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ANALYSIS OF HEAVY ION ELASTIC SCATTERING DATA BY S-MATRIX PARAMETERIZATION

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ABSTRACT

It is shown that the general features of the heavy ion elastic scattering data are reproduced by using a smooth cut-off model for the scattering matrix in which the real nuclear phase shift and the reflection coefficient are used as analytic functions and have a few adjustable parameters. A computer program is described for carrying out the calculations to fit the experimental ratio of elastic differential cross-sections to Rutherford cross-sections. The scattering of 10.375 MeV/ u ^{12}C ions from Fe, Ag and Ta are considered as test cases. The absorption radii and reaction cross-sections predicted by the theory are also calculated.

INTRODUCTION

The subject of heavy ion interactions deals with the study of nuclear reactions/scattering of two atomic nuclei of mass number $A > 4$. There has been an ever increasing interest in this field since the availability of accelerators capable of imparting several MeV per nucleon (MeV/u) energies to nuclei heavier than alpha particles so that their Coulomb barriers could be overcome[1]. At present this is one of the most active areas of Nuclear Physics research both theoretically and experimentally. The special interest in this field lies in the fact that one is capable of observing phenomena which were hitherto inaccessable to laboratory studies e.g. very high angular momentum states of nuclei, nuclear quasimolecules, deep inelastic reactions etc. For energies below 10 MeV/u one is interested in the processes like Coulomb excitation of the interacting nuclei, elastic scattering and quasi-elastic reactions. For higher energies, deep inelastic processes and the fusion reactions are of importance, giving rise to the possibility of the formation of Super Heavy Elements. At relativistic energies[2], (the Bevatron-Superhilac accelerator at Berkeley (BEVALAC) can impart energies upto 2 GeV/u to heavy ions), still different types of phenomena may occur e.g. the production of anamalons, pion condensate, or even quark plasma.

In the present report, we are interested in the elastic scattering of nuclei of $A > 4$ and energy $E \sim 10$ MeV/u from heavy nuclear targets. The study of elastic scattering is important because the total reaction cross-section can be determined from it fairly accurately. This is necessary for taking into account all possible reaction channels, specially in a situation where new modes of reactions are possible[3].

A complete quantum mechanical description of the problem involves the use of ion-nucleus potential in the solution of Schrodinger equation. The scattering amplitude thus calculated is then used to find elastic differential cross-sections. Although this scheme has the advantage that the wave-functions can be used as input for the study of non-elastic processes, however, there

are some difficulties. Firstly it has been found that the optical potential parameters are not uniquely determined from the comparison of theoretical and experimental differential cross-sections[4]. There may be different sets of parameters which produce quite different wave-functions, nevertheless the data is fitted well by all of them. Second difficulty which is of practical nature arising specially in heavy ion scattering is the large number of partial waves which contribute significantly towards scattering e.g. there are in excess of 700 partial waves which must be included in the scattering of ^{238}U from Gold at 10 MeV/u. Under these conditions, the optimization of optical potential parameters becomes excessively extravagent in computer CPU time. It has been, therefore, argued that the simpler methods of analysis[5] may be used by exploiting the special features of heavy-ion interaction viz;

i. Large value of the Coulomb repulsion

ii. Short wavelength of the relative motion of interacting nuclei.

iii. Strong absorptions within nuclear range.

Thus there are various semi-classical approaches[6-9] and closed form experessions[10,11] for the scattering amplitude which have been successfully employed for the calculations of cross-sections. The method adopted in this report avoids the necessity to solve the Schrodinger equation for the determination of S-matrix. Rather the S-matrix (see next section) is itself parameterized. This technique was first used by McIntyre et al[5] for the analysis of α-particle scattering and later extended to heavier ions by Alster et al[12]. The necessary mathematical formulation of this approach is described in the next section. The results obtained for the test cases are discussed in section 3. The program structure and the numerical aspects are described in section 4.

THEORY

2.1 Scattering Problem

The physical situation consists of the scattering of a projectile of mass M_p from a target nucleus of mass M_T at the laboratory energy E_{lab}. The Schrodinger equation for the relative motion of the projectile can be written as

$$\left[\frac{-\hbar^2}{2\mu}\,\underline{\nabla}^2 + V(\underline{r})\right]\psi(\underline{r}) = E\,\psi(\underline{r})$$

where the reduced mass 'μ' is

$$\mu = \frac{M_p M_T}{M_p + M_T}$$

and the centre-of-mass energy E is defined as

$$E = E_{lab.}\,\frac{M_T}{M_p + M_T}$$

These quantities can be written in terms of the reduced mass number

$$A_{red.} = \frac{A_p A_T}{A_p + A_T}$$

where A_p and A_T is the mass number of the projectile and the target respectively i.e.

$$E \simeq \mathcal{E}\,A_{red.} \quad ; \quad \mu \simeq m_N A_{red.}$$

where '$\mathcal{E}$' is the laboratory energy per nucleon for the projectile and m_N = 931.5 is the conversion factor for the units of mass 'a.m.u' to the units 'MeV/c^2.' In eq. (1), $V(\underline{r})$ is the assumed nucleus-nucleus potential. For a spherically symmetric potential, the boundary conditions at large 'r', (i.e. well outside the range of the potential) are represented by an incident plane wave moving along (say) Z-axis and the outgoing spherical wave i.e.

$$\psi(\underline{r}) \xrightarrow[r\to\infty]{} \exp(ikz) + f(\theta)\,r^{-1}\exp(ikr)$$

Here $f(\theta)$ is the relative scattering amplitude of the scattered wave and is related to the elastic differential cross-section as

$$\frac{d\sigma(\theta)}{d\Omega} = |f(\theta)|^2$$

For a short range potential (in general complex), the partial wave expansion for eq. (6) leads to the value of $f(\theta)$ given by,

$$f(\theta) = \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell+1)(1 - S_\ell) P_\ell(\cos\theta)$$

where

$$k = \frac{A_{red.}}{c\hbar} \sqrt{2(m_N c^2)\varepsilon}$$

$P_l(\cos\theta)$ is the Legendre polynomial and S_l is the S-matrix related to the complex nuclear phase shift η_ℓ by

$$S_\ell = e^{2i\eta_\ell} = A_\ell e^{2i\delta_\ell}$$

In eq. (10), S_1 is the real part of the nuclear phase-shift and A_1 is the amplitude of l-th partial wave. The quantity 'A_1', also called the reflection co-efficient, represents the amount of absorption of the incident flux from elastic channel. Thus $0 \leq A_1 \leq 1$, with $A_1 = 1$ corresponding to no absorption and $A_1 = 0$ to complete absorption.

In the above treatment, the long range Coulomb force has not been considered. It is, however, possible to include the Coulomb force and obtain a scattering amplitude similar to eq. (8) with a modified phase shift ($\sigma_\ell + \delta_\ell$), where σ_l is the Coulomb phase shift. The eq. (8) can now be written as

$$f(\theta) = f_c(\theta) + \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell+1) e^{2i\sigma_\ell} (1 - A_\ell e^{2i\delta_\ell}) P_\ell(\cos\theta)$$

where[13)]

$$f_c(\theta) = \frac{i}{2k} \sum_{l=0}^{\infty} (2l+1)\left(1 - e^{2i\sigma_l}\right) P_l(\cos\theta)$$

$$= \frac{-\eta}{2k} \operatorname{cosec}(\theta/2)\, e^{-2i\eta \ln(\sin\theta/2) + 2i\sigma_0}$$

and[13)]

$$\sigma_l = \arg\Gamma(1 + l + i\eta)$$

$$= \sigma_0 + \sum_{l''=1}^{l} \arctan(\eta/l'')$$

In the above equation ' η ' is Sommerfeld parameter which is defined as

$$\eta = \frac{z_1 z_2 e^2 \mu}{\hbar^2 k}$$

The quantity $S_l = e^{2i\delta_l}$ is discussed in the next subsection. The final quantities of interest are calculated from $f(\theta)$ and $f_c(\theta)$ as described in section 3.

2.2 Treatment of S-matrix

The exact determination of the nuclear S-matrix involves the numerical solution of Schrodinger equation in the interaction region. However, an alternate way is to treat it phenomenologically. As an extreme case one may assume that it has a sharp boundary in the angular momentum space i.e. all the partial waves with values $l \leq l_c$ are fully absorbed whereas $l > l_c$ remain unaffected. Thus

$$A_\ell = |S_\ell| = 1 \qquad \text{for } \ell > \ell_c$$
$$= 0 \qquad \text{for } \ell \leq \ell_c$$

The total reaction cross-section, then becomes

$$\sigma_R = \frac{\pi}{k^2}(\ell_c+1)^2$$

In classical terms this situation corresponds to the case where the nuclei with angular momentum 'L' moving along trajectories with $r \leq R_c = L_c/k$ are fully absorbed whereas those with $r > R_c$ undergo pure Coulomb scattering. This is called the sharp cut-off model.

An improvement over this model can be achieved by asuming that the reflection coefficient A_l increases smoothly from zero to one over a definite range of angular momenta. The variation of A_l with respect to l, which is similar to nuclear Fermi distribution seems to be a natural choice. In this way one takes into account the nuclear diffuseness for defining the absorption region. It is usual to assume a similar shape for 'δ_l' although there is no à priori justification for such a choice. Following McIntyre et al[5], we take

$$A_\ell = \left[1 + e^{-(\ell-\ell_A)/\Delta\ell_A}\right]^{-1}$$

and

$$\delta_\ell = \delta_0\left[1 + e^{(\ell-\ell_\delta)/\Delta\ell_\delta}\right]^{-1}$$

where 'l' is now a continuous variable and l_A, Δl_A, δ_0, Δl_δ, l_δ are the adjustable parameters. The physical significance of these parameters is discussed in the next section.

