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Summary

Recent results on intensity and transverse density limitations in the Fermilab 8-GeV Booster are presented. The evidence suggests that the limits are set by incoherent space-charge effects at low energy. Data are interpreted in terms of the space-charge tune shift and possible means of improving performance further are discussed.

Introduction and Overview

The Fermilab Booster is a rapid-cycling (15 Hz) alternating-gradient synchrotron with an average orbit radius of 75.47 meters. It accelerates protons from 204 MeV, the kinetic energy of the Linac beam, to 8 GeV, the normal injection kinetic energy of the Main Ring. The lattice consists of 96 combined-function magnets in 24 periods. The nominal horizontal and vertical tunes are 6.7 and 6.8. Further design features can be found in a comprehensive technical description of the Booster as built[1].

H^- charge-exchange injection is used; a review article which discusses the method and describes the Fermilab system has been published[2]. The H^- current from the Linac is typically 35 mA, or 6×10^{11} particles per Booster turn. Many turns can be injected cleanly into the same transverse phase space in the Booster, which allows the buildup of an intense circulating beam in the limited transverse acceptances of the Booster, starting with a modest Linac beam current. However the fraction of beam lost thereafter is roughly proportional to the number of protons injected, limiting the Booster output intensity to about 3×10^{12} protons per cycle. (The record is 3.55×10^{12} protons per cycle.) Most of the losses occur in the first few milliseconds.

In 1983, Ohnuma[3] reviewed the available information on the dependence of the emittances on energy and intensity, including 8-GeV emittances published in 1981 by Moore et al.[4] Second-order polynomial fits gave a good representation of the intensity dependence of the transverse emittances observed on the first turn in the Main Ring at 8 GeV:

$$\epsilon_H/\pi = 4.03 + 2.42\ N + 1.29\ N^2$$
$$\epsilon_V/\pi = 5.86 + 1.40\ N + 1.09\ N^2,$$

where the intensity N is in units of 10^{12}/cycle. Here and throughout this paper, normalized emittances containing 95% of the beam are quoted and units of mm-mrad are used. A Gaussian transverse distribution of rms width σ observed at a location where the dispersion is zero and the lattice amplitude function is β_L corresponds to a normalized 95% emittance ϵ_N given by

$$\epsilon_N/\pi = 6\beta\gamma\sigma^2/\beta_L \qquad (1)$$

where β and γ are the usual Lorentz factors. At low intensity, the fits agree within the errors with the measured Linac emittances of about 5 π mm-mrad. The fitted data covered intensities up to 2×10^{12}/cycle; at that intensity, the fitted values are 14π mm-mrad and 13π mm-mrad for the horizontal and vertical, respectively. The present paper reports new measurements of Booster emittances obtained by new methods, extended to higher intensities.

Mechanisms for Emittance Growth

The emittance growth due to Coulomb scattering on the 200 microgram/cm² carbon stripping foil can be estimated from multiple-scattering theory. The beam normally passes through the foil about a dozen times; the calculated result is an increase of about 1.2π mm-mrad in the horizontal emittance and 3.8π mm-mrad in the vertical, assuming no H-V coupling during the injection process. These increments are fairly small, but not negligible when compared with the starting values from the Linac of about 5π mm-mrad. Coherent transverse instabilities can also cause emittance growth. Active bunch-by-bunch damping controls coherent vertical motion of the centroid of each bunch.[5] Control of the chromaticities by sextupoles and introduction of Landau damping by octupoles help to suppress transverse instabilities.

Space-charge effects are thought to be the dominant mechanism of intensity-dependent emittance growth. At low energy in the Booster, the incoherent tune shifts caused by the direct forces between beam particles are larger by far than the coherent "wall terms" due to the interaction of the beam with its local environment. Let us then ignore the wall terms and concentrate on the dominant direct term in the subsequent discussion. If one assumes that the distribution of protons in a bunch is Gaussian in all three coordinates and cylindrically symmetric about the beam direction, i.e. "round", and that the bunch length is large compared to the transverse dimensions, then the direct term can be written[6]

$$\Delta\nu = 3r_pN_T/2B\epsilon_N\beta\gamma^2 \qquad (2)$$

where: r_p is the classical radius of the proton, N_T is the total number of protons in the Booster, B is the bunching factor, and β, γ^2 are the kinematic factors.

