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George Thompson

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Ruibin Zhang

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A GAUGE PRINCIPLE YIELDING CONSISTENT CHIRAL THEORIES *

George Thompson and Ruibin Zhang
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We propose a new principle in gauge theories: namely that in a given action, fields should be replaced by gauge invariant equivalents. Using this principle we study anomalous gauge theories and find that the resulting models are anomaly free, unitary and power counting renormalizable.

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Some aspects of gauge theories have, for a long time, been problematic. Among others, the problems of massive Yang-Mills theories without Higgs mechanism, the physical interpretation of (gauge dependent) Green's functions and anomalies in chiral theories have extended the minds of field theorists. We have singled out these three problems as they are different manifistations of the same problem; they bare the same hall-mark, a lack of gauge invariance. Recently a gauge invariant massive Yang-Mills theory was advanced, where the mass term is rendered gauge invariant by employing a gauge invariant vector potential[1]. After this a general method of creating gauge invariant Green's functions from their more usual counterparts was developed[2]. By using the gauge invariant fields it is possible to establish an equivalence between Green's functions and gauge invariant vacuum expectation values in particular gauges[2].

Here we will be concerned with the third subject, that is, chiral anomalous gauge theories. Because of the anomaly these models lack unitarity. However if one employs gauge invariant fields at the outset, then (as we will establish subsequently) the theory is manifestly gauge invariant and anomaly free and unitarity is assured. This success of the procedure of substituting in gauge invariant fields for the conventional fields prompts us to propose the following gauge invariance principle (to be understood in the context of gauge theories as described above): Given an action $S(\Sigma)$, the correct gauge theory to consider is $S(\Sigma^{\#})$, where $\Sigma^{\#}$ is a gauge invariant field associated with Σ.

In the following it is actually simpler to just replace the vector potential and leave the matter fields untouched.

For the sake of concreteness let us concider chiral electrodynamics. The conventional Lagrangian for this theory is,

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}\left(i\partial\!\!\!/ - e\frac{1+\gamma_5}{2}A\!\!\!/\right)\psi \tag{1}$$

with the classical invariance

$$\delta A_\mu = \partial_\mu \Lambda$$
$$\delta\psi = -i\frac{1-\gamma_5}{2}e\Lambda\psi$$
$$\delta\bar{\psi} = i\bar{\psi}\frac{1+\gamma_5}{2}e\Lambda$$

which is spoiled at the quantum level by the anomaly.On using the gauge invariance principle,we instead consider the theory

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}\left[i\partial\!\!\!/ - e\frac{1+\gamma_5}{2}A\!\!\!/^{g}\right]\psi \tag{2}$$

where $A^g_\mu = A_\mu - \partial_\mu\frac{1}{\Box}\partial\cdot A$ [2].(2) posseses the following invariance

$$\delta A_\mu = \partial_\mu\Lambda\,,\quad \delta\psi = \delta\bar{\psi} = \delta A^g_\mu = 0\,,$$

which is clearly preserved at the quantum level.In the parlance of the Fujikawa approach to anomalies,there is no anomaly in the present case because the fermion fields are not transformed [3].Of course the real check of this approach is to determine wether (2) is unitary or not. To do this we express the action in local form (plus gauge fixing and Faddeev-Popov ghosts),

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}\left[i\partial\!\!\!/ - e\left(\frac{1+\gamma_5}{2}\right)(A\!\!\!/ - \partial\!\!\!/\phi)\right]\psi + D\cdot(\Box\phi - \partial\cdot A) + B\,\partial\cdot A + \bar{\omega}\Box\omega. \tag{3}$$

(3) is invariant under the following BRST transformations

$$\delta A_\mu = \epsilon\,\partial_\mu\omega$$
$$\delta\psi = \delta\bar{\psi} = \delta B = \delta D = 0$$
$$\delta\omega = 0\,,\quad \delta\bar{\omega} = -\epsilon B$$

$$\delta\phi = \epsilon\omega \tag{4}$$

It is straightforward to derive the following equations of motion from (3),

$$\partial^\mu F_{\mu\nu} + J_{\nu L} - \partial_\nu B + \partial_\nu D = 0 \tag{5.a}$$
$$\Box\omega = \Box\bar{\omega} = 0 \tag{5.b}$$
$$\Box D + \partial^\mu J_{\mu L} = 0 \tag{5.c}$$
$$\Box\phi = \partial\cdot A = 0 \tag{5.d}$$
$$\Box B = 0 \tag{5.e}$$

where

$$J_{\nu L} = -\frac{1}{2}e\,\bar{\psi}(1+\gamma_5)\gamma_\mu\psi.$$

With the use of equation (5a),the BRST current

$$J^{BRST}_\nu = \partial^\mu\omega\cdot F_{\mu\nu} - \omega J_{\nu L} + B\cdot\partial_\nu\omega - \omega\cdot\partial_\nu D$$

can be greatly simplified;and one arrives at,up to a total divergence

$$J^{BRST}_\mu = B\,\partial_\mu\omega - \omega\,\partial_\mu B. \tag{6}$$

It is obvious that the BRST current is conserved.