RESULTS AND DISCUSSION

3.1 Calculated Quantities

Using the expression for Coulomb scattering amplitude (eq. 12) one can find the Rutherford differential cross-section as,

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_R = \left|f_c(\theta)\right|^2 = \frac{\eta^2}{4k^2}\ \mathrm{cosec}^2(\theta/2)$$

Also from eq. (11) and (12), the ratio of elastic differential cross-section to Rutherford cross-section is determined to be

$$\frac{d\sigma(\theta)/d\Omega}{(d\sigma(\theta)/d\Omega)_R} = \left| -i\ e^{-i\eta \ln \sin^2\theta/2} - \frac{\sin^2(\theta/2)}{\eta}\ S \right|^2$$

where

$$S = \sum_{\ell=0}^{\infty} \left(1 - A_\ell\, e^{2iS_\ell}\right)(2\ell+1)\, P_\ell(\cos\theta)\, e^{2i(\sigma_\ell - \sigma_0)}$$

The experimental values for the same quantity can be fitted to eq. (21) by varying the parameters of A_ℓ and S_ℓ. The subroutine used[13] for the fitting minimizes the quantity,

$$\chi_\nu^2 = \frac{1}{NP-N} \sum_{i=1}^{NP} \left[\frac{d\sigma^i_{exp.}(\theta) - d\sigma^i_{th.}(\theta)}{\Delta\left(d\sigma^i_{exp.}(\theta)\right)} \right]^2$$

where NP are the number of data points and N is the number of parameters. The fits obtained for the cases $^{12}C + ^{56}Fe$, $^{12}C + ^{107}Ag$, $^{12}C + ^{181}Ta$ are displayed in figs. 1-3 and the derived parameters are listed in table 1. The parameter l_A represents the value of 'l' at which $A_l = 1/2$ or the absorption $(1-A_l^2)$ is 75%. The variation of A_l with respect to l is given in fig. 4. Unlike the sharp cut-off model where the absorption radius corresponds to 100% absorption, we define a radius R_a which corresponds to $l = l_A$. By using the conservation of energy requirement

$$E = \frac{Z_1 Z_2 e^2}{r} + \frac{l(l+1)\hbar^2}{2\mu r^2}$$

We find

$$kr = \eta + \sqrt{\eta^2 + l(l+1)}$$

or

$$R_a = k^{-1}\left[\eta + \sqrt{\eta^2 + l_A(l_A+1)}\right]$$

Similarly from eq. (17), it can be seen that A_l goes from 0.9 to 0.1 over the range $4.4\Delta l_A$. The corresponding region of space $\Delta r = t$, is similar to the nuclear surface thickness. For $l \gg \eta$, it is simply

$$t \simeq k^{-1}\Delta l_A$$

In table 2, the values of absorption radius parameter obtained from eq. (25) and the absorption surface thickness from eq. (26) are listed alongwith the total reaction cross-section which is given by[13]

$$\sigma_R = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1 - A_l^2)$$

In eq. (18) the parameter δ_l gives the strength of the nuclear phase shift, however l_δ and Δl_δ do not have any apparent physical significance. The values given in table 2 under heading '1' have been obtained by using the central values of parameter of table 1. These are compared with the values obtained in ref. 12, where a number of different combinations of parameters with similar χ^2 are used to obtain the uncertainties in the quantities r_a, t and σ_R. Most of the values in the two calculations agree, however the small difference in some cases arises because of the restriction $\Delta l_A = \Delta l_\delta$ and $l_A = 1$ imposed in the present calculation. In view of the excellent fits obtained (Fig. 1-3) under this assumption, it is of no physical significance if this condition is relaxed. Another source of discrepancy is the different mass standards: ^{12}C in the present calculation and ^{16}O in reference 12.

3.2 Conclusions

By utilizing an optimization routine from the CPC library[13] and by writing a sub-program for calculating the function of eq. (21), we have obtained good fits to the elastic scattering data[12] of $^{12}C + ^{56}Fe$, $^{12}C + ^{107}Ag$, $^{12}C + ^{181}Ta$ at 10.375 MeV/u. The total reaction cross-section obtained for the test cases agree with the previous calculations. The method described in detail in section 2 serves as a reliable and quick way of analysing elastic scattering data and therefore an operating program on PINSTECH'S local computer is useful for the experimentalists as well as theoretical physicists who wish to compare other elaborate calculations with this simplified approach.

PROGRAM STRUCTURE

The computer program 'HISS' (Heavy Ion Scattering by S-matrix method) has been written in the standard FORTRAN IV language for the PINSTECH'S local computer. The program summary is given in the appendix I. For the purpose of least-square fitting, a subroutine from Computer Physics Communications library program ABMR UNIFIT2 is utilized. The R.H.S. of eq. (20) is calculated through the subprogram FNC(X) during optimization and for writing the final values with optimized parameters at the desired angles of scattering. A flow chart for the calculation done in the main and in FNC(X) is given in appendix II. The input data and output layout are described below while the complete program listing and specimen input/output appears in appendix III-V

4.1 Data Input.

Card A (1 card)
FORMAT (1X, A1, 3X, 10A4)

KATCH - New case identifier : Must be 'Q'
NTIT1 (1-10) - First title

Card B (1 card)
FORMAT (5X, 10A4)

NTIT2 (1-10) - Second Title

Card C (1 card)
FORMAT (I5, F10.5, I5)
NP ---Number of data points : $2 < NP < 100$
DYSTD ---Default uncertainty If DELTY = 0.0 in Card D
NTYPE ---If NTYPE ≤ 12, the input data is scaled. Different scalings may be done by different values of NTYPE. Present provision is for logarithmic transformation only.

Card D (NP cards)
FORMAT (3G 15.5)
X(I) --- Values of scattering angles in C.M frame
Y(I) --- Values of $d\sigma_{EXP}/d\sigma_R$
DELTY(I) --- Uncertainties in Y(I) values

Card E (1 card)
FORMAT (I5, F10.5, I5)
N ---No. of adjustable parameters : $1 < N < 20$, $N \leq NP$
EPSSTD --- Parameter increment ratio (Default 1. D-8)
M --- Total number of parameters : $N \leq M \leq 20$

Card F (1 card)
FORMAT (5I2)
I1, I2, I3, I4, I5 ----Sequence of parameters :
Only the first M parameters are searched

Card G (M cards)
FORMAT (3G15.5)
P(J) --- Parameter values
EPS (J) ---Fractional parameter increment for gradient calculation:
If other than EPSSTD
DP2(J) ---Uncertainty in P(J): May be omitted.

Card H (one card)
FORMAT (I3, 5F10.3)
LL ---l_{max} in the summation over l in eq. (21): If LL is too small, the program increases LL in steps of 10 to reach the correct value.

Z1 ---Atomic number of the projectile nucleus
Z2 ---Atomic number of the target nucleus
A1 ---Mass number of the projectile nucleus
A2 ---Mass number of the target nucleus
EPN ---Energy per nucleon for the projectile

Card I (One Card)
FORMAT (I3, 3F 10.3)
MM ----Print option for the reflection co-eff.
A_1: MM=0 no printing.
MM = 1; 100 values around $l = l_A$ are printed.
XF ---First angle at which the $d\sigma/d\sigma_R$ is required
XL ---Last angle at which the $d\sigma/d\sigma_R$ is required
XD ---Angular increment

4.2 Output

First Page: The first page of the output gives the heading comprising of the identification of heavy ion pair interacting at a specified energy alongwith the source of data and the desired data transformation. The initial and optimized χ^2 alongwith the list of all input parameter guesses and optimized parameters are written. The error code '0' appears when a successful convergence is obtained. The error code '1' shows that the convergence was not obtained in the permitted number of iterations.

Second Page: The angles at which the data is given are written under the heading "Independent" while the transformed data values appear under "Dependent". The "Optimized dependent" are the values obtained by using the model function with fitted parameters. The individual for each data point is also printed.

Third Page: This page is an optional output (If MM = 1). It gives the value of A_l for different 'l' in the vicinity of l_A. For l : 0-l_{max}, the A_l should increase from zero to one.

Fourth Page: Here the final quantities of interest, i.e. the ratio of elastic to Rutherford differential cross-sections and the total reaction cross-section (in Fermi squared units) are printed. The former may be written for any desired range of angles and angle increments (see Input Data).

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Table 1. Parameter of ^{12}C scattering at 10,375 MeV/u for the data[12] displayed in Figs. 1-3.

Target \ Parameter	k (fm^{-1})	η	$\ell_A = \ell_\epsilon$	$\Delta\ell_A = \Delta\ell_\delta$	δ_o	χ^2_ν
$^{56}_{26}Fe$	6.963	7.627	52.571 ± 0.188	3.147 ± 0.080	0.467 ± 0.029	0.49
$^{107}_{47}Ag$	7.602	13.788	60.538 ± 0.180	2.570 ± 0.071	0.686 ± 0.043	0.38
$^{181}_{73}Ta$	7.929	21.415	64.111 ± 0.159	2.573 ± 0.155	0.585 ± 0.036	0.03

Table 2. Values of the absorption radius parameter r_a, absorption thickness t_a and the total reaction cross-section σ_R derived from the central values of the parameters given in table 1. 1: This calculation 2: From ref. 12

Quantity	$r_a = R_a/(A_p^{1/3}+A_T^{1/3})$ fm		t (fm)		σ_R (mb)	
Target	1	2	1	2	1	2
$^{56}_{26}Fe$	1.438	1.42 ± 0.02	0.452	0.43 ± 0.06	2.061	1.96 ± 0.10
$^{107}_{47}Ag$	1.427	1.44 ± 0.02	0.414	0.31 ± 0.05	2.208	2.23 ± 0.07
$^{181}_{73}Ta$	1.420	1.45 ± 0.02	0.324	0.37 ± 0.09	2.263	2.43 ± 0.03

FIGURE CAPTIONS

1- The scattering angle in the centre-of-mass frame θ (C.M.) vs the ratio of elastic differential cross-section to the Rutherford cross-section using the logarithmic scale; for ^{12}C + ^{56}Fe at E_{lab} = 124.5 MeV.