The tune shift as written applies to the dense core of the beam; particles having large betatron amplitudes and those displaced longitudinally far from the bunch center experience considerably smaller shifts. Thus the tune shift at the bunch center also gives a good approximation to the tune spread within the beam. The implications of large horizontal and vertical tune spreads can be understood qualitatively as follows. First of all, the entire distribution of particle tunes should be kept within the region bounded by strong resonances; in particular, the strong half-integer and integer lines certainly ought to be avoided, which limits the allowable tune spread to something less than a half-unit. The nonlinear space-charge forces within the beam may excite resonances as well, although the harmonic content of the resulting driving terms probably includes predominantly only multiples of the lattice periodicity. Furthermore, the tunes of protons undergoing synchrotron oscillations are modulated as

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they oscillate from the dense center to the rarified ends of a bunch. The tune spread and modulation thus result in repeated crossing of higher-order resonance lines, which can lead to emittance growth [7]. The low-energy behavior of the Booster can be explained by assuming that these nonlinear space-charge effects cause the transverse emittances to grow until the Laslett tune-shift parameter is reduced to a tolerable value. As the intensity is raised, a greater proportion of the distribution of particle emittances approaches the limited aperture of the Booster and losses increase. The normal two or three millisecond duration of the losses coincides with the period when the bunching factor in the tune shift formula is changing faster than the kinematic factor $\beta\gamma^2$. In fact, the Booster intensity has been observed to decay at just the rate which keeps the calculated tune shift constant until the kinematic factor "takes over" from the bunching factor a few milliseconds after injection, after which losses are small.

Data and Discussion

New data on the dependence of transverse beam sizes on the Booster beam intensity have been collected for both the horizontal and vertical dimensions. The data presented below are based on measurements of vertical and horizontal beam profiles using different techniques. Vertical profiles are measured using the vertical flying wire profile monitor in the Booster. Measurements of this type can be taken at any time during the Booster acceleration cycle and lead to a direct measurement of the transverse emittance. Since we presently have only a single horizontal flying wire in the Booster it is not possible to separate directly the contributions to the horizontal beam size from transverse and longitudinal emittance using this technique. Thus, horizontal emittance measurements are based on an analysis of multiwire profile monitors in the 8 GeV transfer line between the Booster and Main Ring.

In Figure 1 we show the measured vertical transverse emittance just prior to extraction from the Booster for intensitites in the range 0.5-3.0×10^{12} protons. These data were taken during the August 1986 accelerator startup period at Fermilab. The data show the emittance delivered from the Booster to be about 7π mm-mr for beam intensities up to about 1.2×10^{12}, and that the emittance rises close to linearly with beam intensity above this level. The level observed at low intensities is the emittance delivered from the Linac. The dashed line on the figure is a contour of constant phase space density, $\epsilon_v/N_p = 5.7\pi/10^{12}$. At the same time the data shown in the figure were taken we looked at the evolution of the vertical transverse emittance through the Booster acceleration cycle. To within the resolution of the measurement (about 2 msec) it was found that the blowup observed at higher intensities occurs at injection time.

The analysis of the horizontal data is shown in Figures 2 and 3. These data were taken in December of 1986 during a period of restricted access to study time in the Booster. As a result the data do not extend to as high an intensity as the vertical data. The data show the horizontal transverse emittance to be about 11π mm-mr independent of intensity up to 2.4×10^{12}. The transverse emittance is larger than that delivered from the Linac due to a known horizontal mismatch at the end of the Linac to Booster transfer line.

