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RANDOM ERRORS IN THE MAGNETIC FIELD COEFFICIENTS OF SUPERCONDUCTING QUADRUPOLE MAGNETS*

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Introduction

In a previous publication[1] we have studied the random errors in the multipole coefficients of superconducting dipole magnets that result from asymmetries in their construction. In this paper we shall consider, in a similar way, the random multipole errors of superconducting quadrupoles. Perfect symmetry of construction in such a magnet will allow only higher normal multipoles of 12 pole, 20 pole, etc. to be present in the magnetic field. However, when the conductors are not perfectly positioned, this ideal symmetry is broken and the non-allowed multipoles, normal and skew, appear in the magnetic field. In analyzing the multipoles which arise due to random variations in the sizes and locations of the current blocks, we adopt a model analogous to that used for our analysis[1] of the superconducting dipole. With this approach, based on the symmetries of the quadrupole magnet, we will obtain estimates of the random multipole errors for the arc quadrupoles envisioned for the Relativistic Heavy Ion Collider (RHIC) and for a single-layer quadrupole proposed for the Superconducting Super Collider (SSC).

Field Representation for a Quadrupole

The representation of the magnetic field in the free aperture of a quadrupole magnet (see Fig. 1) can be carried out in terms of the normal, b_n', and skew, a_n', fractional field coefficients according to the expressions

$$B_y = \frac{B_{TH}}{G_{TH}} G \sum_{n=0} b'_n \left(\frac{x}{R_o}\right)^n . \quad (1)$$

and

$$B_x = \frac{B_{TH}}{G_{TH}} G \sum_{n=0} a'_n \left(\frac{x}{R_o}\right)^n . \quad (2)$$

The gradient is G (telsa m^{-1}), while the ratio (B_{TH}/G_{TH}) is that of the dipole bending field to the operating gradient characteristic of the theoretical lattice design of the basic accelerator. R_0 is the reference radius (~2/3 of the inner coil radius). With these choices, one arrives at a set of median plane multipole coefficients (b_n', a_n') for a quadrupole magnet that has the same physical significance as that conventionally employed for the dipole magnet.[1,2]

An alternative field representation, and one which describes the field of the quadrupole in a manner independent of the accelerator for which it may be designed, is given by the expressions

$$B_y = GR_o \sum_{n=0} q'_n \left(\frac{x}{R_o}\right)^n . \quad (3)$$

$$B_x = GR_o \sum_{n=0} p'_n \left(\frac{x}{R_o}\right)^n . \quad (4)$$

These expressions are similar to the usual dipole expressions in the sense that the value of the field at the radius R_0 due to the dominant field, in this case the quadrupole, appears as an overall multiplicative factor. To avoid confusion we have designated this type of normal coefficient as q_n' and the skew one as p_n'. Evidently, this set of coefficients is simply related to the (a_n', b_n'), that is,

$$q'_n = \frac{B_{TH}}{G_{TH}} \frac{b'_n}{R_o} ; \qquad p'_n = \frac{B_{TH}}{G_{TH}} \frac{a'_n}{R_o} . \quad (5)$$

The Quadrupole Model

Since our magnet model for the quadrupole is, except for the specific layout of the conductor blocks, the same as that for a dipole magnet,[1] we will in this section only outline the pertinent equations. Thus the fractional field coefficients are derived from the current distribution, Fig. 1, according to the relation

$$b'_{n-1} - ia'_{n-1} = -\frac{G_{TH}}{B_{TH}} \frac{R_o^{n-1}}{G} \frac{\mu_o}{2\pi} \int \frac{dI}{\rho^n} \left[1 + \frac{\mu-1}{\mu+1} \frac{\rho^{2n}}{R_f^{2n}}\right] e^{in\phi} . \quad (6)$$

For a single keystone-shaped current block, Fig. 2, with N uniformly distributed conductors, we then have

$$b'_{n-1} = \frac{G_{TH}}{B_{TH}} \frac{R_o^{n-1}}{G} \frac{\mu_o NI}{4\pi\rho_o\Delta} \frac{\sin n\omega}{n\omega} \cos n\Phi_o \; M(n) , \quad (7)$$

and

$$a'_{n-1} = -\frac{G_{TH}}{B_{TH}} \frac{R_o^{n-1}}{G} \frac{\mu_o NI}{4\pi\rho_o\Delta} \frac{\sin n\omega}{n\omega} \sin n\Phi_o \; M(n) . \quad (8)$$

where M(n) is the integral

$$M(n) = \int_{\rho_o-\Delta}^{\rho_o+\Delta} \frac{d\rho}{\rho^{n-1}} \left[1 + \frac{\mu-1}{\mu+1} \frac{\rho^{2n}}{R_f^{2n}}\right] . \quad (9)$$

