{R} = \frac{\gamma^2}{\gamma_{tr}^2 - \gamma^2} \cdot \frac{\Delta f}{f} \tag{2}$$

in which γ is the usual relativistic parameter and γ_{tr} is its value at the transition ($\gamma_{tr}^2 \simeq 37$ in the PS).

For example, at high energy ($\gamma >> \gamma_{tr}$) an error of 0.1% on the frequency becomes a radial displacement of 10 cm, which is obviously unacceptable. But then the error of linearity of the oscillator alone is on the order of one percent!

The conclusion is that it is necessary, at least for the PS, to find a method of reducing errors of the frequency program. Note that this requirement applies to the PS, but is not universal (ISR or SPS cases where the relative frequency variation is very small).

<u>Remark</u>

The relationship (2) is valid for an equilibrium state. This is not the case, in particular, at the transition, or we should have an infinite position error for the slightest frequency deviation.

2. ATTEMPTING TO CORRECT THE FREQUENCY PROGRAM

To avoid the need for prodigies of technology in order to obtain the required precision, we could conceive of a feedback correcting system. The most obvious idea would be to detect any deviation in the radial position of the beam and use this signal, duly amplified, to correct the frequency program (Fig. 2).

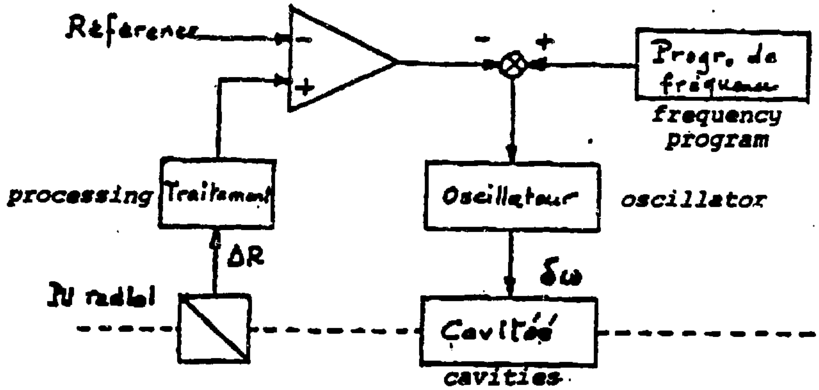

Fig. 2. First attempt at correcting the frequency program.

The radial position of the beam is obtained by one or more PU stations (3, in the PS). The signals from the PU station are processed to convert them into a quasicontinuous voltage proportional to the radial displacement (zero reset of baseline, integration, reference to the intensity, combination of the several PU stations). These operations are represented by the black box *Processing* in Fig. 2.

By the way, we may want to locate the beam elsewhere than in the center of the chamber. This can be done by applying a voltage to the *Reference* input, which in turn amounts to shifting the electrical zero of the PU stations. This signal applied to the reference input is traditionally referred to at the PS as the perturbation signal.

Note in Fig. 2 that our loop system is closed by the beam, whose characteristics we must know in order to determine whether the system is stable or unstable. For purposes of the problems with which we are concerned at the moment, the beam can be represented by a black box: a change $\Delta f = \delta\omega/2\pi$ in accelerator frequency is applied to the input, and a change ΔR in radial beam position is picked up at the output. We shall now try to determine the transfer function of this box.

3. BEAM TRANSFER FUNCTIONS

We try to find the input-output transfer function of the black box representing the beam.

One way to get it is to start from the synchrotron equations

$$\begin{cases} \dfrac{d\Delta\phi}{dt} = a\ \Delta R \\ \dfrac{d\Delta R}{dt} = b\ \Delta\phi \end{cases} \qquad (3)$$

\- linearized - where $\Delta\phi$ is the phase deviation of a particle from the phase of the synchronous particle and ΔR is the deviation of position from the synchronous orbit.

The first of these equations is simply an expression of the frequency deviation

$$\frac{d\Delta\phi}{dt} = 2\pi\ \Delta f = 2\pi\ \frac{\Delta f}{f} \cdot f = 2\pi f \left(\frac{\gamma_{tr}^2 - \gamma^2}{\gamma^2}\right) \frac{\Delta R}{R} \quad \text{(cf. equation 2)}$$

whence

$$a = -2\pi\ \frac{f}{R}\ \gamma_{tr}^2\ \eta \qquad (4)$$

with:

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}\ .$$

The second equation expresses the deviation in energy gain. A phase deviation $\Delta\phi$ yields a gain deviation per revolution of $eV \cos \phi_s\ \Delta\phi$ (V is the accelerator voltage), or an energy deviation per unit time of:

$$\frac{dE}{dt} = eV \cos \phi_s \cdot \Delta\phi \cdot \frac{\beta c}{2\pi R}\ .$$

This energy deviation becomes a radial deviation

$$dR = -\frac{1}{\gamma_{tr}^2}\frac{1}{\beta^2}\frac{R}{E}\,dE ,$$

which finally gives:

$$b = -\frac{eV\cos\phi_s\, c}{2\pi\beta\,\gamma_{tr}^2\,E} . \tag{5}$$

Now if the accelerator frequency suffers a perturbation $\delta\omega$, only the first of these equations is affected; the equations (3) are then written

$$\begin{cases} \dfrac{d\Delta\phi}{dt} = a\,\Delta R + \delta\omega \\ \dfrac{d\Delta R}{dt} = b\,\Delta\phi \end{cases} \tag{6}$$

where $\Delta\phi$ is now the change in phase between beam and accelerator voltage.

Replacing the derivatives with respect to time, as in electric circuits, with multiplications by $j\omega$, we obtain

$$\begin{cases} j\omega\,\Delta\phi = a\,\Delta R + \delta\omega \\ j\omega\,\Delta R = b\,\Delta\phi \end{cases} \tag{7}$$

whence, eliminating $\Delta\phi$, and putting $ab = -\Omega^2$ ($\Omega/2\pi$ is of course the synchrotron frequency):

$$\Delta R = \frac{b}{\Omega^2 - \omega^2}\,\delta\omega = \mu_2\,\delta\omega . \tag{8}$$

We can also calculate the quantity $\Delta\phi$:

$$\Delta\phi = \frac{j\omega}{\Omega^2 - \omega^2} \; \delta\omega = \mu_1 \; \delta\omega \quad . \tag{9}$$

Thus the beam is represented by the following black box (Fig. 3), in which the output $\Delta\phi$ has also been shown (being none other, except for a factor 1/b, than the derivative of ΔR).

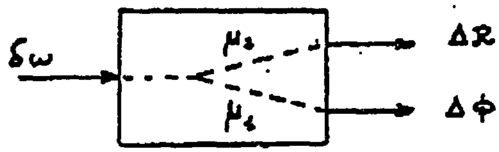

Fig. 3. Beam represented by equivalent black box.

The reader familiar with the notation of transfer functions will easily recognize the characteristics of an oscillating circuit without damping (for $\omega = \Omega$, the amplitude of the oscillation becomes infinite).

<u>Remark</u>

The so-called adiabatic damping of oscillations, due to the fact that the parameters of the oscillation vary with time, is here neglected. It in fact has much less effect than the one we are seeking to create by the beam control system.

We may also represent the black box of Fig. 3 by an equivalent electric circuit:

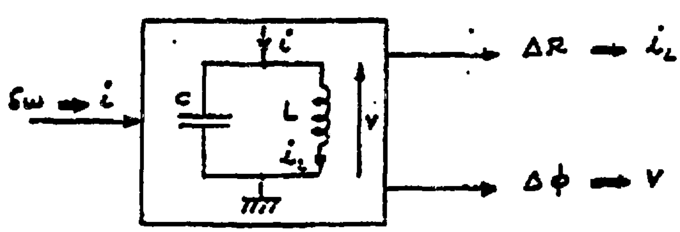

Fig. 4. Oscillating circuit equivalent to the beam.

For we have:

voltage at terminals of oscillating circuit

$$v = \frac{1}{C} \cdot \frac{j\omega}{\Omega^2 - \omega^2} \; i$$

current in inductance L

$$i_L = \frac{1}{LC} \cdot \frac{1}{\Omega^2 - \omega^2} \; i \qquad (10)$$

– putting $\Omega^2 = 1/LC$. If we take the correspondence $(v, \Delta\phi)$ and $(i_L, \Delta R)$, we indeed come back to the form of equations (8) and (9).

Another way to get the transfer functions is to start out from trajectories in the longitudinal phase plane. The synchrotron oscillation of small amplitude is represented in the phase plane by an elliptical trajectory (Fig. 5a).