The vertical emittance data shown in Figure 1 suggest that the transverse phase space density

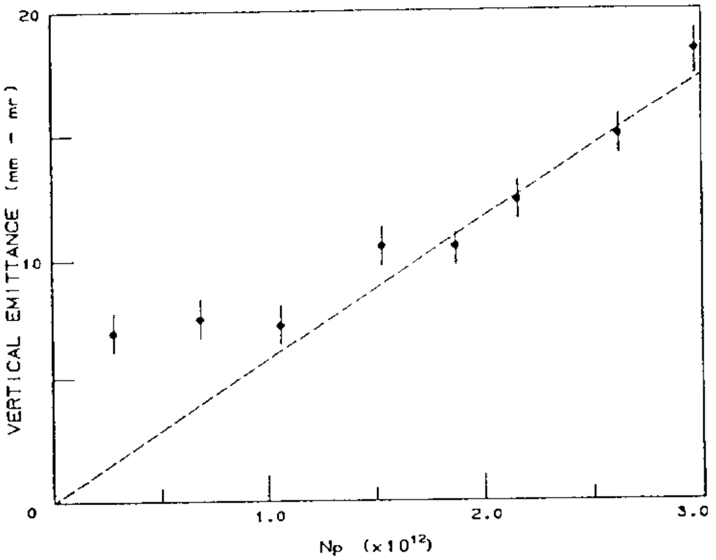

Figure 1. Measured vertical transverse emittance in the Booster as a function of delivered intensity. The dashed curve is described in the text.

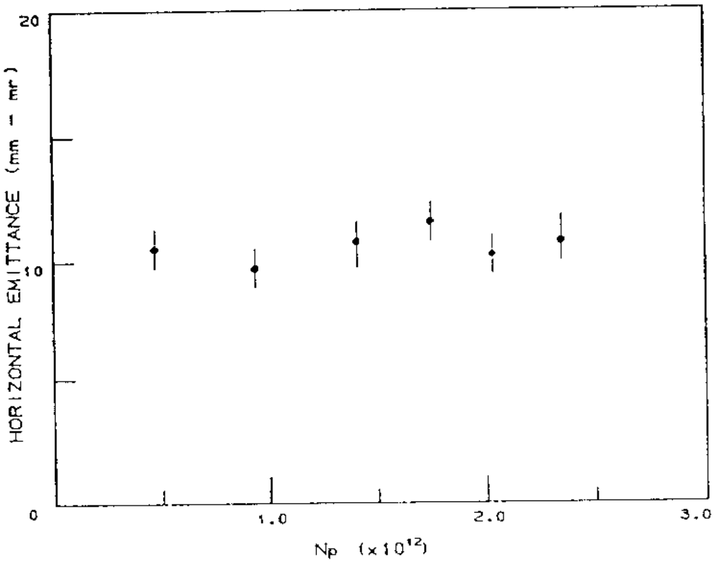

Figure 2. The measured horizontal transverse emittance delivered from the Booster as a function of intensity.

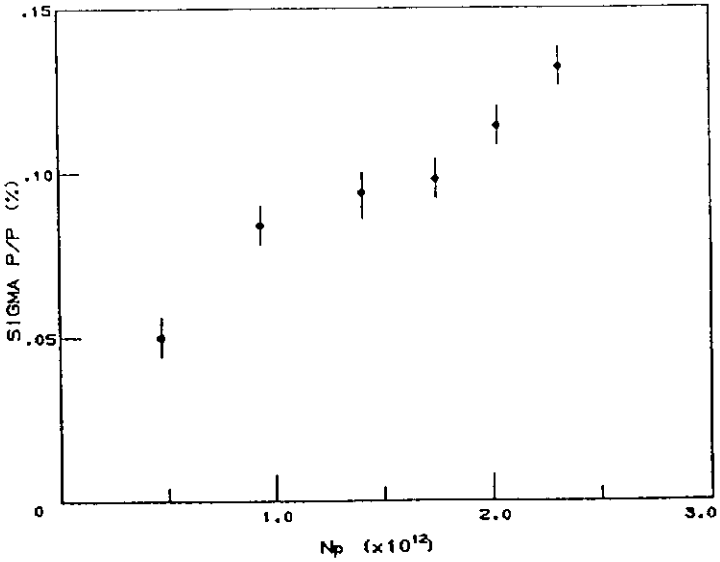

Figure 3. Measured momentum spread delivered from the Booster as function of intensity.