Because we are interested in the variation of the field coefficients for small changes $(\delta\phi_o, \delta\omega, \delta\rho_o, \delta\Delta)$ in the positions and sizes of the blocks, we write

$$\delta b'_{n-1} = X(n,\phi_o)\,\delta\phi_o + X(n,\rho_o)\,\delta\rho_o + X(n,\omega)\delta\omega + X(n,\Delta)\,\delta\Delta \quad (10)$$

where the X's are the partial derivatives, such as $\partial b'_{n-1}/\partial\phi_o = X(n,\phi_o)$. An expression similar to Eq. (10) holds for the variation in the skew coefficients $(\delta a'_{n-1})$ with the partial derivations $Y(n,\phi_o) = \partial a'_{n-1}/\partial\phi_o$, etc. Assuming initially that the variations $\delta\phi_o$, $\delta\rho_o$, $\delta\omega$, and $\delta\Delta$ for the N_B current blocks are uncorrelated, we find that the total rms variation in the normal coefficients is

$$(\delta b'_{n-1})^2 = \sum_{l=1}^{N_B} \left\{ (X_l(n,\phi_o))^2 (\delta\phi_o)_l^2 + (X_l(n,\rho_o))^2 (\delta\rho_o)_l^2 + (X_l(n,\omega))^2 (\delta\omega)_l^2 + (X_l(n,\Delta))^2 (\delta\Delta)_l^2 \right\} . \quad (11)$$

A similar expression applies to the skew coefficients.

Random Coefficients and Quadrupole Symmetries

A perfectly built superconducting quadrupole magnet exhibits three symmetries.[3] First, the current distributions above and below

*Work performed under the auspices of the U.S. Department of Energy the median plane are the same, that is, top bottom symmetry $(TB)_S$ exists; second, the current distributions to the left and right of the central vertical plane are the same, that is, left-right symmetry $(LR)_S$ exists; and lastly, the current distributions about the four (45°, 135°, 225°, and 315°) pole axes are antisymmetric, that is, "quadrupole-coil" symmetry $(QC)_A$ exists. When these symmetries are satisfied, only the normal coefficients b'_1, b'_5, b'_9, etc. have values which do not vanish. All the other normal multipoles as well as all the skew multipoles vanish. In the model we have developed, it is the unsymmetrical changes in the block variables which break these symmetries and permit the forbidden multipoles to appear. Consistent with this idea we arrive at a combination of partial derivatives ($\mathfrak{X}$) which exhibits the basic four-fold symmetry of a quadrupole. Thus for the set of blocks numbered 1, 6, 7, 12, 13, 18, 19, and 24 (see Fig. 3), we write

$$\begin{aligned}\mathfrak{X}_1(n,\phi_o) = {} & (1+\alpha)(1+\sigma)(1+\tau)X_1(n,\phi_o) - (1+\alpha)(1+\sigma)(1-\tau)X_6(n,\phi_o) \\ & + (1+\alpha)(1-\sigma)(1-\tau)X_7(n,\phi_o) - (1+\alpha)(1-\sigma)(1+\tau)X_{12}(n,\phi_o) \\ & + (1-\alpha)(1-\sigma)(1+\tau)X_{13}(n,\phi_o) - (1-\alpha)(1-\sigma)(1-\tau)X_{18}(n,\phi_o) \\ & + (1-\alpha)(1+\sigma)(1-\tau)X_{19}(n,\phi_o) - (1-\alpha)(1+\sigma)(1+\tau)X_{24}(n,\phi_o) \; .\end{aligned} \tag{12}$$

The expression for $\mathfrak{X}_1(n,\omega)$ is like Eq. (12) except that the second, fourth, sixth, and eighth terms of the rhs all have positive signs. Similarly $\mathfrak{X}_1(n,\rho_o)$ and $\mathfrak{X}_1(n,\Delta)$ are obtained from $\mathfrak{X}_1(n,\omega)$ by letting $\omega \to \rho_o$ and $\omega \to \Delta$ in turn. In these combinations of derivatives, it is the parameter τ, which, when different from zero, break the "quadrupole-coil" symmetry. In the same way, the parameter σ breaks the $(LR)_S$, while α breaks the $(TB)_S$. Treating the quadrupole block arrangement shown in Fig. 3 as made up of three sets of eight blocks each, we can write for the rms variation in the normal coefficients

$$\begin{aligned}(\delta b'_{n-1})^2 = \sum_{l=1,2,3} \Big\{ & (\mathfrak{X}_l(n,\phi_o))^2 (\delta\phi_o)_l^2 + (\mathfrak{X}_l(n,\omega))^2 (\delta\omega)_l^2 \\ & + (\mathfrak{X}_l(n,\rho_o))^2 (\delta\rho_o)_l^2 + (\mathfrak{X}_l(n,\Delta))^2 (\delta\Delta)_l^2 \Big\} \; .\end{aligned} \tag{13}$$