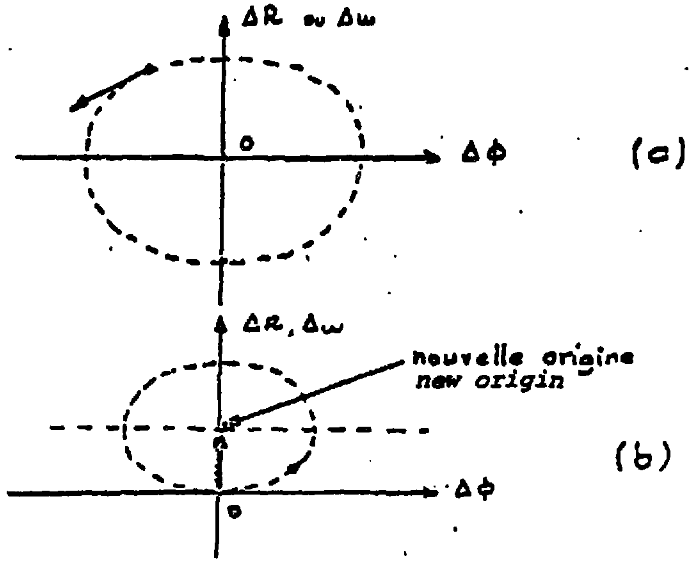

Fig. 5. RF frequency step represented in the phase plane.

The origin corresponds to the synchronous particle, and the vertical axis is graduated in energy, or radial position, or frequency, which for given magnetic field comes to the same thing. Applying a frequency step $\Delta\omega/2\pi$ thus amounts to shifting the origin along the vertical axis. The particle initially in equilibrium at 0 will then oscillate on the elliptical trajectory around the new origin (Fig. 5b). If we represent this oscillation as a function of time for the two variables $\Delta\phi$ and ΔR, we obtain Fig. 6.

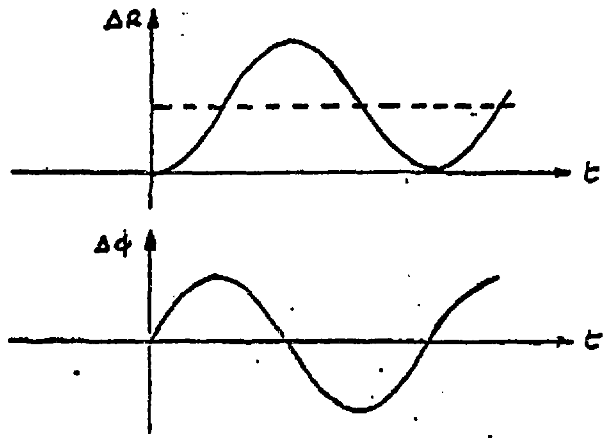

Fig. 6. Beam responses to a frequency step.

We indeed come back to the behavior of an undamped resonator as described by the equivalent diagram of Fig. 4 when acted upon by a step in current.

4. RESPONSE OF LOOP SYSTEM

Having obtained the transfer functions of the beam, we can now determine the response of the servo mechanism of Fig. 2. For the diagram of our servo principle is the following:

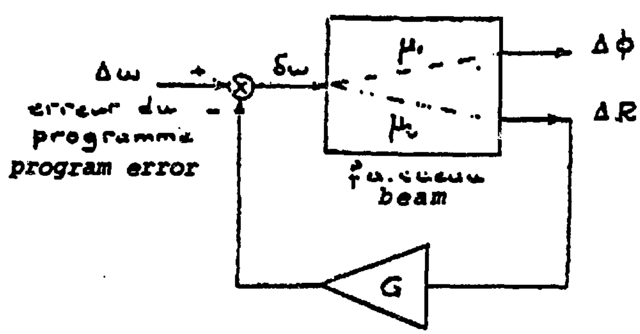

Fig. 7. Diagram of principle corresponding to Fig. 2.

The gain G converts the radial displacement of the beam into a change in frequency of the oscillator. We are looking for the deviation in radial position corresponding to an error $\Delta\omega$ in the frequency program. We have

$$\Delta R = \mu_2 \; \delta\omega = \mu_2 \; (\Delta\omega - G\Delta R) \quad ,$$

whence

$$\Delta R = \frac{b}{\Omega^2 + bG - \omega^2} \, \Delta\omega \quad . \tag{11}$$

For static program errors ($\omega = 0$), we reduce the deviation in radial position considerably if bG is large and positive ($bG >> \Omega^2$). In that case, we have substantially

$$\Delta R \simeq \frac{1}{G} \, \Delta\omega$$

- a familiar result for servo mechanisms. Note that the quantity

$$b = - \frac{eV\frac{c}{C}\cos\phi_s}{2\pi\beta \, \gamma_{tr}^2 \, E}$$

changes sign at the transition. For as a changes sign, owing to η, b must also change to $-b$ ($\phi_s \to \pi - \phi_s$) so that we will still have $-ab = \Omega^2$, a positive quantity. Hence we must change the sign of G at the transition so that we still have a negative and not a positive feedback.

How does the servo mechanism behave in transient mode? We can answer this question immediately by comparing equation (11) with equation (8). We see that they have the same form; Ω^2 has simply been replaced by $\Omega^2 + bG$, a much greater quantity. Hence the response is still that of an undamped resonator, but one resonating at a much higher frequency this time.

We may also say - which comes to the same thing - that the servo mechanism is just at its limit of stability, as one may convince one's self by drawing the Nyquist diagram of the curve of complex open-loop gain as a function of frequency (Fig. 8).

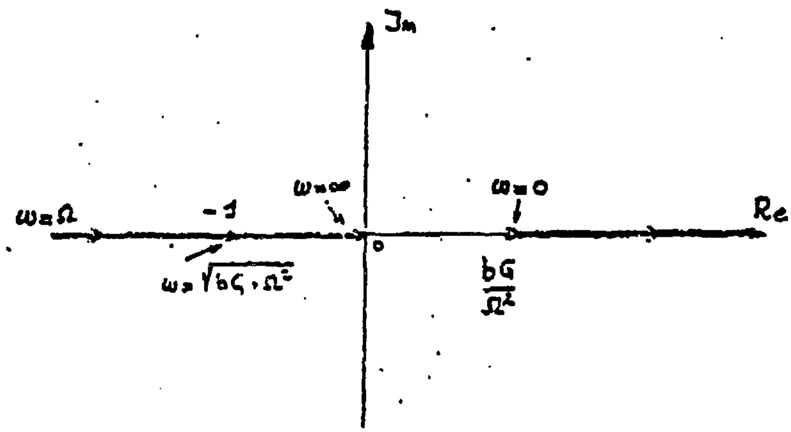

Fig. 8. Nyquist diagram for servo mechanism of Fig. 7.

The open-loop gain is

$$G\mu_2 = \frac{bG}{\Omega^2 - \omega^2} .$$

For zero frequency ($\omega = 0$), we find the abscissa point bG/Ω^2. The representative point then travels along the axis of reals through increasing values, goes to infinity, and changes sign for $\omega = \Omega$, then passing through the point -1 for $\omega = -\sqrt{bG + \Omega^2}$. So we are just at the limit of stability for a pure real gain G. If we allow for the unavoidable delay in the electronics between PU station and cavity, the complex gain curve will pass slightly above the point -1, and in that case, as we know, Nyquist's criterion tells us that the system is unstable.

While it is always possible to avoid this catastrophic situation, for example by means of a phase advance network, we see that the transient response will still be highly oscillatory, and this of course is not very satisfactory for a servo mechanism.

It is interesting to follow up the electrical analogy of Fig. 4 in the case of the looped system. The loop amplifier adds a current into the circuit, of value $-G\ i_L$ (Fig. 9). This generator in turn is equivalent to an induction of value $L' = L/G$. For in both cases, the voltage V at the terminals of the dipole will cause a current $Gv/jL\omega$ to flow. The two inductances in parallel are equivalent to another of much smaller value, which explains why the apparent resonance frequency of the circuit is considerably increased.

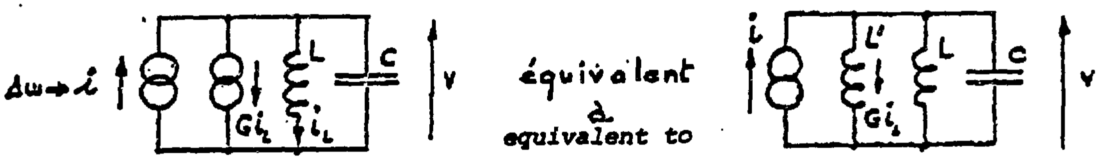

Fig. 9. Electrical equivalents of the servo of Fig. 7.

5. A FREQUENCY PROGRAM DERIVED FROM THE BEAM

When the beam is already bunched, that is, some time after injection, we can easily measure the frequency of revolution of the beam, for example by connecting a frequency-voltage converter to the output of a so-called phase PU (suitably filtered) (Fig. 10). In this way, we manufacture a frequency program that can have better precision than the digital program. For we free ourselves from the approximation to the theoretical function by straight-line segments. Only the non-linearities of the oscillator and frequency-voltage converter remain.