2- Ibid; for ^{12}C + ^{107}Ag

3- Ibid; for ^{12}C + ^{181}Ta

4- Variation of the parameterized reflection co-efficient 'A_l' with respect to the angular momentum 'l' for the three cases of fig. 1,2 and 3. The l_A and Δl_A are the fitted parameters.

Fig. 1.

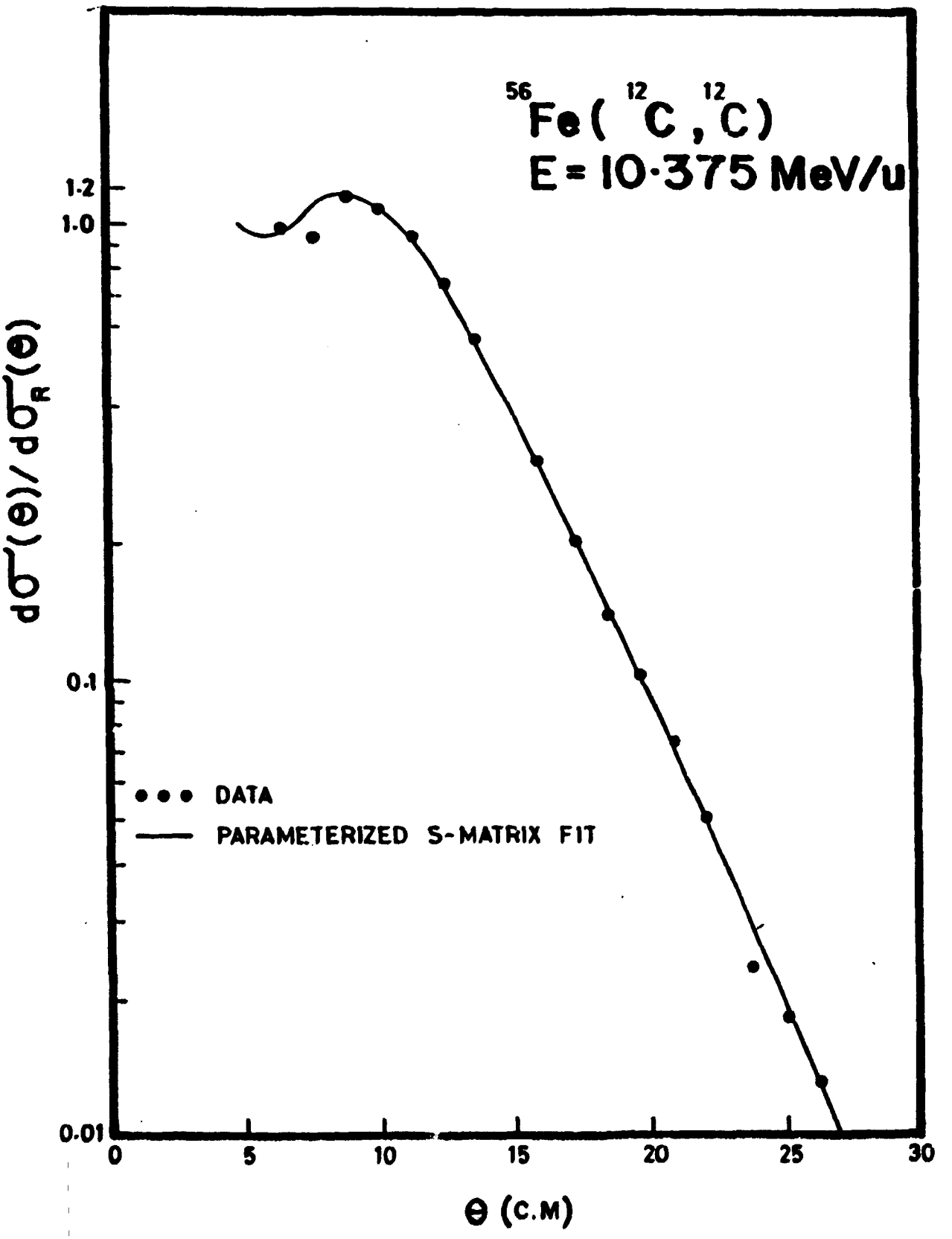

56Fe(12C , 12C)
E = 10·375 MeV/u
dσ(θ)/dσR(θ)
1·2
1·0
0·1
0·01
DATA
PARAMETERIZED S-MATRIX FIT
0
5
10
15
20
25
30
θ (C.M)

Fig. 2.

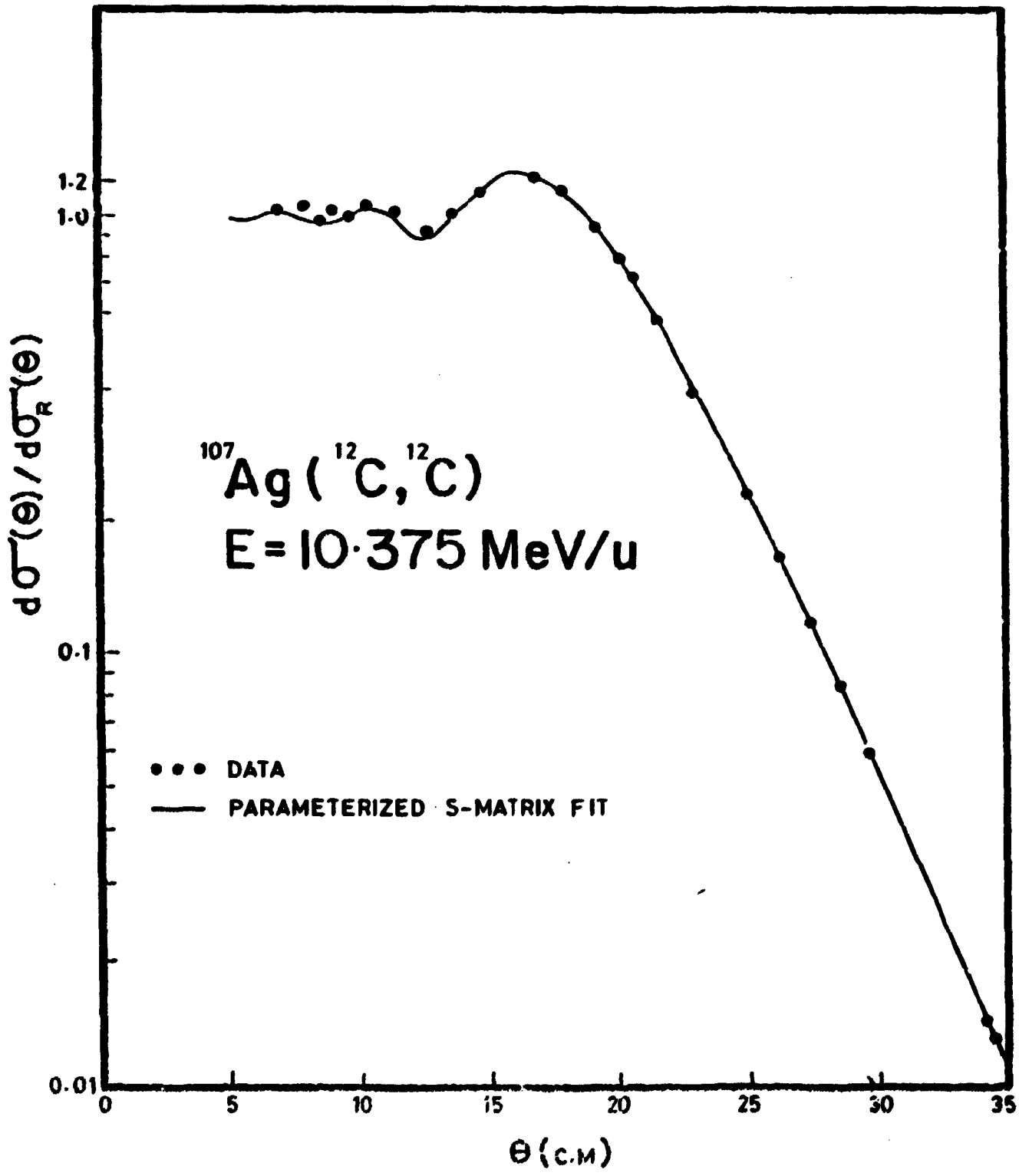

1·2
1·0
0·1
0·01
dσ(θ)/dσR(θ)
107Ag (12C, 12C)
E = 10·375 MeV/u
DATA
PARAMETERIZED S-MATRIX FIT
0
5
10
15
20
25
30
35
θ (c.m)

Fig. 3.