delivered from the Booster is limited by the incoherent space charge tune spread at injection. This tune spread arises from the electromagnetic self-interaction of the beam and is given by the equation (2). The dashed line shown in Figure 1 is the contour of constant space charge tune spread, $\Delta\nu=0.38$ (calculated at injection). The observation that the emittance blowup seems to occur at the earliest stages of the acceleration cycle is consistent with this interpretation. Although the horizontal emittance measurements do not confirm this interpretation, they are at least consistent with it. The horizontal beam size in the Booster is naturally larger than the vertical due to both the dilution of the beam as it comes into the machine and due to the momentum spread of the injected beam. These effects delay the onset of an increase of emittance with intensity to higher intensities than in the vertical case. Unhappily, we find that the contour, $\Delta\nu=0.38$, starts to rise above the observed constant horizontal emittance at an intensity of about 2.4×10^{12}--just where our present measurements end. Confirmation of a rise in the horizontal emittance consistent with that observed vertically awaits more measurements at higher intensities and/or an improvement in the Booster horizontal injection matching. We hope these will both be done in the near future.

Methods for Improvement

The Laslett tune-shift formula implies various ways to improve the performance of a machine limited by space-charge tune shift effects. An obvious way to achieve a significant reduction in the incoherent tune spread is to increase the injection energy. An upgrade of the Fermilab Linac to 400 MeV is currently under active consideration; this would increase $\beta\gamma^2$, which appears in the denominator of the tune shift, by a factor of 1.72.

Other measures might apply to the existing Booster without significant upgrades. The strengths of resonances can be reduced by means of correction magnets of the appropriate multipolarity and harmonic distribution, thereby enlarging the allowed region of tune space and making large tune shifts more tolerable. In the Booster, DC dipoles are routinely used to correct the closed orbit, thereby reducing the effect of the integer lines at low energy. Centering the orbit in the good-field region of the lattice magnets should also help to reduce the strengths of higher-order resonances. After the dipole steering corrections, the most important improvement in normal operation is usually achieved by careful and largely empirical tuning of the correction quadrupole ramps which control the machine tunes; the optimal curves are very much intensity-dependent, which suggests the importance of intensity-related tune shifts and spreads. These ramped quadrupoles also have individually controlled DC offsets which are used to reduce the strength of the half-integral resonance driving terms at injection. Recently, a project to correct the third-order lines with harmonic sextupoles has been initiated.

According to equation (2), increasing the transverse emittance reduces the tune shift proportionally. However, the small transverse acceptances of the Booster and of the Main Ring severely limit the emittances that can be transmitted. Whether increasing the transverse emittances is advisable depends also on the intended use of the beam: for those protons destined to strike fixed targets, larger transverse emittances have no serious detrimental effect provided the acceptances of downstream machines are adequate, but for those protons destined to collide with antiprotons, larger emittances result in lower luminosity. Besides increasing the transverse emittances, the transverse distribution can also be modified in principle; any reduction of the central density, where the tune shift is largest, ought to pay dividends. No systematic procedure to tailor the transverse distributions has been developed; in practice, tuning to minimize the initial transverse sizes usually results in the best transmission.

Finally, the bunching factor can be improved; any reduction of the peak bunch current will reduce the tune shift. The bunch can be lengthened by increasing the longitudinal emittance; just as in the transverse case, the feasibility and advisability of this measure depends on the longitudinal acceptance, that is, on the amount of RF voltage available and the momentum acceptance of the ring, and on the intended use of the beam. The bunch can also be lengthened without increasing the emittance, by carefully tailoring the amount of accelerating voltage as a function of time to the longitudinal emittance or by modifying the distribution of particles within the longitudinal phase space. Some recent performance improvements have resulted from efforts in this direction [8].

Acknowledgments

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