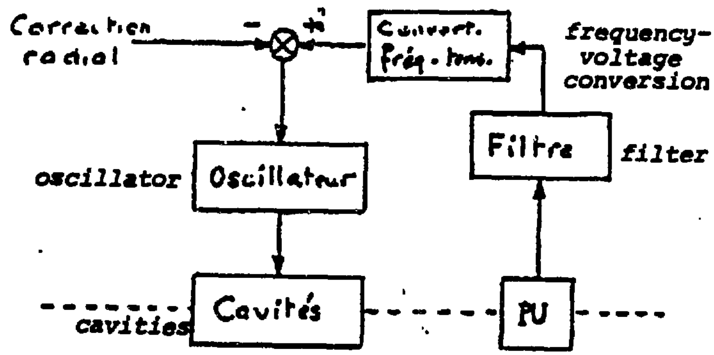

Fig. 10. A frequency program derived from the beam.

In the PS, this so-called beam-derived program is used instead of the digital program a few msec after injection. By doing this, we introduce a new negative feedback loop, as may be seen in Fig. 10. Note that this loop acts in the same way as the one in Fig. 2. Thus it converts a change in beam frequency (or, what amounts to the same thing, a change in radial position) into a change in frequency of the oscillator. These two changes will of course be equal, so that the frequency program will be correctly calibrated. Hence the gain of this loop (between beam and oscillator) is unity. If this gain is expressed in the same units as G (unit frequency/unit radial position), we obtain Ω^2/b. This is in fact simply the conversion factor between radial displacement and frequency change (see for example equation (8) at the point of equilibrium $\omega = 0$).

Thus we can simply allow for this auxiliary loop by replacing G with $G - \Omega^2/b$ in the preceding results. Having just chosen $G \gg \Omega^2/b$, we see that the change is minute. For example, the new resonance frequency is now $\omega = \sqrt{bG}$ instead of $\omega = \sqrt{bG + \Omega^2}$.

Note that since the information is taken from the frequency and not from the position, the gain does not change sign at the transition (the gain of the radial loop is always decreased).

6. HOW TO DAMP THE SYSTEM? THE PHASE LOOP

We should like our equivalent oscillating circuit of Fig. 9 to be strongly damped, by placement of a small resistance between its terminals (Fig. 11a). By the same technique we used to manufacture the fictive self induction L', let us add a current generator of value G'v, which is exactly equivalent to placing a resistance 1/G' in parallel in the circuit (Fig. 11b).

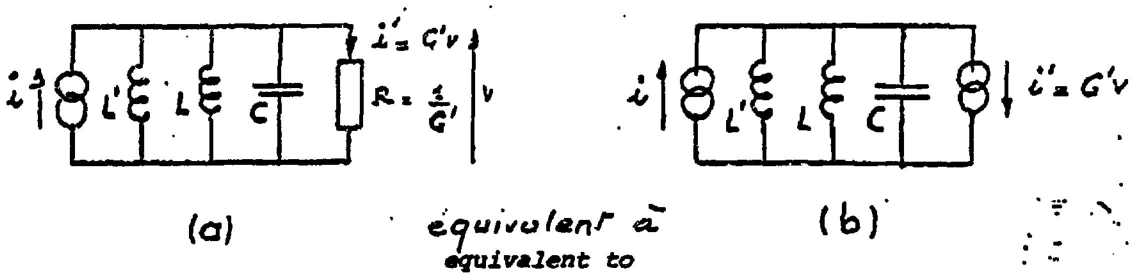

Fig. 11. Damping the oscillating circuit with a resistance or an equivalent current generator.

This additional generator of a current proportional to the voltage expresses itself in terms of feedback by an additional loop acting on the oscillator from the phase of the beam relative to the accelerator voltage (Fig. 12).

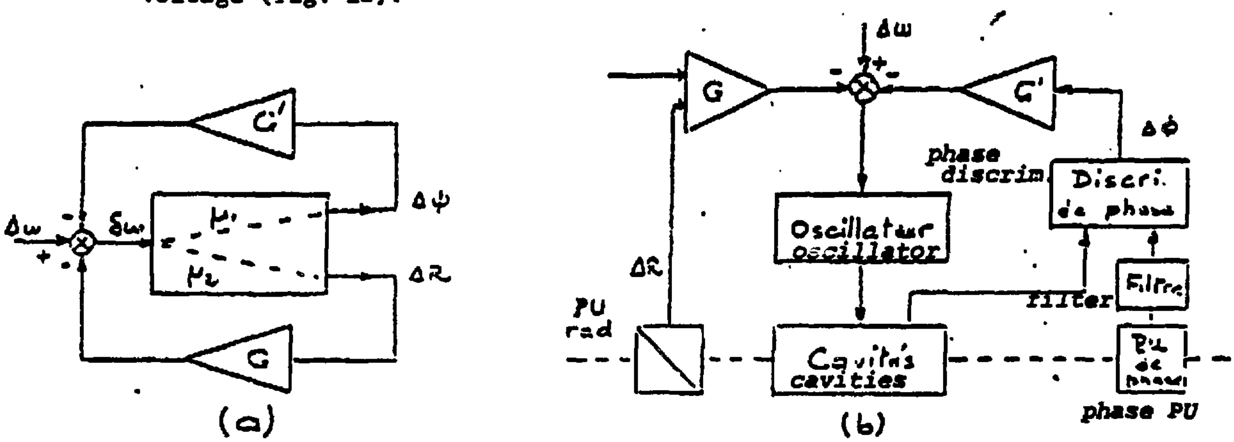

Fig. 12. Damping the system by means of an auxiliary loop.

For in our analogy, a source of current corresponds to a change in frequency of the oscillator, and the voltage v at the terminals of the circuit is proportional to a phase deviation between beam and RF. The information on the phase of the beam is taken by means of a suitably filtered phase PU. The phase of the beam and that of the accelerator voltage are compared in a phase discriminator whose output, amplified by G', corrects the frequency of the oscillator.

A variation on this scheme is indicated in Fig. 13. The radial correction signal now acts on the oscillator through a dephaser, the phase discriminator and the gain amplifier G', and this does not change the principle of the circuit in any way. We now define a gain G" of the radial loop (in degrees or radians per unit radial displacement), and of course we get the relationship $G_{(\text{Fig. 12})} = G'G''$.

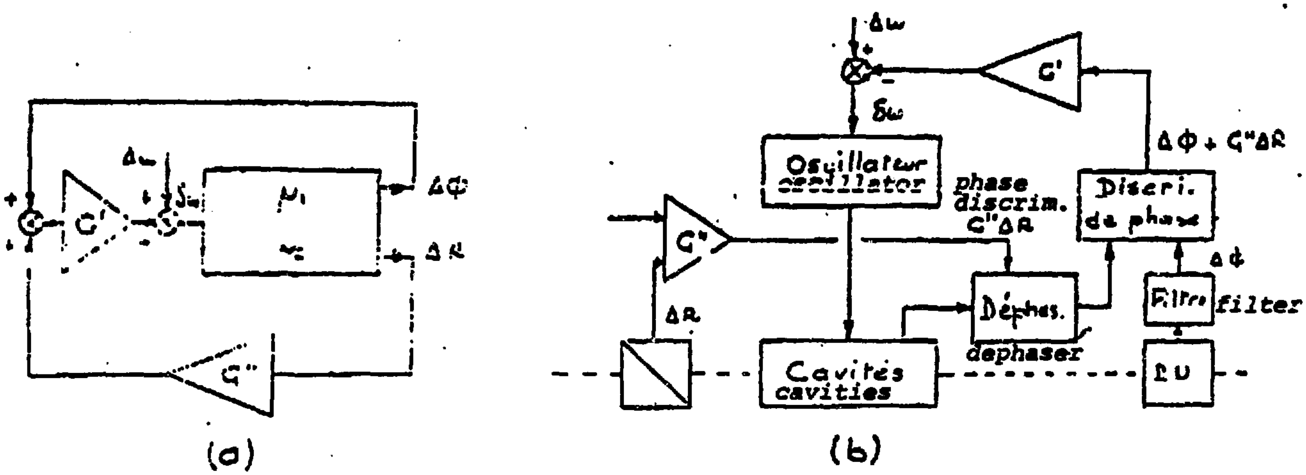

Fig. 13. Variation on the scheme of Fig. 12.

This in fact is the scheme used at the PS, and the one with which we are going to continue. First we shall verify more directly that the analogy of the oscillating circuit does give us the damping we are looking for.