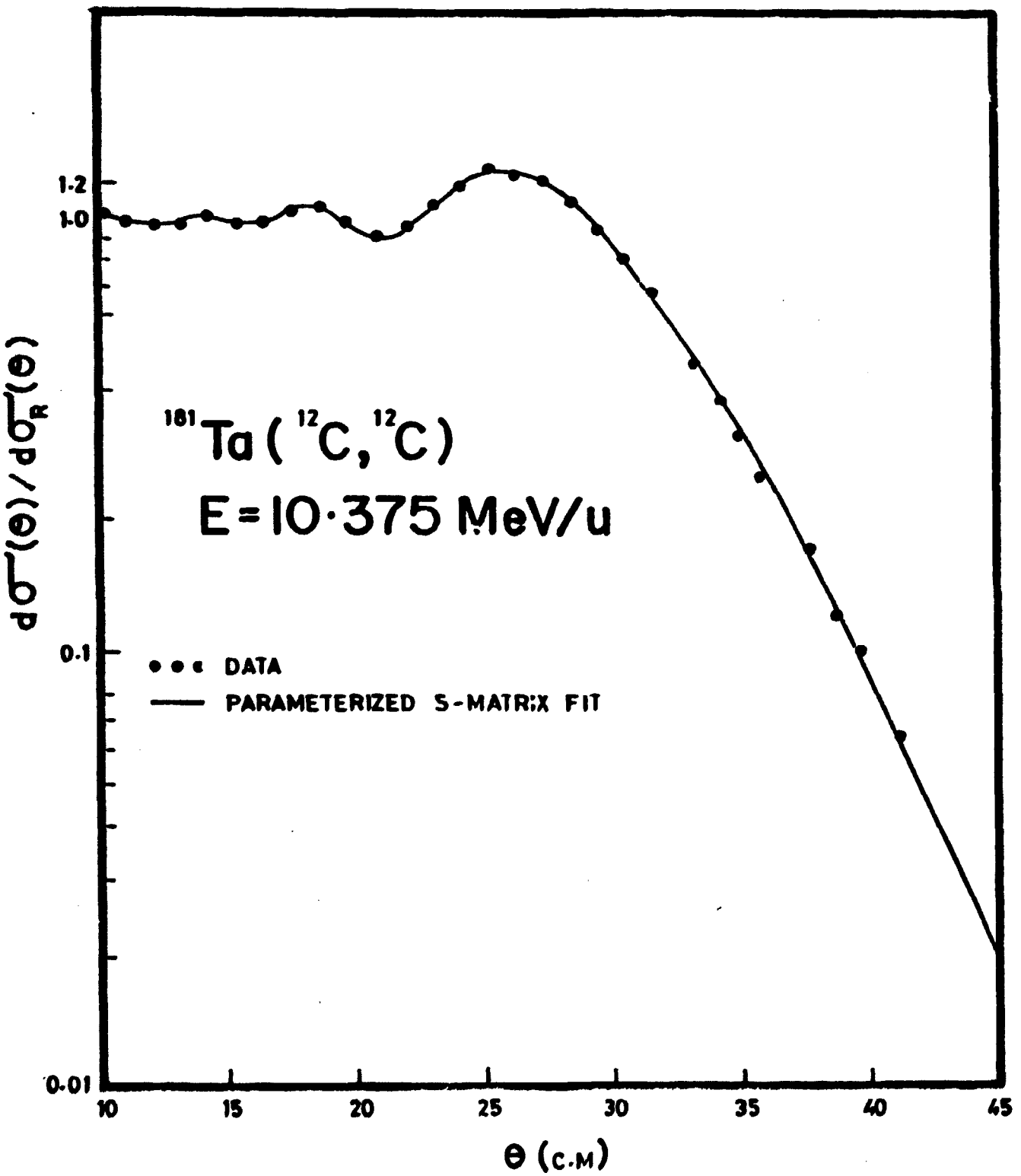
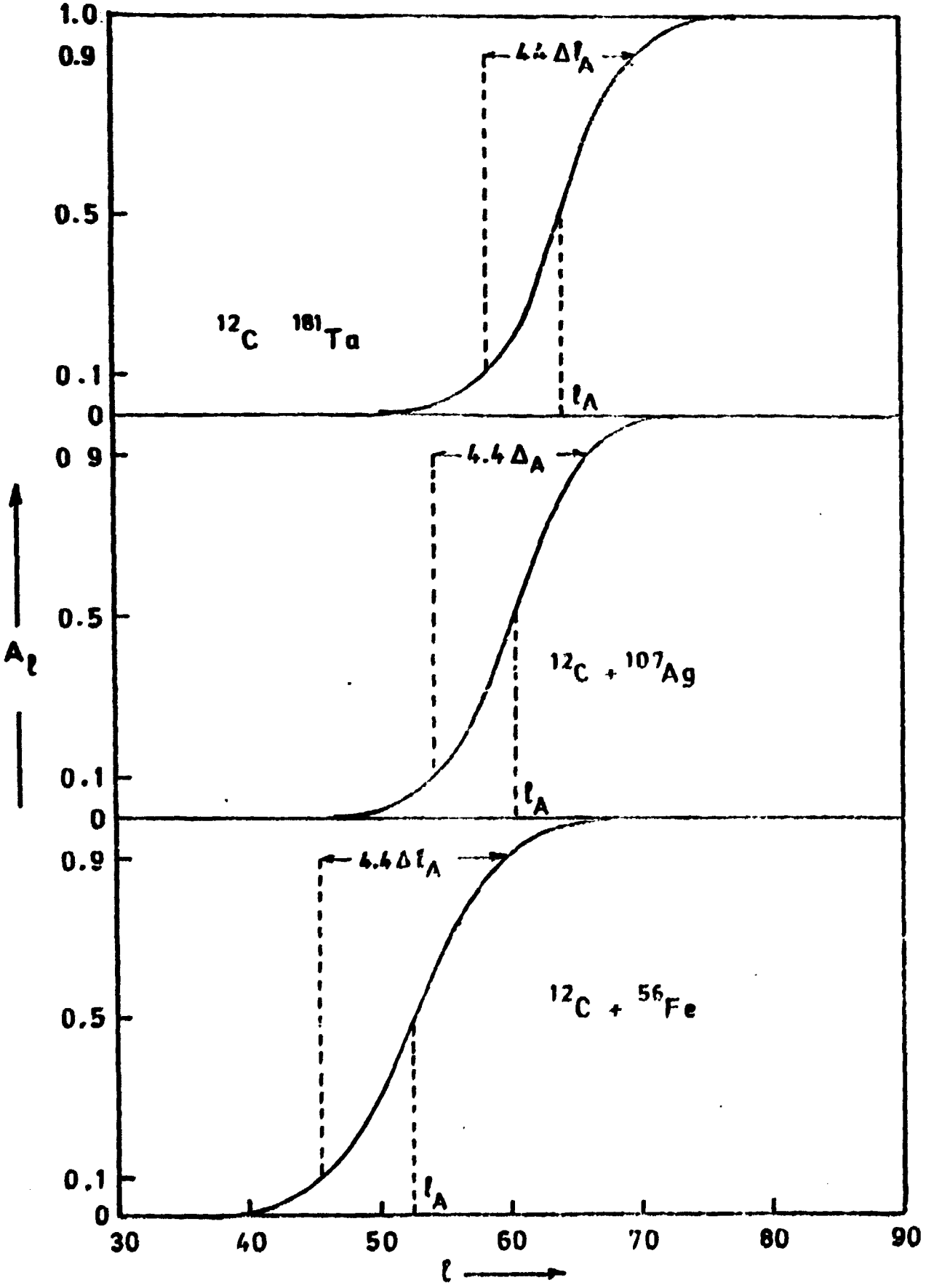

1·2
1·0
0·1
0·01
dσ(θ)/dσR(θ)
181Ta (12C, 12C)
E=10·375 MeV/u
DATA
PARAMETERIZED S-MATRIX FIT
10
15
20
25
30
35
40
45
θ (c.m)

Fig. 4.