Random Multipole Errors of the RHIC and SSC Quadrupoles

A design for the superconducting quadrupole to be used in the regular arcs of the RHIC storage rings is specified in Ref. 4. On the basis of this design we have calculated the random errors in the magnetic multipoles, equivalent to the following linear rms variations: 2 mils (0.002 inch) for ($\delta\phi_O$) and ($\delta\rho_O$), 1 mil for ($\delta\omega$) and ($\delta\Delta$). For the asymmetry parameters we have chosen the set: $\tau = 0.39$, $\sigma = 0.67$, and $\alpha = 0.67$. This choice was based on the asymmetry parameters found for the CBA dipoles.[1] Thus the quadrupole-coil parameter τ is the (LR) parameter (dipole coil) obtained from the fits to the CBA dipole. Similarly, the other two values, σ and α, are equal to the $(TB)_S$ parameter of the CBA dipoles. In short these asymmetry parameters, which we note are also close to those determined for the Tevatron dipoles,[1] reflect the techniques in coil construction and magnet assembly achieved in the past.

Figures 4 and 5 show the results of our calculation for the RHIC quadrupole. As expected, the plot of the normal random coefficients ($\delta b'_n$) shows a high-low pattern: higher values for the allowed multipoles (n=1, 5, 9, etc.) and lower values for the non-allowed multipoles. For the skew multipoles ($\delta a'_n$) the pattern is the opposite one: lower for the allowed, higher for the non-allowed. Thus, it appears that the symmetry breaking, which caused the well defined zig-zag pattern in the case of a dipole magnet,[1] manifests itself in a "softened zig-zag" effect in the case of the random widths of a quadrupole magnet. In Table I we give a summary of these random multipoles. In the interest of completeness, we have also listed the alternative coefficients ($\delta q'_n, \delta p'_n$).

A design for a single layer quadrupole for the SSC has been developed at Brookhaven National Laboratory.[5] Starting with the same linear errors that we previously assumed for the RHIC quadrupole (2 mils for $\delta\phi_O$ and $\delta\rho_O$, 1 mil for $\delta\omega$ and $\delta\Delta$) and adopting the same asymmetry parameters, we have calculated the associated random multipole errors for the design. The results are presented in Table II. We note that these values ($\delta b'_n, \delta a'_n$) for the quadrupole are, in general, larger than those estimated earlier for the SSC dipoles.[1] However, it is important to point out that, since there are fewer arc quadrupoles than dipoles (about 1 to 5) in the SSC rings and since, in addition, they are each shorter than a dipole (about 1 to 5), the contribution of these random errors to the possible emittance growth of the proton beam will be considerably less than that due to the dipoles.[6]

References

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TABLE I. Multipole RMS Widths (R_o = 2.5 cm) For RHIC Arc Quadrupole Q763.

MULTIPOLE NUMBER	REFERRED TO DIPOLE FIELD Eqs. 1&2*		REFERRED TO QUAD. FIELD Eqs. 3&4	
	NORMAL RMS WIDTH	SKEW RMS WIDTH	NORMAL RMS WIDTH	SKEW RMS WIDTH
n	$\delta b'_n(10^{-4})$	$\delta a'_n(10^{-4})$	$\delta q'_n(10^{-4})$	$\delta p'_n(10^{-4})$
0	2.7	4.3	5.6	8.7
1	4.9	1.3	10.1	2.6
2	2.1	2.3	4.3	4.7
3	1.4	1.6	2.9	3.3
4	1.3	1.3	2.7	2.7
5	1.7	0.29	3.4	0.60
6	0.55	0.55	1.1	1.1
7	0.27	0.31	0.56	0.64
8	0.22	0.22	0.46	0.46
9	0.27	0.05	0.56	0.10
10	0.09	0.09	0.18	0.18

*G_{TH}/B_{TH} = 19.536m^{-1}

TABLE II. Multipole RMS Widths (R_o = 1 cm) For the SSC Arc Quadrupole QSSC 72.