$$\partial^\mu J^{BRST}_\mu = B\Box\omega - \omega\Box B = 0\,,$$

hence the BRST charge satisfies

$$\dot{Q}_{BRST} = 0. \tag{7}$$

Canonical quantization of the theory can be carried out in the ordinary way,however,we will not do this explicitly here.We only want to say that rewriting Q_{BRST} in terms of independant fields and using the canonical commutation relations,one can easily prove that Q_{BRST} does generate the BRST transformations (4) and satisfy the condition

$$Q_{BRST}^2 = 0, \tag{8}$$

thus establishing unitarity.

To get a feel of what (2) corresponds to with regards to (1), we may rewrite it as (now in four dimensions, previously there was no such restriction)

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}\left[i\partial\!\!\!/ - e\frac{1+\gamma_5}{2}A\!\!\!/\right]\psi - a\frac{\partial\cdot A}{\Box}F\tilde{F} \tag{9}$$

where a is the coefficient of the anomaly in the theory (1). The extra term exactly compensates the anomaly that arises from the transformation of the fermionic fields. (9) is in form very similar to the model advocated in [4], except that there is no new field involved. It appears reasonable that in the gauge $\partial\cdot A = 0$ that the nonlocal terms vanish, that the anomaly is consistently eliminated and therefore the model (2) is renormalizable, though we have not checked this supposition in great detail. (Indeed the high momentum behaviour of the boson propagator at the bare level is good in all covariant gauges).

Two dimensional chiral theories, for example, the chiral Schwinger model [5], have also been recieving some attention. Here we simply quote some results concerning these theories in terms of gauge invariant fields. The general action (fermionic) considered is

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - e\frac{1+\gamma_5}{2\sqrt{\pi}}A\!\!\!/^{g}\right]\psi. \tag{10}$$

Any b leads to the same effective theory

$$\mathcal{L} = \frac{ae^2}{2}A_\mu\left(g^{\mu\nu} - \frac{1}{\Box}\partial^\mu\partial^\nu\right)A_\nu$$

with a the undetermined parameter in defining the fermionic determinant. So for a > 0 and all b these models are equivalent to the Schwinger model.

Let us now turn our attention to non-abelian chiral theories. In [2] the gauge invariant fields for non-chiral fields have been given. Here we present the relevant construction for chiral fields. The starting point is

$$\mathcal{L} = \bar{\psi}_L iD\!\!\!\!/_L \psi_L \qquad D\!\!\!\!/_L = \partial\!\!\!/ - igA\!\!\!/_L \tag{11}$$

with ψ_L a left-handed spinor. The gauge invariant construction of the theory is then

$$\mathcal{L} = \bar{\psi}_L\left(i\partial\!\!\!/ - U_L^{-1}[gA\!\!\!/_L - i\partial\!\!\!/]U_L\right)\psi_L \tag{12}$$

with U_L determined by

$$\left[D_L^\mu, \left(A_\mu\cdot T - iU_L^{-1}\partial_\mu U_L/g\right)\right] = 0$$

and the fermions are blind to the gauge transformations. As in the abelian case this is gauge invariant. It has not been possible to write (12) in a form equivalent to (9) in four dimensions. The best we have been able to do is to write (12) as a conventional theory and a Wess-Zumino type term as a five dimensional integral. Perhaps two dimensions will prove more forthcoming. These and related topics will be discussed in more detail elsewhere.

In conclusion we would like to reiterate that by applying the gauge principle advocated in the introduction, it is possible to eliminate the anomalies in chiral theories consistently and the resulting theory is unitary and power counting renormalizable(this is true at least in the abelian case).

After completing this work references [6,7] were brought to our attention.The final results of [6] coincide with [4],except for that now the extra field arises naturaly from the group volume.Their final action is identical to our equation (3) but without the constraint on ϕ imposed by the field D.Clearly the combination $A_\mu - \partial_\mu \phi$ represents a gauge invariant vector potential ($\delta A_\mu = \partial_\mu \Lambda$, $\delta\phi = \Lambda$).Hence just as in massive vector theories there are two possible ways to construct a gauge invariant thoery:either as a true Stueckelberg theory,with the inclusion of an extra (possibly) dynamical degree of freedom,or as a mimic of the Stueckelberg model where the gauge group volume field ϕ is determined by a constraint (the equation of motion [1]).The theory proposed in [7] is a generalization of [4,6] to include a mass term for the vector potential (again in Stueckelberg form).It is claimed that the mass arises " on quantization ".It is hard to accept this,as the theory without the mass term has all the properties required in [7],namely a conserved and nilpotent BRST charge.

A totaly different approach to dealing with anomalies in Weyl theories may be found in [8].

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