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GRAND UNIFIED THEORIES IN COSET SPACE DIMENSIONAL REDUCTION *

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ABSTRACT

We examine particle content of the effective four-dimensional GUT's arising in the coset space dimensional reduction of 10-dimensional E_8 supersymmetric Yang-Mills theory.

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1. INTRODUCTION

Considerable efforts has been devoted to derive physically interesting four-dimensional models starting from theories defined in multidimensional space-time. There is a lot of attractive ways for dimensional reduction procedure. For example, it is possible to avoid some problems of pure gravitational theory by introduction of Yang-Mills fields with gauge group G coupled to fermions. This will bring certain arbitrariness, but on the other hand, it is possible to find a natural role for the new fields in string theories . Such kind of arguments force us to choose E_8 supersymmetric theory in ten dimensions as a fundamental theory of particle interactions. An important step in the investigation of such a composite Einstein-Yang-Mills theory is to determine the effective theory emerging from the Yang-Mills sector. An appropriate formalism, namely Coset Space Dimensional Reduction (CSDR) has been developed by Manton and his collaborators [1-3] .

Even in this frameworks we have still a large degree of freedom. Namely, one may try to obtain only the Weinberg-Salam model [4,5] or to get [2,5,6] Grand Unified Theories after the geometrical symmetry breaking. One may also require the four-dimensional theory to have gauge group SU(3) x SU(2) x U(1), with the Higgs sector which breaks the symmetry down to SU(3) x U(1) in the usual way. In this paper we will examine various GUT's which could arise after CSDR if the gauge and fermion fields in ten dimensions are placed in the lowest 248 representation of E_8 group.

2.DESCRIPTION OF THE CSDR SCHEME.

Let us briefly recall the coset space dimensional reduction procedure [1,3] . One starts with a pure gauge field theory with gauge group G coupled to fermions in 4+N dimensional space-time M which is assumed to be the direct product of Minkowski space and a compact coset space S/R. S is a compact Lie group and R some fixed Lie subgroup of S. For the coordinates on this space-time we write $z^M = (x^i, y^\alpha)$, where x^i is the Minkowski coordinates and y^α label extra dimensions. We would like to get the lagrangian which is independent of the extra coordinates. This can be achieved by demanding that the dependence of the fields on the extra coordinates be a gauge transformation. Thus the lagrangian, being gauge invariant, is independent of the extra coordinates and when it is inserted in the action on M, one may integrate over the coset space coordinates, and obtain a Yang-Mills-Higgs theory in four-dimensional Minkowski space.

S acts naturally as a symmetry group on S/R, and hence on M, via (right) multiplication producing a mapping of M onto itself $y^\alpha \to \bar{y}^\alpha$. One can restrict attention to an infinitesimal mapping defined by the vector fields : $\bar{y}^\alpha = y^\alpha + \varepsilon \xi^\alpha$. To every generator of S there corresponds one field ξ^α_m, so ξ^α_m represent Lie algebra of S. Vector field B_M which is independent on the extra coordinats possesses $L_\xi B_M = 0$, where L_ξ is the Lie derivative with respect to ξ^α_m . For the S symmetric gauge field one have instead $L_\xi A_M = D_M W_m$ where $D_M W_m$ is infinitesimal gauge transformations $A^{g}_M(z) = A_M(z) + \varepsilon D_M W$ with some $W = W^a(z) T^a$ in the Lie algebra of G. The complete solution of this equation can be expressed, in terms of arbitrary fields $A^a_i(x)$ and $\varphi^a_m(x)$, as

$$
\begin{aligned}
A^a_i &= A^a_i(x) \,, \\
A^a_\alpha &= \varphi^a_m(x)\, \xi^m_\alpha(y) \,,
\end{aligned}
\tag{1}
$$

where A, φ should obey the constraints [1]