The frequency shift $\delta\omega$ applied to the oscillator is written

$$\delta\omega = \Delta\omega - G'(\Delta\phi + G''\Delta R) \tag{12}$$

or, observing that $\Delta\phi = (j\omega/b)\ \Delta R$ (equations (8) and (9))

$$\delta\omega = \Delta\omega - G'(G'' + \frac{j\omega}{b})\ \Delta R \tag{13}$$

and this combined with

$$\Delta R = \mu_1\ \delta\omega = \frac{b}{\Omega^2 - \omega^2}\ \delta\omega$$

gives finally

$$\Delta R = \frac{b\ \Delta\omega}{\Omega^2 - \omega^2 + bG'G'' + j\omega G'} \ . \tag{14}$$

So by means of this additional loop, the phase loop, we have transformed equation (11) by adding to the denominator a term $j\omega G'$, which is nothing other than a damping term (the amplitude never becomes infinite, no matter what the frequency). By choosing G' large enough, we can damp the resonance as much as we like. On the other hand, the correction of static errors ($\omega = 0$) is not affected by this new loop.

As before, let us plot the Nyquist diagram in this new configuration. The open-loop gain is the product of the beam transfer function (μ_2) by electron transfer function: $G'(G'' + \frac{j\omega}{b})$ (see equation (13)). We thus find:

$$\text{open-loop gain} = \frac{bG'G'' + j\omega G'}{\Omega^2 - \omega^2} \ . \tag{15}$$

The plot of this function in the complex plane is traced in Fig. 14.

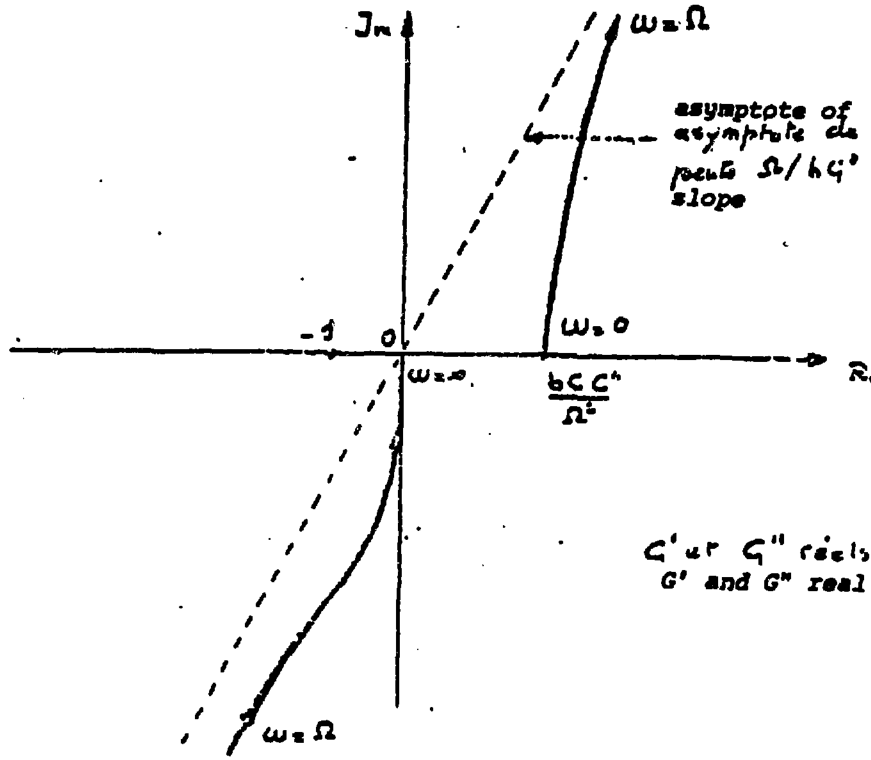

Fig. 14. Nyquist diagram for servo mechanism of Fig. 13.

We observe in this diagram that the curve is tangent to the vertical axis for large values of ω, and consequently distant from the point -1. The effect of the phase loop has been to shift the curve about 90° in the neighborhood of the critical point, and therefore to damp the resonance strongly. Rather than continue to reason in terms of the over-all system, we shall now examine the behavior of each loop separately, which comes down to the same thing in principle but gives us a more physical interpretation of the phenomena.

7. STABILITY OF THE PHASE LOOP

In Fig. 13a, the phase loop is the upper loop. The loop gain is simply $G'\mu_1$:

$$G'\mu_1 = G' \frac{j\omega}{\Omega^2 - \omega^2} \quad .$$

We must now take account of the real transfer function of the amplifier G'. The latter involves unavoidable delays due to the propagation time along the cables (about 1.5 μsec in the PS). Besides, we must take account of the limited pass band of the accelerator cavities, which for a small frequency modulation behave like a low-pass filter. Furthermore, the amplifier itself has a non-infinite pass band.

Thus the curve of the complex gain $G'\mu_1$, a pure imaginary if G' is real, will be rotated clockwise because of all these delays (Fig. 15).

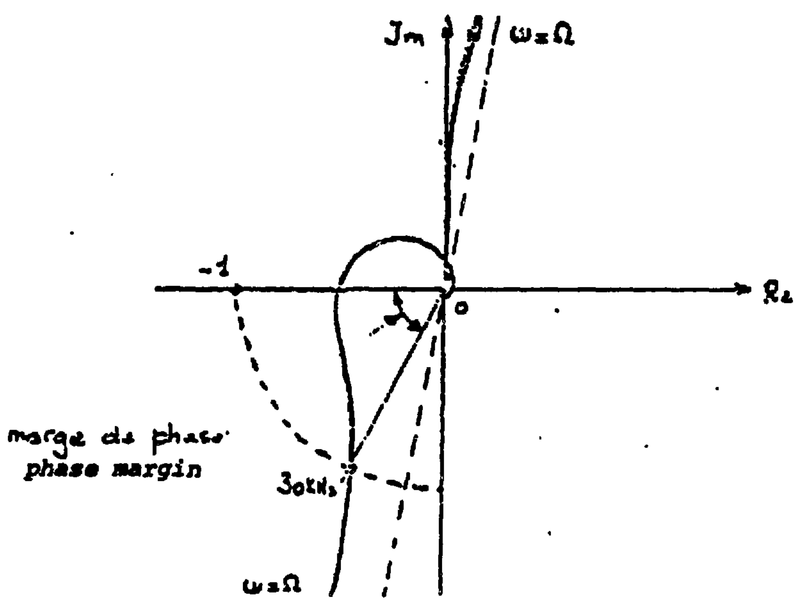

Fig. 15. Nyquist diagram of the phase loop alone.

The cable delay corresponds to a rotation proportional to the frequency. For a delay τ, the rotation is 90° at frequency $f = 1/4\tau$ (160 kHz in the PS). At this frequency, the loop gain, which is substantially G'/ω, must necessarily be less than unity to ensure stability, or in our case, $G' < 160$ kHz/rad. To maintain a sufficient margin of safety, considering the other possible phase shifts, the choice for the PS was $G' \simeq 30$ kHz/rad (0.5 kHz/°) at the frequency of 30 kHz. At this frequency, the loop gain is unity, but the phase margin is sufficient (Fig. 15). Besides, the total gain $G'G''$ must be great, at least for the low frequencies, for proper correction of the static errors in the frequency program. We are thus induced to increase the gain G' considerably towards low frequencies (we shall see later why it is rather undesirable to increase G''), and in the PS the static gain of the phase loop is 50 kHz/°, or 100 times more than for 30 kHz. This result is obtained by means of a suitable corrector network placed in the loop amplifier.

Now that we have a stable phase loop, we can calculate its input-output transfer function. We shall take the reference phase ϕ_r as input parameter and the radial position of the beam as output parameter (Fig. 16).

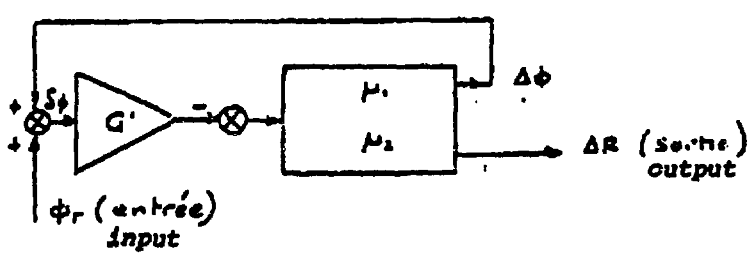

Fig. 16.

We write the circuit equations

$$\delta\phi = \Delta\phi + \phi_r$$
$$[\Delta\phi = -\mu_1 G' \,\delta\phi] \qquad (16)$$

whence

$$\Delta\phi = -\frac{\mu_1 G'}{1 + \mu_1 G'}\,\phi_r = -\frac{j\omega}{\frac{\Omega^2 - \omega^2}{G'} + j\omega}\cdot\phi_r \tag{17}$$

From this we find ΔR:

$$\Delta R = \frac{\mu_2}{\mu_1}\Delta\phi = -\frac{\mu_2 G'}{1 + \mu_1 G'}\,\phi_r = -\frac{b}{\frac{\Omega^2 - \omega^2}{G'} + j\omega}\,\phi_r \quad . \tag{18}$$

To illustrate these results, we can plot the response to a unit step applied to the input ϕ_r. Throughout a frequency band centered around Ω, the quantity $(\Omega^2 - \omega^2)/G'$ is negligible, giving us a unit response for the phase, and an integrator response for the radial position (Fig. 17). The frequencies neighboring upon 30 kHz (loop gain $G'/\omega = 1$) determine the initial transient (response time on the order of 100 μsec).