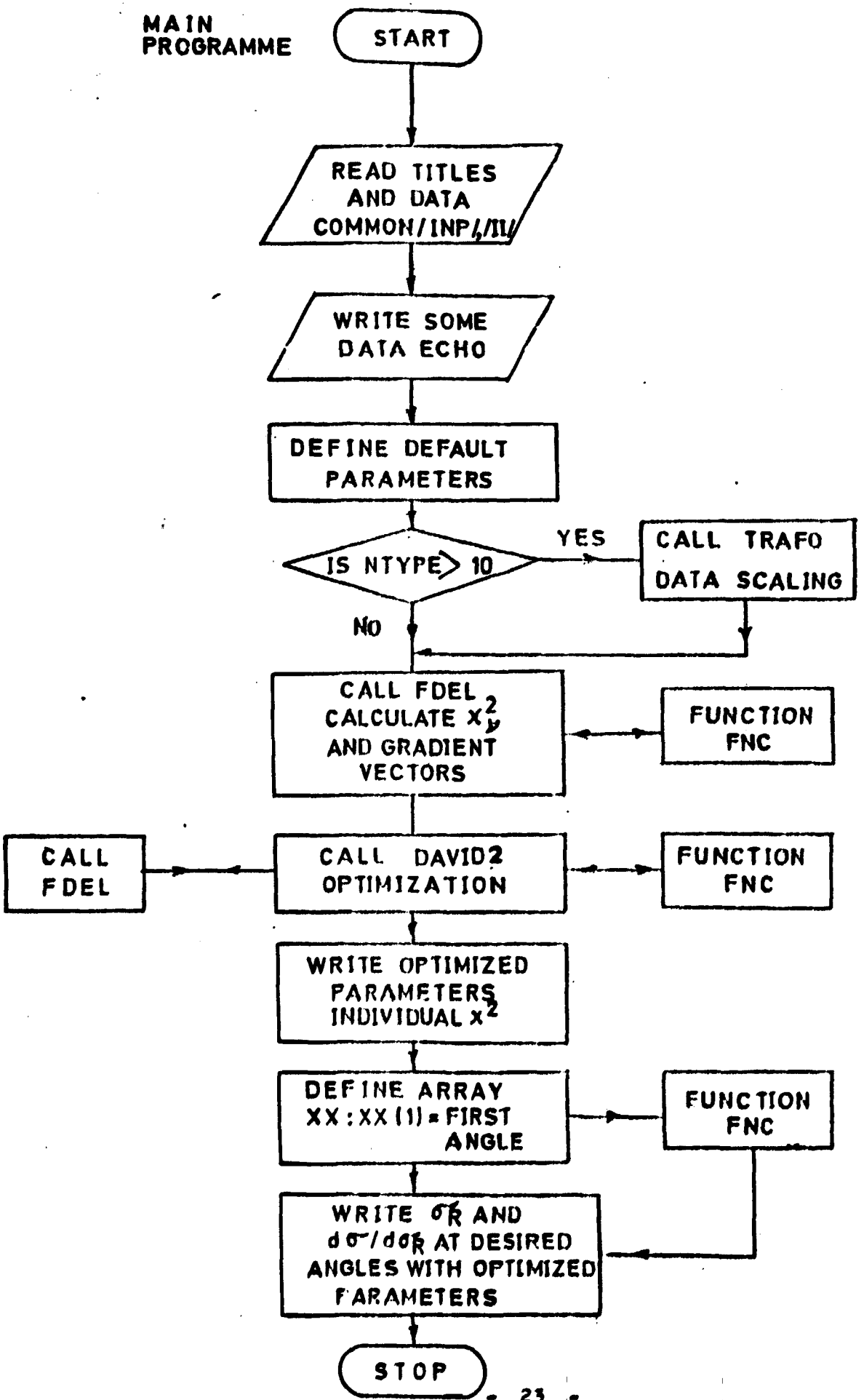

1.0
0.9
0.5
0.1
0
4.4 Δl_A
12C 181Ta
l_A
0.9
0.5
0.1
0
4.4 Δ_A
12C + 107Ag
l_A
A_l
0.9
0.5
0.1
0
4.4 Δl_A
12C + 56Fe
l_A
30
40
50
60
70
80
90
l

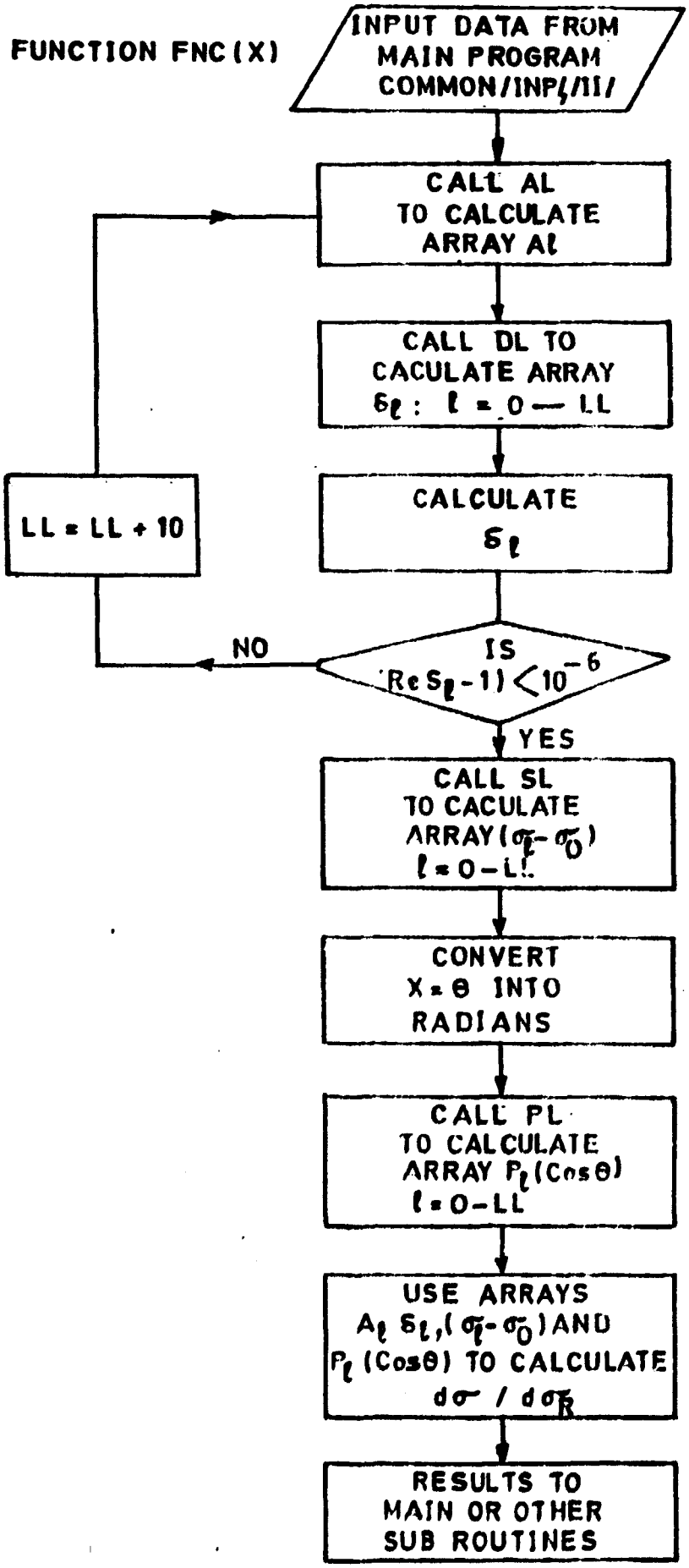
Appendix I

PROGRAM SUMMARY

Title of Program	:	HISS
Directory	:	DRA1: [HITIHAN]
Program Obtainable from	:	T.P.G. of N.P.D.
Computer	:	VAX 11/780 Installation PINSTECH
Operating System	:	VMS/MOS
Programming Language	:	FORTRAN IV (G)
High Speed Storage Required	:	94 K Bytes
Number of bits in a word	:	32
Overlay Structure	:	None
Tape Code	:	EBCDIC
Nature of Physical Problem	:	Calculation of $d\sigma/d\sigma_R$ for heavy ions and fitting the parameters of S-matrix.
Method of Solution	:	Davidson's non-linear minimization technique programmed by E. von-Meerwall[13]
Typical Running Time	:	6-10 min. for fitting 3-5 parameters
Program Features	:	1- Upto 100 data points with uncertainities can be read. 2- The number of parameters should be less than 20. 3- Only first M parameters out of N are fitted. ($M \leq N$) 4- The parameter guesser may be given alongwith their ranges.

APPENDIX II
(1)




FUNCTION FNC(X)
INPUT DATA FROM MAIN PROGRAM COMMON/INPℓ/II/
CALL AL TO CALCULATE ARRAY Aℓ
CALL DL TO CACULATE ARRAY δℓ : ℓ = 0 — LL
CALCULATE δℓ
IS Re Sℓ - 1) < 10^-6
NO
LL = LL + 10
YES
CALL SL TO CACULATE ARRAY (σℓ - σ0) ℓ = 0 - LL
CONVERT X = θ INTO RADIANS
CALL PL TO CALCULATE ARRAY Pℓ (Cosθ) ℓ = 0 - LL
USE ARRAYS Aℓ δℓ, (σℓ - σ0) AND Pℓ (Cosθ) TO CALCULATE dσ / dσR
RESULTS TO MAIN OR OTHER SUB ROUTINES