$$\partial_i \varphi_n = [A_i, \varphi_n] \,, \tag{2a}$$

$$f_{mnp} \varphi_p + [\varphi_m, \varphi_n] = 0, \tag{2b}$$

$$\forall m, \forall n, \forall p = 1, \ldots, \dim R,$$

$\forall i$,

where f_{mnp} are the structure constants of $R \subset S$. Because of the constraints (2a) , the fields φ_n would have no kinetic term and one may fix them to be constants, and from (2b) we see that φ_n generate an R subalgebra of the gauge group algebra. That is R had to be a subgroup of G, the original gauge group, for such solutions to exist . Corresponding generators in G we shell denote by T^m, m = 1, ..., dim R. Further, in view of (2a), that subgroup H of G ,all of whose elements commute with all elements of R (namely the centralizer of R in G) was the resultant gauge symmetry after the dimensional reduction

The second constraint can be rewritten as

$$f_{mnp} \varphi^a_p + g_{abc} \varphi^b_m \varphi^c_n = 0, \tag{3}$$

$$\forall m, \forall n \le \dim R,$$

where g_{abc} are structure constants of G. Adjoint representation of S (ad S), decomposes into irreducible representations of R according to the branching rule

$$ad\, S = ad\, R + \sum_i \underline{n}_i \;. \qquad (4)$$

Similarly, the branching rule for ad G into irreps of R_G x H (where R_G is the embedding of R into G) is

$$ad\, G = \sum_j (\underline{n}_j' \otimes \underline{m}_j) \;. \qquad (5)$$

Applying now Schur's lemma to the Eq.(3) one obtains that for each pair $(\underline{n}_i, \underline{n}_j')$ where $\underline{n}_i$, and $\underline{n}_j'$ are identical irreps, there is an m_j multiplet of Higgs fields in the four-dimensional theory.

The Higgs potential is determined by the primary gauge interaction and does not contain any uncertainty but the explicit minimization of the potential requires hard work. However, in the case when $S \supset R$ has an isomorphic image in G, the gauge group K after Higgs symmetry breaking precisely coincides with the centralizer of S in G [7]. Moreover, V is zero in this case, which is the minimum possible. Such mechanism of symmetry breaking naturally avoids the problem of non-zero cosmological constant, at least classically.

Four-dimensional fermion multiplets can be identified in the similar way [3]. One first notes that R is naturally embedded into the SO(N) of S/R in such a way that the N of SO(N) has the

branching rule

$$\underline{N} = \sum_i \underline{n}_i \;, \qquad (6)$$

where $\underline{n}_i$ are the same irreps as occur in (4). This follows from the way R acts on tangent vectors at the point which R leave fixed. In this way the embedding of R into SO(N) is uniquely determined. Next one takes the spinor $\underline{S}$ of SO(N) and decomposes it into irreps of R

$$\underline{S} = \sum_i \underline{S}_i \;. \qquad (7)$$

Then one has to decompose the representation $\underline{F}$ of the gauge group G to which the fermions are assigned under R_G x H

$$\underline{F} = \sum_j (\underline{s}_j', \underline{h}_j) \;. \qquad (8)$$

It turns out that for each pair of $(\underline{s}_i, \underline{s}_j')$ where $\underline{s}_i$ and $\underline{s}_j'$ are equivalent irreps, there is an $\underline{h}_j$ multiplet of spinor fields in the four-dimensional theory. It is possible to obtain parity violation in the dimensionally reduced theory provided one starts with Weyl spinors and rank S = rank R.

In principle, exhaustive search should be made for a groups G,S and R , where the centralizer of R in G contain SU(3) x U(1). But in general, there is too many ways for searching. For the reasons outlined in the introduction we choose G = E_8 and for the coset space we assume that dim (S/R) = 6 and rank S = rank R. All such six-dimensional coset spaces with S being simple are given in table 1 [5]. After the groups G, S,R are fixed one may still obtain different H depending on the embeddings of R into G and S. We shall consider successively the geometrical symmetry breaking resulting in E_6 , SO(10) and SU(5) GUT's.