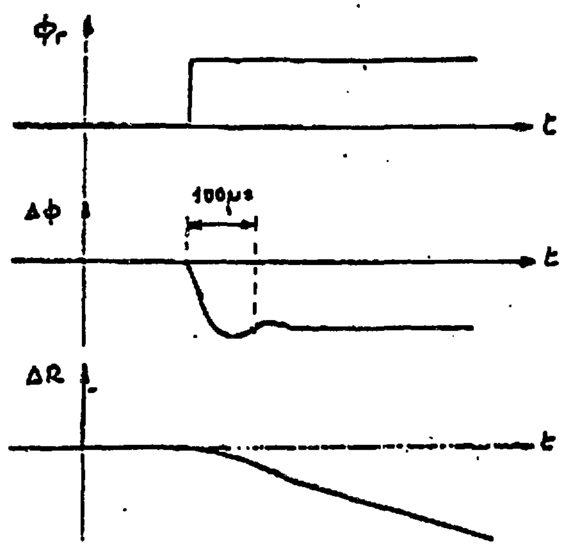

Fig. 17. Transient response of phase loop.

Thus the phase loop **imposes** the reference phase between beam and RF after a transient period that is short compared to the synchrotron period, which again means a radial displacement of the beam at constant velocity. Likewise, an abrupt program variation (frequency error) gives the same result.

8. STABILITY AND RESPONSE OF THE COMPLETE SYSTEM

Having determined the closed-loop transfer function of the phase loop ($\phi_r \rightarrow \Delta R$), we can at once write the open-loop gain of the complete system. It is simply

$$\xi = G'' \times \frac{b}{\frac{\Omega^2 - \omega^2}{G'} + j\omega} . \quad \text{(see equation 18)} \qquad (19)$$

(see equation (13)). The corresponding Nyquist curve is shown in Fig. 18 for a pure real radial loop gain G''.

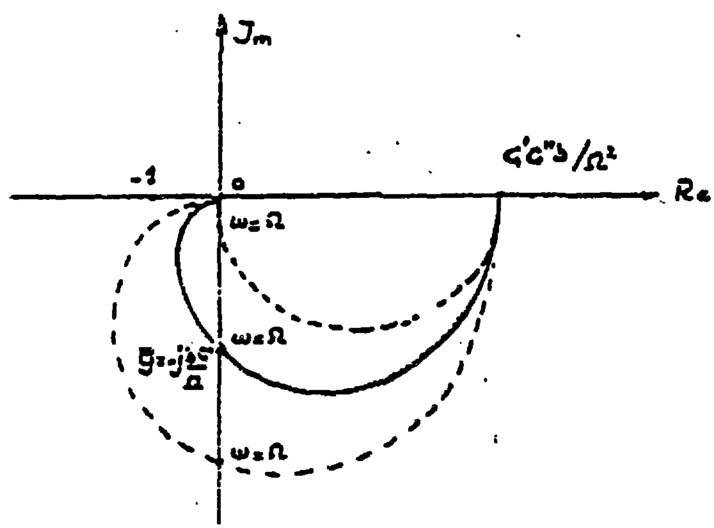

Fig. 18. Nyquist diagram of complete system for various values of G'' (with G'G'' constant).

Several curves have been plotted in this figure, corresponding to several values of G", the total gain G'G" being held constant. (For that is what serves to correct the static errors in the frequency program.) We indeed see that the stability of the system is the better assured, the smaller the gain G", thus justifying our correction network for G'. In this case, the gain curve approaches a semicircle, characteristic of a system with just one time constant. This enables us to obtain the transient response of the system directly. It is simply an exponential response (Fig. 19), the approximate time constant of which (1/bG") is obtained by neglecting the terms of second order in equation (14).

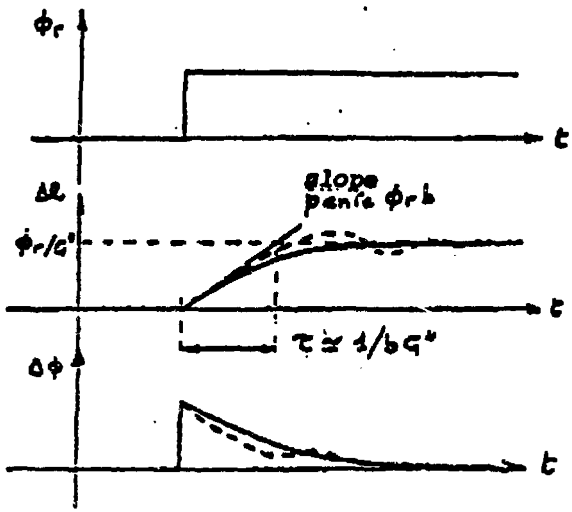

Fig. 19. Transient response of complete system.

Now if we take account of the limited pass band of the amplifier G", we see that we have a response with two time constants, namely a transient with overshoot and damped oscillation (dotted curves). However, there are here no such technical limitations as for the phase loop, since the frequencies involved are much lower.

An interesting limiting case is where the gain G" is a pure imaginary (pure integrator). This is in fact the situation that arises when the beam is synchronized on an outside oscillator (see Appendix I).

Note that from the expression $\mathcal{G}$ we can calculate the closed-loop response of the system, for example the transfer function $\delta\omega \rightarrow \Delta R$. We find quite simply

...

which is none other than equation (14).

9. BENEFIT OF A PHASE PROGRAM

When we introduced the phase loop in section 6, we implicitly assumed that the output of the phase discriminator (Fig. 12) vanished when the beam is in equilibrium (synchronous phase), so that the correction applied to the oscillator would vanish. In reality, the circuits acting on the phase discriminator do involve errors, are sensitive to the intensity, to the frequency, and introduce unavoidable deviations. Besides, the equilibrium phase (stable phase) between beam and cavities depends on the state of the machine (accelerator voltage and derivative of the magnetic field). For we have the relationship

$$V \sin \phi_s = 2\pi R \rho \frac{dB}{dt} \qquad (20)$$

expressing the energy gain per revolution in two ways. (V is the accelerator voltage and ϕ_s the stable phase.)

We see in Fig. 12 that all the phase errors are equivalent to frequency errors of the program, except for a factor G', and that they will finally manifest themselves as errors of static radial position, which are easily calculated from equation (14) (with $\omega = 0$):

$$\Delta R = \frac{b\Delta\omega}{\Omega^2 + bG'G''} = \frac{bG'\Delta\phi}{\Omega^2 + bG'G''} \approx \frac{\Delta\phi}{G''} \quad .$$

Since we cannot increase G", for reasons of loop stability, as we have just seen, we must try to reduce the static phase errors as far as possible. In particular, we can already take account of those resulting from equation (20) by programming the dephaser of Fig. 13b using the function: Arc sin $[2\pi \; R \; \rho \; (dB/dt)/V]$. This work is done by the phase program, the purpose of which is to reduce the variations in radial position corresponding to variations in V or dB/dt. It should be noted that according as we are above or below the transition, we choose the one or the other determination of the Arc sin function. Besides, we also use this phase program for particular operations (for example, the 180° leap on the unstable phase, for "debunching").

The residual phase errors, due to the electronics, difference in length of cables, errors in the phase program itself, have now been corrected at the cost of a radial displacement, which in the PS is of the order of 1 mm for a phase error of 2°.

The following question may now be raised. After all this work, we have succeeded in properly correcting the deviations of the frequency program, but we have had to introduce a phase program whose errors (including those of the electronics) likewise affect the radial position. So it seems that we have only shifted the problem, and we cannot be sure that with present techniques it will be much simpler to create a phase program correct within a few degrees rather than a frequency program with a precision on the order of 10^{-4}. Then why a "beam control" system? The technological justification that led us to enter upon this project (see section 2) seems much less serious, and we must now see whether we have gained anything with all our machinery.

10. CONSERVATION OF LONGITUDINAL EMITTANCE

In the equilibrium state, the beam, which we have hitherto represented by a point (Fig. 5), occupies a certain area in the phase plane, bounded by a particular trajectory (Fig. 20). Measured in suitable units, this area, the longitudinal emittance of the beam, is preserved if we vary the parameters of the oscillation (f, V_{RF} etc.) slowly relative to the synchrotron period. This is what we call the adiabatic condition. We may also say that the beam remains always adapted to the trajectories, which is to say that its front permanently remains a particular trajectory in the phase plane.