APPENDIX III

PROGRAM LISTING

```
C                    ***************
C                    *             *
C                    *     HISS    *
C                    *             *
C                    ***************
C
C       THIS PROGRAM UTILIZES AN ADAPTED SUBROUTINE FROM  CPC LIBRARY
C       PROGRAM   '  UNIFIT2  '  BY  ' E. VON MEERWALL ' (CPC 11(1976)211
C       TO FIT THE DATA ON HEAVY ION ELASTIC DIFFERENTIAL CROSS-SECTIONS
C       WITH THE HELP OF THE EXPRESSION DEFINED IN EQ.13 ,20 & 21 OF THE
C       N.P.D INTERNAL REPORT  "  ANALYSIS OF HEAVY ION ELASTIC SCATTERING
C       DATA BY S-MATRIX PARAMETERIZATION ".
C                                                          I.E.QURESHI
C                                                          25-10-1984
C
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON XK,A,X,Y,DELTY,W,P,EPS,GRAD,CH2,NP,N,NTYPE
        COMMON/INP/LL,Z1,Z2,A1,A2,EPN
        COMMON/II/I1,I2,I3,I4,I5
        DIMENSION A(1000),X(100),Y(100),W(100),EPS(20),P(20),GRAD(20),
     * DELTY(100),DP(20),NTIT1(10),NTIT2(10),LABELX(5),LABELY(5),
     * DP2(20),KE(2),KV(2),FCTR(9),SRT(100),XX(100)
       DATA FCTR/1.D2,1.D-4,1.D6,1.D-8,1.D9,1.D-10,1.D11,1.D-12,1.D6/
       DATA IRD/1/,IWT/2/,KUE/1H0/,KE/4H EPS,4HILON/,KV/4H +/-,4H DP /
C
    2 FORMAT (I5,F10.5,I5)
    5 FORMAT (3G15.5)
   11 FORMAT (1X,A1,3X,10A4)
   12 FORMAT (5X,10A4)
   13 FORMAT (45X,10A4/)
   14 FORMAT (/10X,26HABORT THIS CASE SINCE NP =,I3,5H, N =,I3)
   16 FORMAT (//15X,23HINITIAL CHI-SQUARE (PER,I3,
     * 20H DEGREES OF FREEDOM),//)
   17 FORMAT (23X,F20.5)
   18 FORMAT (//15X,13HPARAMETER NO.,14X,15HPARAMETER VALUE,15X,2A4,
     * 18X,8HGRADIENT,/)
   19 FORMAT (/34X,14HRESULTS OF FIT)
   20 FORMAT (///15X,23HNO. (MAX) OF ITERATIONS,16X,
     * 16H      ERROR CODE,16X,20HOPTIMIZED CHI-SQUARE)
   21 FORMAT (/24X,I2,1H(,I2,1H),30X,I2,23X,F20.6)
   22 FORMAT (/20X,35HTHERE WILL BE NO FIT FOR PARAMETERS,I3,
     * 8H THROUGH,I3/)
   23 FORMAT (21X,I2,15X,F20.10,13X,D12.5,13X,D12.5)
   26 FORMAT (/10X,19HMULTIPLY ALL EPS BY,1PD8.1)
   28 FORMAT (3X,I3,4X,F10.5,6X,F10.6,5X,D12.4,10X,
     * F10.6,9X,D13.5,7X,D13.6)
   29 FORMAT (1H1,////10X,11HINDEPENDENT,6X,9HDEPENDENT,7X,6HWEIGHT,
     * 7X,19HOPTIMIZED DEPENDENT,5X,15HFNC(P(OPT),X)-Y,4X,
     * 22HCONTRIB. TO CHI-SQUARE,//)
C
C    INITIALIZATION OF PROGRAM
C
       NCASE=0
C
C    INPUT OF CASE INFORMATION: TITLES AND LABELS
C
    1 READ(IRD,11,END=39) KATCH,(NTIT1(I),I=1,10)
      IF (KATCH.NE.KUE) GO TO 1
      READ(IRD,12) (NTIT2(I),I=1,10)
      NCASE=NCASE+1
      O=-1.D37
        PI=3.14159265D0
```

```
C
C    INPUT OF DATA: X, Y, DELTY, AND DEFAULT VALUES
C
      READ(IRD,2) NP,DYSTD,NTYPE
      DYSTD=DABS(DYSTD)
      NBAR=0
      IF(DYSTD.GT.1.D-7) GO TO 3
      DYSTD=1.D0
      NBAR=1
    3 DO 4 L=1,NP
      READ(IRD,5,END=39) X(L),Y(L),DELTY(L)
      IF(X(L).GT.Q) Q=X(L)
      DELTY(L)=DABS(DELTY(L))
      IF(DELTY(L).EQ.0.D0.AND.NBAR.EQ.1) GO TO 6
      IF(DELTY(L).EQ.0.D0) DELTY(L)=DYSTD
      W(L)=(1.D0/DELTY(L))**2
      GO TO 4
    6 W(L)=1.D0
    4 CONTINUE
C
C    INPUT OF PARAMETERS, EPSILONS, PARAMETER ERRORS, AND DEFAULTS
C
      READ(IRD,2) N,EPSSTD,M
      IF(M.LT.N.OR.M.GT.20) M=N
      IF(NP.LT.3.OR.NP.GT.100) GO TO 35
      IF(N.GE.1.AND.N.LT.NP.AND.N.LE.20) GO TO 36
   35 WRITE(IWT,14) NP,N
      GO TO 1
   36 IF(EPSSTD.EQ.0.D0) EPSSTD=1.D-8
        READ(IRD,60)I1,I2,I3,I4,I5
60      FORMAT(5I2)
      DO 9 L=1,M
      READ(IRD,5,END=39) P(L),EPS(L),DP2(L)
      IF(EPS(L).LT.1.D-10) EPS(L)=EPSSTD
      Q=DABS(P(L))
      IF(Q.NE.0.D0) EPS(L)=EPS(L)*Q
      Q=1.D-3*DABS(DP2(L))
      IF(Q.NE.0.D0) EPS(L)=DMIN1(EPS(L),Q)
      IF(L.LE.N) GO TO 9
      EPS(L)=0.D0
      DP(L)=0.D0
      DP2(L)=0.D0
      GRAD(L)=0.D0
    9 CONTINUE
      NPN=NP-N
      ANPN1=1.D0/NPN
        READ(IRD,61)LL,Z1,Z2,A1,A2,EPN
61      FORMAT(I3,5F10.3)
        READ(IRD,62)MM,XF,XL,XD
62      FORMAT(I5,3F10.3)
C
C    SOME ECHO OUTPUT
C
      WRITE(IWT,13) (NTIT1(I),I=1,10),(NTIT2(I),I=1,10)
C
C    FIT INITIALIZATION
C
      IF(NTYPE.GE.10) CALL TRAFO
      CALL FDEL
      WRITE(IWT,16) NPN
      WRITE(IWT,17) CH2
      WRITE(IWT,18) KE
      WRITE(IWT,23) (I,P(I),EPS(I),GRAD(I),I=1,M)
      IF(M.EQ.N) GO TO 10
      N1=N+1
```

```
      WRITE(IWT,22) N1,M
   10 LIMIT=4*N+6
      IF(LIMIT.GT.60) LIMIT=60
C
C    FIT, EPS ADJUSTMENT FOR GRADIENT TROUBLE, RESULTS
C
      NERR=0
   34 NERR=NERR+1
      DO 38 L=1,M,1
   38 DP(L)=DP2(L)
      CALL DAVID2(DP,1.D-9,KOUNT,LIMIT,IER)
      IF(IER.NE.-1) GO TO 31
      IF(NERR.GT.9) GO TO 31
      FM=FCTR(NERR)
      WRITE(IWT,26) FM
      DO 32 L=1,N,1
   32 EPS(L)=EPS(L)*FM
      GO TO 34
   31 WRITE(IWT,19)
      WRITE(IWT,20)
      WRITE(IWT,21) KOUNT,LIMIT,IER,CH2
      WRITE(IWT,18) KV
      WRITE(IWT,23) (I,P(I),DP(I),GRAD(I),I=1,N)
C
C    TABLE OF VALUES AND FIT
C
      WRITE(IWT,29)
      DO27 JN=1,NP
      R=X(JN)
      Z=FNC(R)
      D=Y(JN)-Z
      D2=ANPN1*D*D*W(JN)
      WRITE(IWT,29) JN,X(JN),Y(JN),W(JN),Z,D,D2
   27 CONTINUE
C
C       PRINT RATIO OF THE DIFFERENTIAL CROSS-SECTIONS AT DESIRED
C       ANGLES BY USING THE OPTIMIZED PARAMETERS.
C
        J1=P(1)-50
        J2=P(1)+50
        XX(1)=XF
        NN=(XL-XF)/XD
        DO 50 J=1,NN
        SR=FNC(XX(J))
        SRT(J)=10.0**SR
50      XX(J+1)=XX(J)+XD
        IF(MM.EQ.1)WRITE(2,53)
        IF(MM.EQ.1)WRITE(2,54)(J-1,A(J),J=J1,J2)
        WRITE(IWT,52)
        WRITE(IWT,51)(XX(J),SRT(J),J=1,NN)
51      FORMAT(3(10X,F5.2,G15.5,6X))
52      FORMAT(1H1,//10X,3(9X,'RATIO OF    ',16X),/10X,3('ANGLE',4X,
     X          'DIFF. X-SECS',15X),//)
53      FORMAT(1H1,///5X,3(1X,'L',10X,'A(L)',6X),//)
54      FORMAT(3(2X,I5,G15.5))
C
C       CALCULATION OF THE TOTAL REACTION X-SECTION.
C
        SIGR=0.D0
        DO 70 J=1,LL
70      SIGR=SIGR+(2.*J-1)*(1.-A(J)**2)
        SIGR=SIGR*PI/XK**2
        WRITE(IWT,80)SIGR
80      FORMAT(//10X,'TOTAL REACTION X-SEC =',G15.5,'  F**2')
```

```
C
C
        GO TO 1
39      STOP
      END
C
C-----------------------------------
      SUBROUTINE FDEL
C
C   COMPUTES CHI-SQUARE AND GRADIENT VECTOR FOR DAVID2
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON XK,XA,X,Y,DELTY,W,P,EPS,GRAD,CH2,NP,N,NITYPE
        COMMON/INP/ LL,Z1,Z2,A1,A2,EPN
        COMMON/II/I1,I2,I3,I4,I5
      DIMENSION XA(1000),X(100),Y(100),DELTY(100),W(100),P(20),EPS(20)
     X          ,GRAD(20)
      ANPN1=1.D0/(NP-N)
      ALFA=0.1D0/N-0.98D0
C
C   CHI-SQUARE
C
      CH=0.D0
      DO 1 I=1,NP,1
      R=X(I)
      A=FNC(R)-Y(I)
    1 CH=CH+A*A*W(I)
      CH2=CH*ANPN1
C
C   GRADIENT VECTOR
C
      DO 3 K=1,N,1
      P(K)=P(K)+EPS(K)
      C=0.D0
      DO 2 I=1,NP,1
      R=X(I)
      A=FNC(R)-Y(I)
    2 C=C+A*A*W(I)
      GRAD(K)=ANPN1*(C-CH)/EPS(K)
      P(K)=P(K)-EPS(K)
    3 EPS(K)=EPS(K)*ALFA
C
      RETURN
      END
C
C
```

```
C
C
C
      SUBROUTINE DAVID2 (DP,TERM,KOUNT,LIMIT,IER)
C
C   VARIABLE METRIC NON-LINEAR MINIMIZATION OF FUNCTION