MULTIPOLE NUMBER	REFERRED TO DIPOLE FIELD Eqs. 1&2*		REFERRED TO QUAD. FIELD Eqs. 3&4	
	NORMAL RMS WIDTH	SKEW RMS WIDTH	NORMAL RMS WIDTH	SKEW RMS WIDTH
n	$\delta b'_n(10^{-4})$	$\delta a'_n(10^{-4})$	$\delta q'_n(10^{-4})$	$\delta p'_n(10^{-4})$
0	6.1	9.5	19.0	29.5
1	8.4	2.1	26.3	6.6
2	2.7	3.0	8.5	9.2
3	1.3	1.5	3.9	4.5
4	0.83	0.83	2.6	2.6
5	0.77	0.14	2.4	0.44
6	0.19	0.20	0.58	0.61
7	0.06	0.08	0.19	0.25
8	0.04	0.04	0.12	0.13
9	0.03	0.006	0.10	0.02
10	0.007	0.007	0.02	0.02

*G_{TH}/B_{TH} = 13.121m^{-1}

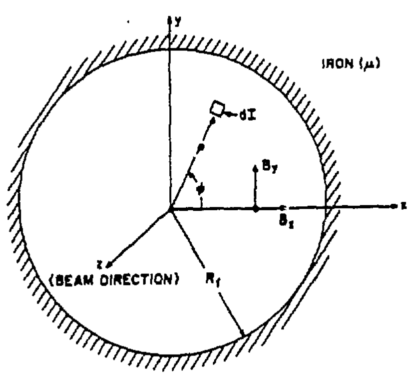

Figure 1. Coordinate system for the free aperture of an accelerator magnet.

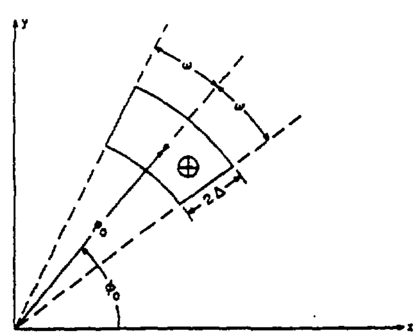

Figure 2. Notation for location and size of a current block.

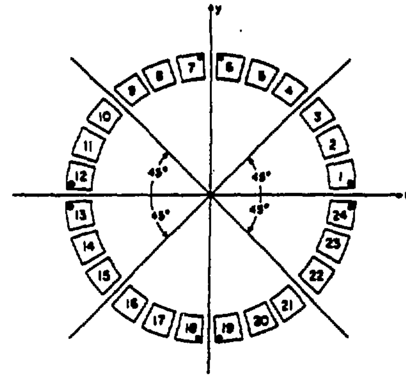

Figure 3. Locations and numbering arrangement for the current blocks.

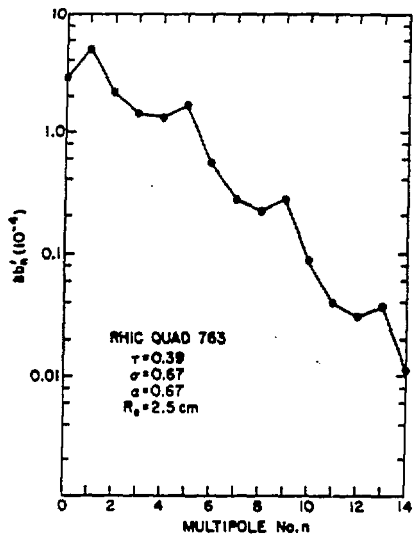

Figure 4. Normal multipole random widths ($\delta b'_n$) for the RHIC Quadrupole 763.

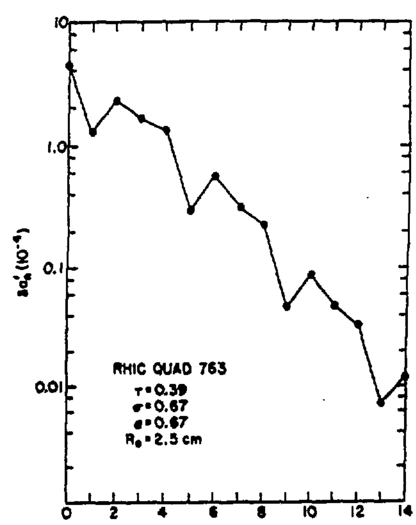

Figure 5. Skew multipole random widths ($\delta a'_n$) for the RHIC Quadrupole 763.