In the case of the linearized oscillation we have so far been considering, this adaptation is achieved if the center of gravity of the beam (moment of 1st order) is located in the center of the phase plane, and if the ratio of the axes of the trajectory ellipse (moment of 2nd order) corresponds to the ratio of the axes of the beam ellipse. If we are to take account of the non-linearity of the oscillation, it will of course have to satisfy other conditions upon the moments of higher order.

Now let us apply a strongly non-adiabatic perturbation to the synchrotron oscillation, for example a unit step on the accelerator frequency. We know that the center of gravity will then oscillate at the synchrotron frequency, as is seen in Fig. 6. If we apply this same perturbation to our loop system (namely a unit step on the frequency program, or what amounts to the same thing, on the phase program), the center of gravity of the beam will follow an exponential trajectory (Fig. 19), from which the synchrotron frequency component has disappeared entirely. It is all as though the process had become completely adiabatic for the motion of the center of gravity of the beam.

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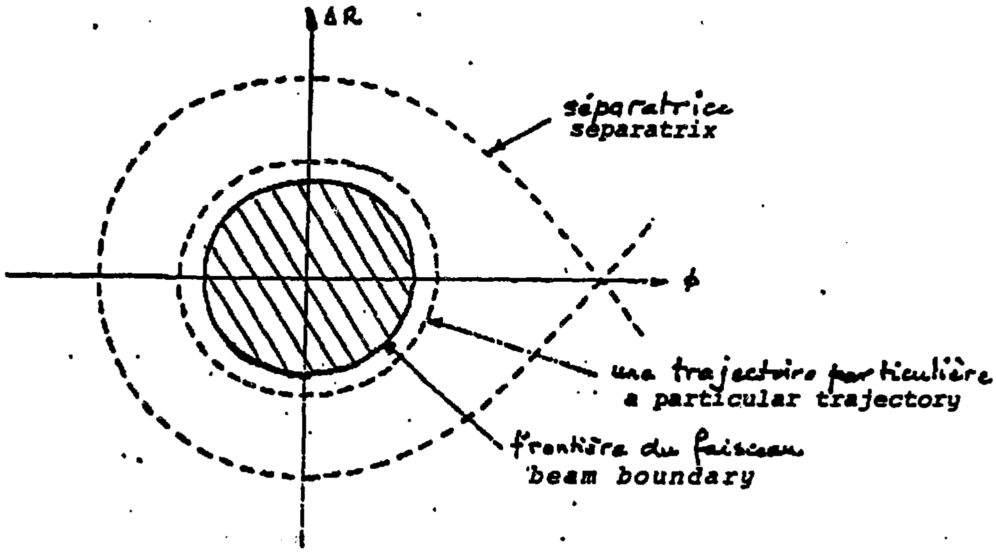

Fig. 20. Representation of the beam in the longitudinal phase plane.

Note that the variation of the frequency applied to the beam through our loop system is not necessarily very slow relative to the synchrotron period (otherwise the process would be adiabatic, even for the moments of higher order).

We can say, in other words, that the "beam control" system always imposes placement of the beam at the center of the trajectories in the phase plane. That is what we noted before in section 7: the phase loop imposes the reference phase between beam and accelerator voltage, and so eliminates any possible oscillation at the synchrotron frequency. This reference phase is therefore nothing else but the stable phase (the one for which a particle does not oscillate) of our loop system.

Let us go back to our step $\delta\phi$ applied to the phase program. The phase loop reacts quasi-instantaneously and shifts the phase of the accelerator voltage relative to the beam by the amount $\delta\phi$. We now have a new equilibrium situation, but with a different stable phase, which means that the beam will drift radially, as we already know. To this new situation, a new separatrix and a new family of trajectories correspond (Fig. 21). However, these new trajectories are still constructed around the center of gravity of the beam, because of the phase loop. To be sure, the stable phase is afterwards slowly restored to its initial value by the radial loop (Fig. 19).

11. PROBLEMS OF SECOND ORDER

Now let us examine the evolution of the initial ellipse, after application of our step $\delta\phi$, in more detail.

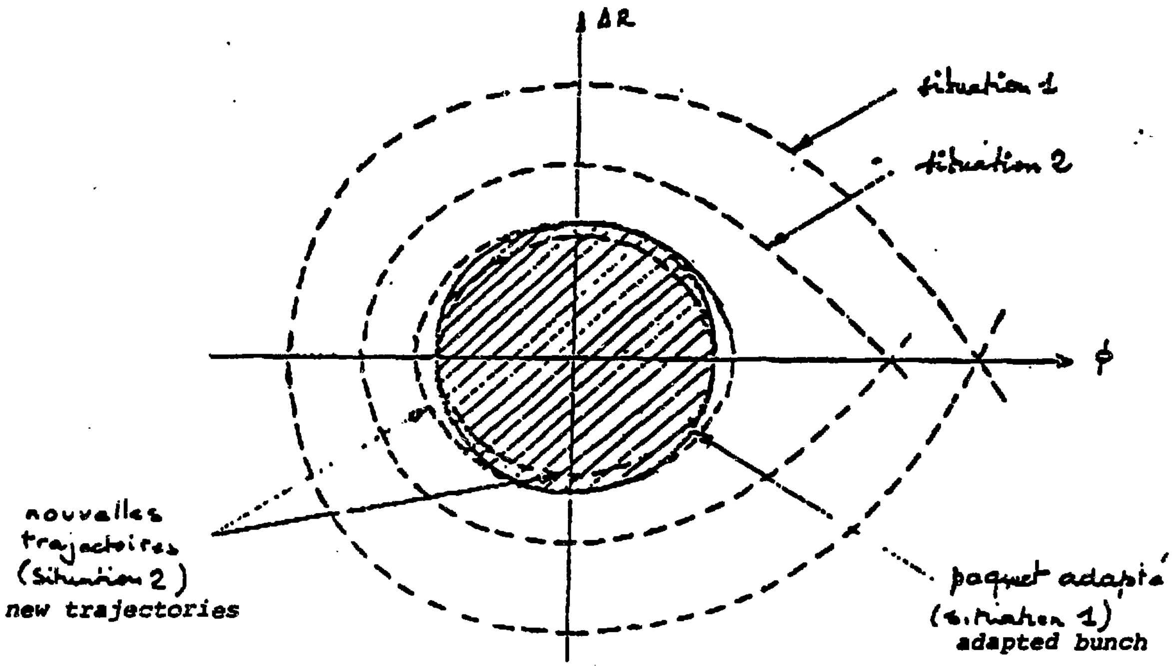

Fig. 21. Influence of an abrupt variation of the stable phase in the phase plane.

The new trajectories around it are slightly different because, the synchrotron frequency having varied, the axial ratio of the trajectory ellipses, which is proportional to it, is likewise altered (dotted curves), which means that our beam is no longer exactly in equilibrium (no longer adapted). It will execute oscillations of shape, as indicated in Fig. 22, where the deviations have been deliberately exaggerated.

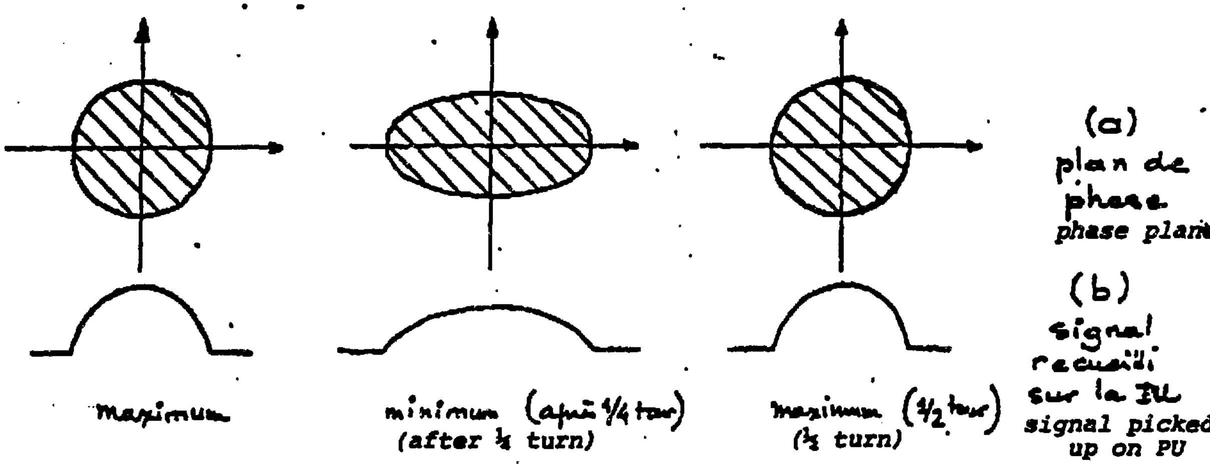

Fig. 22. Shape oscillations of the beam.