C   UNCONSTRAINED SIMPLIFIED DAVIDON METHOD
C   US AEC R AND D REPORT ANL 5990, APPENDIX (1959, REV.)
C   NEEDS FDEL FOR CHI-SQUARE (F) AND GRADIENT (G)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON XK,XA,X0,Y0,Z0,W0,P,EPS,G,F,NP,N,NTYPE
      DIMENSION X0(100),Y0(100),Z0(100),W0(100),P(20),EPS(20),G(20)
      DIMENSION DP(20),PB(20),GB(20),HU(20),HV(20),H(20,20),XA(1000)
C
C   INITIALIZATION
C
      ANPN=(NP-N)*0.5D0
      T=5.D0
      KOUNT=0
      IER=0
      N1=0
      N2=4
      IF(N2.LE.N) N2=N+1
      N5=0
      IF(LIMIT.LE.N2) LIMIT=N2+1
      CALL FDEL
C
C   PREPARATION OF ERROR MATRIX H
C
      DO 1 J=1,N,1
      IF(N.EQ.1) GO TO 3
      DO 2 K=1,N,1
    2 H(J,K)=1.D-10
    3 H(J,J)=1.D-2
      IF(DP(J).NE.0.D0) GO TO 7
      IF(P(J).EQ.0.D0) GO TO 9
      DP(J)=P(J)*3.D-2
    7 A=ANPN*(DP(J)**2)
      B=A
      IF(DABS(G(J)).GT.5.D-3) B=0.3D0*DABS(DP(J)/G(J))
      H(J,J)=0.3D0*DSQRT(A*B)
    9 PB(J)=P(J)
      GB(J)=G(J)
    1 DP(J)=H(J,J)
      FB=F
C
C   ITERATIVE FIT
C
   60 KOUNT=KOUNT+1
      IF(KOUNT.GT.LIMIT) GO TO 50
C
C   GO TO NEW MINIMUM
C
      DO 10 K=1,N,1
      HV(K)=0.D0
      DO 11 J=1,N,1
   11 HV(K)=HV(K)+H(K,J)*G(J)
   10 P(K)=P(K)-HV(K)
      U=0.D0
      V=0.D0
```

```
C
C    CLOSENESS TO MINIMUM
C
       CALL FDEL
       IF(F.GE.FB) GO TO 21
       IER=2
       N4=KOUNT
       N5=NG+1
       FB=F
       DO 20 J=1,N,1
       DP(J)=H(J,J)
       IF(DP(J).LT.0.D0.AND.KOUNT.GE.N2) GO TO 75
       PB(J)=P(J)
   20  GB(J)=G(J)
       IER=0
   21  DO 22 K=1,N,1
       HU(K)=0.D0
       DO 23 J=1,N,1
   23  HU(K)=HU(K)+H(K,J)*G(J)
       V=V+G(K)*HV(K)
   22  U=U+G(K)*HU(K)
C
C    RESCUE FROM BAD GUESSES AND/OR EARLY GRADIENT TROUBLE
C
       IF(KOUNT.GT.3) GO TO 27
       A1=1.2D0*FB
       IF(F.LT.A1.AND.U.GT.0.D0) GO TO 27
       DO 26 J=1,N,1
       IF(U.EQ.0.D0) EPS(J)=1.D1*EPS(J)
       H(J,J)=1.D-1*H(J,J)
   26  P(J)=PB(J)-5.D-2*H(J,J)*G(J)
       CALL FDEL
       GO TO 60
C
C    TESTS OF CONVERGENCE AND STABILITY
C
   27  IF(KOUNT.LE.N2) GO TO 29
       IER=-1
       IF(U.EQ.V.OR.U.EQ.0.D0) GO TO 75
       IER=0
       IF(U.LE.TERM) GO TO 70
C
C    COEFFICIENT FOR UPDATING H
C
   29  A1=-U/(T-1.D0)
       A2=U/(T+1.D0)
       A3=T*A2
       A4=-T*A1
       IF(V.GE.A1) GO TO 41
       ALFA=1.D0/(V-U)
       GO TO 30
   41  IF(V.GE.A2) GO TO 42
       ALFA=1.D0/(T*U)-1.D0/U
       GO TO 30
   42  IF(V.GE.A3) GO TO 43
       ALFA=(U-2.D0*V)/(U*(V-U))
       GO TO 30
   43  IF(V.GE.A4) GO TO 44
       ALFA=(T+1.D0)/U
       GO TO 30
   44  ALFA=1.D0/(V-U)
```

```
C
C    UPDATE H, CYCLE
C
   30 DO 31 J=1,N,1
      J1=J
      DO 32 K=1,J1,1
      H(J,K)=H(K,J)+ALFA*HU(K)*HU(J)
   32 H(K,J)=H(J,K)
   31 CONTINUE
      GO TO 60
C
C    TERMINATION
C
   50 N1=N1+1
      IF(N1.GE.2) GO TO 69
      LIMIT=LIMIT+6
      GO TO 60
   69 IER=1
C
C    COPY BEST RESULTS
C
   70 IF(N5.EQ.0) IER=-1
      IF(IER.EQ.-1.OR.IER.EQ.2.OR.N4.LT.N) N1=-1
   75 F=FB
      DO 71 J=1,N,1
      IF(N1.EQ.-1) DP(J)=0.00
      DP(J)=DSQRT(DP(J)/ANPN)
      P(J)=PB(J)
   71 G(J)=GB(J)
      RETURN
      END
```

```
C
C**********************************************************************
C
C       THIS SUBPROGRAM CALCULATES THE FUNCTION DEFINING      *
C       THE RATIO OF ELASTIC DIFFERENTIAL CROSS-SECTION       *
C       DIVIDED BY THE RUTHERFORD CROSS-SECTION               *
C
C**********************************************************************
C
        FUNCTION FNC(X)
        IMPLICIT REAL*8(A-H,O-Z)
        REAL*4 UL
        COMPLEX*16 XI,F1,F31,F32,F3,CHECL
        COMMON/INP/ LL,Z1,Z2,A1,A2,EPN
        COMMON/II/ I1,I2,I3,I4,I5
        COMMON XK,A,Q1,Q2,Q3,Q4,P,Q5,Q6,Q?,NPO,N,NTYPE
        DIMENSION A(1000),D(1000),PP(1000),S(1000)
        DIMENSION Q1(100),Q2(100),Q3(100),Q4(100),P(20),Q5(20),Q6(20)
        PI=3.141592654 D0
        CH=197.32892 D0
        ALP=7.2973515D-03
        XM=?31.5D0
        XI=DCMPLX(0,D0 ,1,D0)
        RC=1.4*((A1**0.33333D0)+(A2**0.33333D0))
        ARED= (A1*A2)/(A1+A2)
        XMU=XM*ARED
        XK=DSQRT(2.*XMU*EPN*ARED)/CH
        ETA=Z1*Z2*ALP*XMU/(CH*XK)
        UL=SNGL(XK*RC)
998     CALL AL(LL,P(I1),P(I2),A)
        CALL DL(LL,P(I1),P(I2),P(I5),D)
        CHECL=A(LL)*CDEXP(2.*XI*D(LL))
        IF(DREAL(CHECL-1.).LE.1.D-6)GO TO 990
        LL=LL+10
        GO TO 998
990     CALL SL(LL,ETA,S)
        SINE=DSIN(X*PI/360.)
        SINS=SINE*SINE
        F1=-XI*CDEXP(-XI*ETA*DLOG(SINS))
        F2=SINS/ETA
        CALL PL(LL,X,PP)
        F3=DCMPLX(0.D0,0.D0)
        DO 30 L=1,LL
        XL=DFLOAT(L-1)
        F31=CDEXP(2.*XI*S(L))
        F32=A(L)*CDEXP(2.*XI*D(L))
        F3=F3+(1.-F32)*(2.*XL+1.D0)*PP(L)*F31
30      CONTINUE
        FNC=CDABS(F1-F2*F3)**2
        FNC=DLOG10(FNC)
C40     CONTINUE
        RETURN
        END
        SUBROUTINE AL(LL,XLP,DXL,A)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(LL)
        DO 10 I=1,LL
        L=I-1
        XL=DFLOAT(L)
        P=(XL-XLP)/DXL
        IF(P.LT.-50.)GO TO 2
        IF(P.GT.+50.)GO TO 3
        Q=DEXP(-P)
        A(I)=1.D0/(1.D0+Q)
        GO TO 10
2       A(I)=0.
```

```
          GO TO 10
3         A(I)=1.D0
10        CONTINUE
          RETURN
          END
          SUBROUTINE DL(LL,XLD,DXL,DLD,D)
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION D(LL)
          DO 10 I=1,LL
          L=I-1
          XL=DFLOAT(L)
          P=(XL-XLD)/DXL
          IF(P.LT.-50.)GO TO 2
          IF(P.GT.+50.)GO TO 3
          Q=DEXP(P)
          D(I)=DLD/(1.D0+Q)
          GO TO 10
2         D(I)=DLD
          GO TO 10
3         D(I)=0.D0
10        CONTINUE
          RETURN
          END
          SUBROUTINE SL(LL,ETA,S)
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION S(LL)
          SS=0.D0
          S(1)=SS
          DO 10 I=2,LL
          L=I-1
          THET=ETA/L
          SS=SS+DATAN(THET)
          S(I)=SS
10        CONTINUE
          RETURN
          END
          SUBROUTINE PL(LL,XC,P)
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION P(LL)
          P(1)=1.D0
          P(2)=DCOS(XC*3.1415926/180.D0)
          DO 10 I=3,LL
          L=I
          XL=DFLOAT(L)
          P(L)=(2.*(XL-2.D0)+1.)*P(2)*P(L-1)-(XL-2.D0)*P(L-2)
          P(L)=P(L)/(XL-1.)
10        CONTINUE
          RETURN
          END
        SUBROUTINE TRAFO
C
C   PERFORMS INDICATED TRANSFORMATION ON THE DATA, AND/OR THE
C   PARAMETER GUESSES, FOR NTYPE.GE.10
C
          IMPLICIT REAL*8(A-H,O-Z)
          COMMON XK,A,X,Y,DELTY,W,P,EPS,GRAD,CH2,NP,N,NTYPE
          DIMENSION A(1000),X(100),Y(100),DELTY(100),W(100),P(20),EPS(20)
         X          ,GRAD(20)
          DATA IWT/2/
C
```

```
C
      IF(NTYPE.GT.12) GO TO 10
      DO 2 I=1,NP,1
C     X(I)=2.D0*X(I)
      IF(Y(I).LT.1.D-5) GO TO 3
        IF(DELTY(I).LT.1.D-05)GO TO 3
      DELTY(I)=DELTY(I)/Y(I)
      W(I)=(1.D0/DELTY(I))**2
      Y(I)=DLOG10(Y(I))
      GO TO 2
    3 Y(I)=-4.D0
      DELTY(I)=0.D0
      W(I)=0.D0
    2 CONTINUE
   10 RETURN
C
      END
$
```

APPENDIX IV

INPUT DATA

```
0   12C + 181TA  AT  10.375 MEV/N
    ALSTER AND CONZET DATA (LOG SCALE)
  31                10
 7.9                0.99              0.0099
 9.0                1.01              0.0101
10.2                1.01              0.0101
11.2                0.98              0.0098
12.2                0.96              0.0096
13.3                0.97              0.0097
14.4                1.01              0.0101
15.5                0.98              0.0098
16.5                0.99              0.0099
17.6                1.04              0.0104
18.7                1.05              0.0105
19.7                0.98              0.0098
20.9                0.91              0.0091
22.1                0.96              0.0096
23.2                1.09              0.0109
24.2                1.19              0.0119
25.3                1.29              0.0129
26.3                1.25              0.0125
27.4                1.22              0.0122
28.5                1.09              0.0109
29.5                0.94              0.0094
30.6                0.81              0.0081
31.6                0.67              0.0067
33.3                0.46              0.0138
34.3                0.38              0.0038
34.9                0.312             0.00312
35.8                0.25              0.0025
37.6                0.166             0.00166
38.7                0.122             0.00244
39.6                0.103             0.00309
41.2                0.063             0.00189
   3                 5
1 2 4 5 3
   66.0
   3.0
   0.3
  66.0
   3.0
120 6.0        73.0        12.0        181.0     10.375
    1 10.0       50.0         1.0
$
```

APPENDIX V

OUTPUT PAGE(1)

12C + 181TA AT 10.375 MEV/N

ALSTER AND CONZET DATA (LOG SCALE)

INITIAL CHI-SQUARE (PER 28 DEGREES OF FREEDOM)

7.83349

PARAMETER NO.	PARAMETER VALUE	EPSILON	GRADIENT
1	66.0000000000	-0.62480D-06	0.91348D+01
2	3.0000000000	-0.29400D-07	0.49721D+01
3	0.3000000000	-0.28400D-08	0.22873D+02
4	66.0000000000	0.00000D+00	0.00000D+00
5	3.0000000000	0.00000D+00	0.00000D+00

THERE WILL BE NO FIT FOR PARAMETERS 4 THROUGH 5

RESULTS OF FIT

NO. (MAX) OF ITERATIONS	ERROR CODE	OPTIMIZED CHI-SQUARE
20(24)	0	0.903868

PARAMETER NO.	PARAMETER VALUE	+/- DP	GRADIENT
1	64.1107382029	0.15914D+00	-0.44941D-05
2	2.5726808455	0.15531D+00	-0.73537D-07
3	0.5847672986	0.36075D-01	-0.95264D-06

APPENDIX V

OUTPUT PAGE (2)

	INDEPENDENT	DEPENDENT	WEIGHT	OPTIMIZED DEPENDENT	FNC(P(OPT),X)-Y	CONTRIB. TO CHI-SQUARE
1	7.90000	-0.004365	0.1000D+05	0.000135	-0.44993D-02	0.723000D-02
2	9.00000	0.004321	0.1000D+05	0.004328	-0.66037D-05	0.155748D-07
3	10.20000	0.004321	0.1000D+05	-0.004278	0.85991D-02	0.264086D-01
4	11.20000	-0.009774	0.1000D+05	0.007169	-0.15943D-01	0.907734D-01
5	12.20000	-0.017729	0.1000D+05	-0.008920	-0.88088D-02	0.277122D-01
6	13.30000	-0.013228	0.1000D+05	0.004914	-0.18142D-01	0.117551D+00
7	14.40000	0.004321	0.1000D+05	0.007957	-0.36352D-02	0.471951D-02
8	15.50000	-0.009774	0.1000D+05	-0.019005	0.92312D-02	0.304339D-01
9	16.50000	-0.004365	0.1000D+05	-0.005964	0.15995D-02	0.913752D-03
10	17.60000	0.017033	0.1000D+05	0.026475	-0.94415D-02	0.318365D-01
11	18.70000	0.021189	0.1000D+05	0.018513	0.26763D-02	0.255802D-02
12	19.70000	-0.008774	0.1000D+05	-0.018921	0.10147D-01	0.367736D-01
13	20.90000	-0.040959	0.1000D+05	-0.051893	0.10934D-01	0.426975D-01
14	22.10000	-0.017729	0.1000D+05	-0.029139	0.10410D-01	0.387046D-01
15	23.20000	0.037426	0.1000D+05	0.026565	0.10861D-01	0.421302D-01
16	24.20000	0.075547	0.1000D+05	0.070915	0.46316D-02	0.766141D-02
17	25.30000	0.110590	0.1000D+05	0.099017	0.11573D-01	0.478320D-01
18	26.30000	0.096919	0.1000D+05	0.100060	-0.31500D-02	0.354396D-02
19	27.40000	0.086360	0.1000D+05	0.077266	0.90942D-02	0.295373D-01
20	28.50000	0.037426	0.1000D+05	0.032498	0.49283D-02	0.867417D-02
21	29.50000	-0.026872	0.1000D+05	-0.024255	-0.26174D-02	0.244666D-02
22	30.60000	-0.091515	0.1000D+05	-0.099817	0.83023D-02	0.246172D-01
23	31.60000	-0.173925	0.1000D+05	-0.179943	0.60177D-02	0.129331D-01
24	33.30000	-0.337242	0.1111D+04	-0.332139	-0.51034D-02	0.103352D-02
25	34.30000	-0.420216	0.1000D+05	-0.430804	0.10588D-01	0.400352D-01
26	34.90000	-0.505845	0.1000D+05	-0.491745	-0.14100D-01	0.710027D-01
27	35.80000	-0.602060	0.1000D+05	-0.586074	-0.15986D-01	0.912739D-01
28	37.60000	-0.779892	0.1000D+05	-0.785403	0.55108D-02	0.108462D-01
29	38.70000	-0.913640	0.2500D+04	-0.912160	-0.14799D-02	0.195539D-03
30	39.60000	-0.987163	0.1111D+04	-1.019910	0.32747D-01	0.425542D-01
31	41.20000	-1.200659	0.1111D+04	-1.215917	0.15258D-01	0.923797D-02

APPENDIX V

OUTPUT PAGE (3)

L	A(L)	L	A(L)	L	A(L)
13	0.23550E-08	14	0.34738E-08	15	0.51240E-08
16	0.75583E-08	17	0.11149E-07	18	0.16445E-07
19	0.24258E-07	20	0.35782E-07	21	0.52780E-07
22	0.77854E-07	23	0.11484E-06	24	0.16940E-06
25	0.24987E-06	26	0.36857E-06	27	0.54367E-06
28	0.80194E-06	29	0.11829E-05	30	0.17449E-05
31	0.25738E-05	32	0.37965E-05	33	0.56000E-05
34	0.82604E-05	35	0.12185E-04	36	0.17973E-04
37	0.26511E-04	38	0.39105E-04	39	0.57680E-04
40	0.85080E-04	41	0.12549E-03	42	0.18510E-03
43	0.27301E-03	44	0.40265E-03	45	0.59382E-03
46	0.87568E-03	47	0.12911E-02	48	0.19033E-02
49	0.28050E-02	50	0.41321E-02	51	0.60831E-02
52	0.89471E-02	53	0.13142E-01	54	0.19264E-01
55	0.28159E-01	56	0.40987E-01	57	0.59304E-01
58	0.85080E-01	59	0.12062	60	0.16829
61	0.22985	62	0.30567	63	0.39371
64	0.48924	65	0.58556	66	0.67576
67	0.75455	68	0.81932	69	0.86994
70	0.90797	71	0.93571	72	0.95549
73	0.96939	74	0.97904	75	0.98569
76	0.99026	77	0.99337	78	0.99550
79	0.99694	80	0.99793	81	0.99859
82	0.99905	83	0.99935	84	0.99956
85	0.99970	86	0.99980	87	0.99986
88	0.99991	89	0.99994	90	0.99996
91	0.99997	92	0.99998	93	0.99999
94	0.99999	95	0.99999	96	1.0000
97	1.0000	98	1.0000	99	1.0000
100	1.0000	101	1.0000	102	1.0000
103	1.0000	104	1.0000	105	1.0000
106	1.0000	107	1.0000	108	1.0000
109	1.0000	110	1.0000	111	1.0000
112	1.0000	113	1.0000		

APPENDIX V

OUTPUT PAGE (4)

ANGLE	RATIO OF DIFF. X-SECS	ANGLE	RATIO OF DIFF. X-SECS	ANGLE	RATIO OF DIFF. X-SECS
10.00	0.98857	11.00	1.0143	12.00	0.98604
13.00	0.99692	14.00	1.0307	15.00	0.97972
16.00	0.96210	17.00	1.0249	18.00	1.0692
19.00	1.0219	20.00	0.93030	21.00	0.88706
22.00	0.92885	23.00	1.0378	24.00	1.1570
25.00	1.2414	26.00	1.2654	27.00	1.2248
28.00	1.1365	29.00	1.0130	30.00	0.87731
31.00	0.73998	32.00	0.61089	33.00	0.49689
34.00	0.39744	35.00	0.31476	36.00	0.24679
37.00	0.19136	38.00	0.14764	39.00	0.11277
40.00	0.85461E-01	41.00	0.64447E-01	42.00	0.48026E-01
43.00	0.35572E-01	44.00	0.26144E-01	45.00	0.18957E-01
46.00	0.13676E-01	47.00	0.97505E-02	48.00	0.68590E-02
49.00	0.48056E-02				

TOTAL REACTION X-SEC = 226.32 F**2

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