These deviations in bunch length are easily observed at a sum PU station (specifically, the wide-band station), the crest amplitude of which is detected. We thus obtain a response of the type of that in Fig. 23. Note that we have tentatively assumed the radial loop to be without effect, since the deviation $\delta\phi$ has not been corrected.

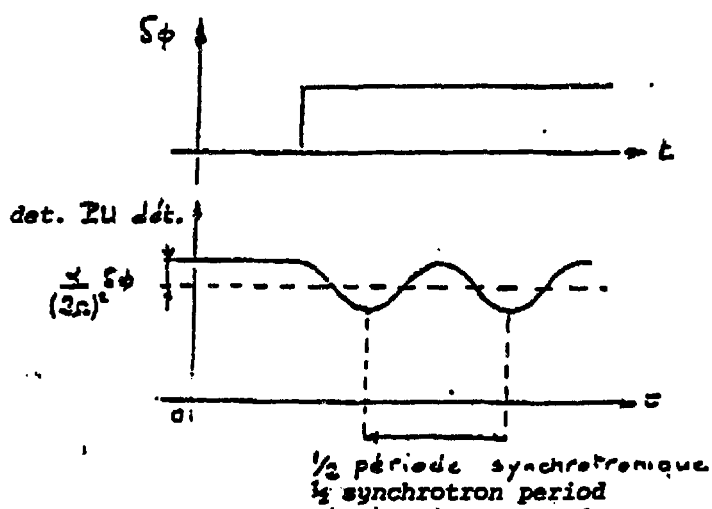

Fig. 23. Transient response of moment of second order.

If we are interested only in the variation of the PU signal, we obtain something very much like the curve in Fig. 6 for the deviation ΔR, which we can express in the following mathematical form (same as equation (8) representing the curve of Fig. 6):

$$\xi = \frac{\alpha}{(2\Omega)^2 - \omega^2} \delta\phi = \mu_3 \delta\phi \quad . \tag{21}$$

In this equation ξ is the signal variation picked up by the detector from the PU station, which is nothing else but the measure of the de-adaptation of the beam to the trajectories, or again a quantity proportional to the moment of second order. Note also that in the denominator, we have replaced Ω with 2Ω, since the frequency of the shape oscillation is double the synchrotron frequency, as we can gather from Fig. 22. Thus we have established a new transfer function linking the second order moment of the beam to an excitation of the stable phase.

We can apply the same argument to a variation in the accelerator voltage. Assume an abrupt variation of V in the form of a unit step. The phase loop keeps the RF - center of gravity of beam phase constant, so the beam drifts radially. The stable and unstable points of the "fish" do not move, only the height of the fish varies, so the axial ratio of the new trajectories is modified. We eventually find the same result as before, namely another transfer function linking the second-order moment to a variation of the RF voltage.

$$\xi = \frac{\alpha'}{(2\Omega)^2 - \omega^2} \delta V = \mu_4 \delta V \quad . \tag{22}$$

12. CORRECTING THE MOMENT OF SECOND ORDER. "HEREWARD DAMPING"

As for the oscillations in radial position, or phase (first-moment oscillations), we shall try to transform the oscillatory response of the beam into a strongly damped response. We have already seen how this result was reached for the first moment (movement of center of gravity), and we shall now apply the same technique. An additional negative feedback loop of gain H is introduced, which, from the detected wide-band PU, corrects the stable phase (Fig. 24a).

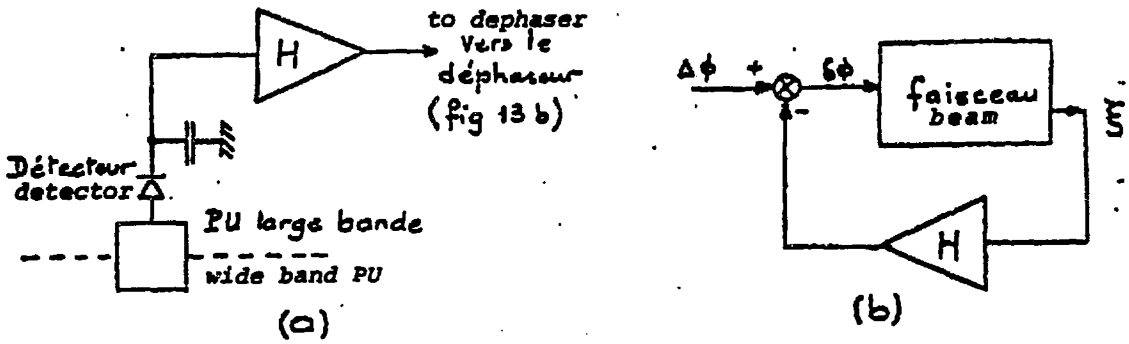

Fig. 24. "Hereward damping" acting on the stable phase.

The equivalent diagram of this additional loop has been indicated in Fig. 24b, and we can write its equations immediately:

$$\begin{cases} \delta\phi = \Delta\phi - H\xi \\ \xi = \mu_s \delta\phi = \dfrac{\alpha}{4\Omega^2 - \omega^2} \; \delta\phi \end{cases} \quad . \tag{23}$$

We immediately infer

$$\xi = \frac{\alpha \, \Delta\phi}{4\Omega^2 - \omega^2 + \alpha H} \quad . \tag{24}$$

If a non-oscillating response is wanted, it suffices that H have an <u>imaginary component</u> (of suitable sign). Since we are interested in variations of ξ only, we take H to be an amplifier with alternating coupling, having a differentiator characteristic in the frequency range (around the synchrotron frequency) with which we are concerned. Thus we have substantially

$$H = j\omega k$$

which gives the desired damped response. For example, for the critical damping:

$$k = 4\Omega/\alpha$$

As a matter of fact, in our diagram of Fig. 24b we forgot the radial loop, which also acts upon the dephaser and tends to oppose the perturbation introduced by H: The equations of the complete system are written:

$$\left\{\begin{aligned} \delta\phi &= \Delta\phi - H\xi - G''\Delta R \\ \xi &= \mu_3\delta\phi \\ \Delta R &= \frac{b}{j\omega}\,\delta\phi \quad \text{(cf. equation 18)} \end{aligned}\right. \tag{25}$$

We infer the exact expressions for ξ and ΔR:

$$\xi = \frac{\alpha\Delta\phi}{4\Omega^2 - \omega^2 + \alpha H + \frac{G''b}{j\omega}(4\Omega^2 - \omega^2)} \qquad \Delta R = \frac{b\Delta\phi}{G''b + j\omega + \frac{H\alpha j\omega}{4\Omega^2 - \omega^2}} \tag{26}$$

We can verify, taking account of the orders of magnitude of the parameters, that the functioning of each of these loops is not much altered. For the radial loop, of which only the response at very low frequencies concerns us, the additional term $-\omega^2 k\alpha/(4\Omega^2 - \omega^2)$, of the second order in ω is negligible. Contrariwise, the response at frequencies on the order of $(2\Omega)/2\pi$ determines the damping contributed by the Hereward damping loop, and we see that the radial loop has a negligible effect in this frequency range.

13. CORRECTION ON THE ACCELERATOR VOLTAGE

Another possibility of damping is suggested to us by equation (22). This is to close the loop by way of the accelerator voltage, as indicated in the diagram of Fig. 25.

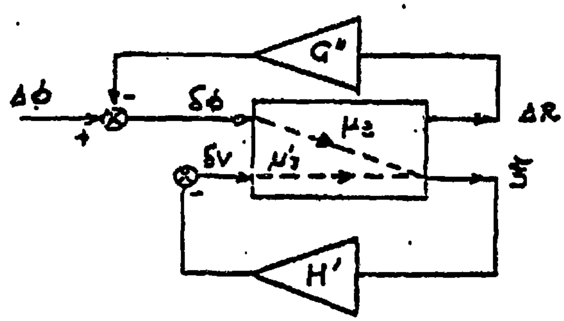

Fig. 25. "Hereward damping" acting on the accelerator voltage.

The equations of the system are

$$\begin{cases} \delta\phi = \Delta\phi - G''\Delta R \\ \xi = \mu_3\delta\phi + \mu_3'\delta V = \dfrac{\alpha\delta\phi + \alpha'\delta V}{4\Omega^2 - \omega^2} \\ \delta V = -H'\xi \end{cases} \tag{27}$$

whence

$$\xi = \frac{\alpha(\Delta\phi - G''\Delta R)}{4\Omega^2 - \omega^2 + \alpha' H'} \quad . \tag{28}$$

As before, we give H' the characteristic of a differentiator, which introduces the desired damping in the response. The numerator simply represents the response of the radial loop. In reality, the perturbation δV of the RF voltage likewise yields a perturbation of the stable phase, because of the relationship (20). We can very readily take account of this effect, at the cost of a notational complication, and we find that the results remain qualitatively identical.

Why two types of negative feedback? Only because the first type, the one acting on the phase, does not work on the plateaus of the magnetic field. This is because the quantity α in equation (21) characterizes the <u>variation</u> of the axial ratio of the trajectories when the stable phase is changed. Now the axial ratio is nothing else, except for a scale factor, but the synchrotron frequency

$$f_s = \frac{\Omega}{2\pi} = \frac{\beta c}{2\pi R}\left(\frac{|n|\ eV\cos\phi_s}{2\pi\ E\ h}\right)^{\frac{1}{2}} . \tag{29}$$

The quantity $df_s/d\phi_s$, proportional to the variation of the axial ratio, hence to α, vanishes for $\phi_s = 0$ or π, that is, when we are on a plateau of the magnetic field. The negative feedback loop is then open and no longer functions. Note also that we must change the sign of H according as the beam is accelerated or decelerated (sign of $\sin\phi_s$) and according as we are above or below the transition (this does not show explicitly in (29)).

To illustrate how the Hereward damping loop works, we choose the following simple case. We are on a plateau of the magnetic field, so the loop acting on the voltage is the one that operates. We suddenly perturb the amplitude of the RF voltage by an amount ΔV (Fig. 27). Since we are on a plateau, the stable phase does not change ($\phi_s = \pi$), enabling us to use the simplified diagram of Fig. 26.

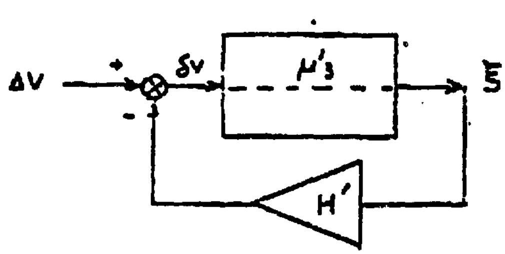

Fig. 26. "Hereward damping" on voltage (plateau of magnetic field).

Its equations are:

$$\begin{cases} \delta V = \Delta V - H'\xi & \text{with} \quad H' = j\omega k' \\ \xi = \mu j \Delta V \end{cases} \tag{30}$$

From this we get

$$\delta V = \frac{\Delta V}{1 + \frac{j\omega k'\alpha'}{4\Omega^2 - \omega^2}} . \tag{31}$$

The form of the transient response of δv is given in Fig. 27. For the very high frequency components, we have $\delta V = \Delta V$, whereas below the synchrotron frequency the response is substantially exponential and tends asymptotically towards ΔV.

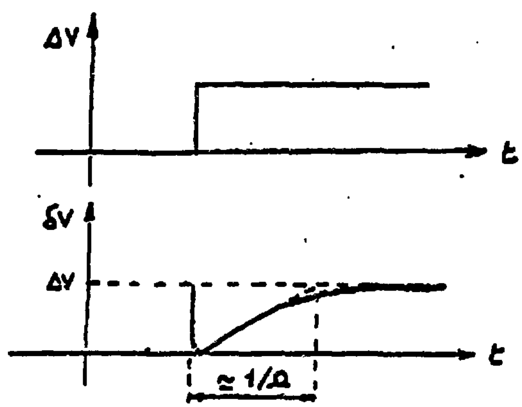

Fig. 27. Transient response of "Hereward damping" on RF voltage. The time constant is substantially $1/\Omega$ for the critical damping ($k'\alpha' = 4\Omega$).

The very brief transient (much quicker than $1/2\Omega$) has no effect on the beam; on the other hand, our step ΔV has been transformed into a much slower rise, in the same way as a frequency step had been transformed into an exponential rise. Now it is all as though the variation in accelerator voltage were adiabatic for the moment of second order (although this variation is not very slow relative to the synchrotron period).

This result ca be extended to the other type cases by more :licated calcula ions. If for example an abrupt RF voltage jump ..pplied when the phase loop is working, this will instantly shift stable phase to keep the trajectories adapted to the beam.

LUSION

.e close of thi study, we learned that the essential function .e beam control system was to preserve the longitudinal emit- e during accele ation. This function has been performed for first- and seco d-order moments of the beam by means of the e loop and the Hereward damping" loop. Note that for pertur- ons external to the system, the first and second orders suffice he case of a li ear oscillation. When we are dealing with a ly non-linear o cillation (as is the case for example if the h" is almost fu 1), another damping mechanism, completely for- to this descri tion, intervenes. This is Landau damping, re- .d to the fact that the synchrotron frequency is not the same all the particl s in the bunch. So it would not seem very worth e to extend our system in the direction of orders higher than second, since s me other mechanism might take over.

We have assume throughout that the perturbations came from <u>outside</u> - varia ions of frequency, of stable phase, of RF volt- amplitude - and have tried to reduce these perturbations. It be realized th .t this approach is only a method of presenting problem. We al . know that in a looped system, the perturbations <u>rnal</u> to the sys em are likewise corrected by the negative feed- effect. This is true here also, and we can regard the "beam rol" system as a method of correcting the possible beam oscil- .ons (related fc example to parasite elements in the machine) he absence of c itside excitation. By way of example, the

"Hereward damping" loop was installed in the PS, not to limit the rapid variations in RF voltage (which could be done directly), but to suppress an instability (negative damping) the precise cause of which has still not been determined.

Another hypothesis, implicitly entertained in this exposition, is that the motion of all the bunches is the same (coherent oscillations). This is more or less the case if we are dealing with an outside perturbation not too rapid relative to the period of rotation. However, experience with the PS has shown us that the bunches can oscillate relative to each other because of parasite elements present in the machine. Our "beam control" is helpless in these cases; it simply corrects the mean oscillation of all the bunches. One could indeed wish to have a system that might act on each of the bunches more or less individually. But that is another story!

* * *

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APPENDIX I

SYNCHRONIZATION ON AN OUTSIDE FREQUENCY

The static radial position of the beam in a system such as has just been described depends on the frequency and phase errors of the programs used, as well as on the quality of the electronics. We can try to free ourselves of this difficulty by indefinitely increasing the gain of the radial loop for low frequencies, in other words by giving G" the characteristic of a pure integrator.

One way to achieve this objective is to compare the frequency of the beam (which is equivalent to its radial position for a given magnetic field) and an outside reference (reference oscillator) in a phase discriminator. Such a discriminator yields a voltage proportional to the phase deviation, that is, to the integral of the frequency (or radial position) deviation. The corresponding diagram is that of Fig. 28 (compare with Fig. 13b).

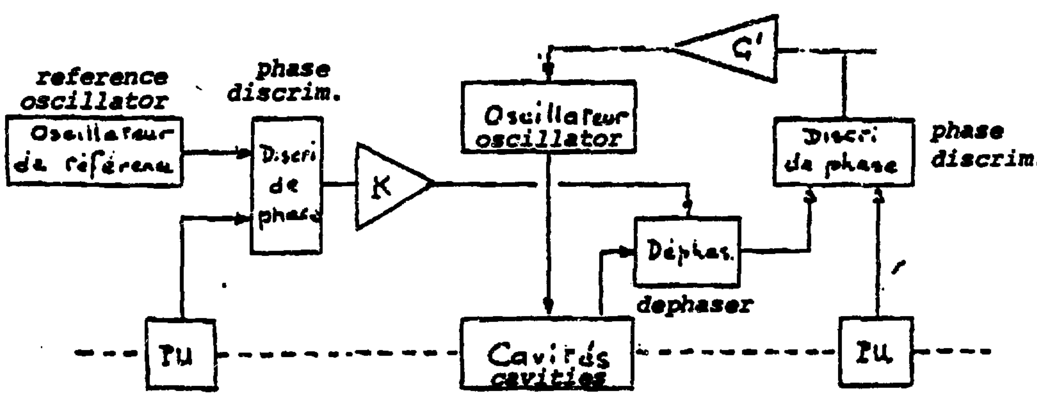

Fig. 28. Synchronization on an outside frequency. (Note that the two PU's may be merged; they have both been shown here only for the sake of clarity of the diagram.)

Now we must calculate the value of G" corresponding to this diagram. First of all, a frequency deviation must be converted into a deviation of position: factor Ω^2/b (cf. equation (8)), then introduce the integrator $(1/j\omega)$. We thus obtain

$$G'' = \frac{K}{j\omega}\frac{\Omega^2}{b} \quad .$$

Referring to Fig. 18, we see that the integrator will rotate the entire figure by 90° (K is assumed pure real). In particular, the point located on the axis of imaginaries $(\omega = \Omega)$ will be shifted to the negative real half-axis. The condition of stability obviously is that the abscissa of this point be within the segment [0, -1], which gives immediately

$$\left|\frac{bG''}{\Omega}\right| < 1 \; , \qquad \text{for} \qquad \omega = \Omega$$

whence

$$\left(\frac{K}{\omega}\frac{\Omega^2}{b}\right)\frac{b}{\Omega} < 1 \quad .$$

We finally obtain a limiting stability condition that is expressed very simply by the inequality $K < 1$.