3. EXAMPLES OF UNIFIED THEORIES

3.1. E_6 GUT's in 4-dimensions

3.1.1. $S/R = G_2/SU(3)$ and $H = E_6$ (see also [5,8]).

The R is going to be identified with the one appearing in the decomposition $E_8 \supset SU(3) \times E_6$

$$\underline{248} = (\underline{8},\underline{1}) + (\underline{1},\underline{78}) + (\underline{3},\underline{27}) + (\underline{3}^*,\underline{27}^*) \quad .$$

Using Table 1 one find that the geometrical Higgs are assigned to $\underline{27} + \underline{27}^*$ and left fermions lie in $\underline{27} + \underline{78}$. This example possesses simple supersymmetry in both 10 and 4 dimensions. The group $G_2 = S$ has an isomorphic image within E_8 . The centralizer of S is $K = F_4$ .Thus, the E_6 theory is broken by the Higgs field to a vectorlike F_4 theory.

3.1.2. $S/R = Sp(4)/SU(2) \times U(1)$ and $H = E_6 \times U(1)$ (see also[8]).

The adjoint E_8 has $SU(2) \times U(1) \times E_6$ branching rule

$$\underline{248} = (\underline{1},\underline{27})(-2) + (\underline{2},\underline{27})(1) + (\underline{2},\underline{1})(3) + \text{h.c.} + (\underline{3},\underline{1})(0) + (\underline{1},\underline{78})(0) + (\underline{1},\underline{1})(0) \quad .$$

The Higgs fields are $\underline{27}(-2) + \underline{27}(1)$ + h.c. and f_L is $\underline{27}^*(2) + \underline{78}(0) + \underline{27}^*(-1) + \underline{1}(0)$. Explicit calculations of Higgs potential show [8] that E_6 can be broken down to F_4 or SO(10) but in the former case the minima of the potential have the lower value.

3.2. SO(10) GUT's in 4-dimensions.

3.2.1. $S/R = SO(7)/SO(6)$ and $H = SO(10)$ (see also [5]).

The decomposition of the adjoint E under $SO(6) \times SO(10)$ is

$$\underline{248} = (\underline{15},\underline{1}) + (\underline{1},\underline{45}) + (\underline{6},\underline{10}) + (\underline{4},\underline{16}) + (\underline{4}^*,\underline{16}^*) \quad .$$

Using table 1 one may easily find that there will be one Higgs field $\underline{10}$ and one $\underline{16}$-plet of fermions. The Higgs field break SO(10) down to SO(9), but one may think [5] that this Higgs is responsible for the electroweak breaking (then the inverse radius of coset space should be of order M_W) leaving SO(10) breaking on the GUT scales for the other mechanisms (such as radiative corrections a la Coleman-Weinberg [5]).

3.2.2. $S/R = SU(4)/SU(3) \times U(1)$ (or $Sp(4)/SU(2) \times U(1)$) with $H = SO(10) \times U(1)$.

For the first coset space we identify SU(3) with the same factor in decomposition $E_6 \times SU(3) \subset E_8$ and U(1) as that in SO(10) x

$U(1) \subset E_6$. The branching rules for the adjoint are ($E_8 \supset SU(3) \times U(1) \times SO(10)$)

$$\underline{248} = (\underline{1},\underline{16})(-3) + (\underline{3},\underline{1})(4) + (\underline{3},\underline{10})(-2) + (\underline{3},\underline{16}^*)(1)$$

$$+ \text{h.c} + (\underline{8},\underline{1})(0) + (\underline{1},\underline{1})(0) + (\underline{1},\underline{45})(0).$$

We have two Higgs multiplets $\underline{10}(-2) + \underline{10}(2)$ and one fermion multiplet $\underline{16}^*(3)$. One may obtain almost the same decompositions if one embeds SU(2) of the second coset space in SU(3) in such a way that the branching rules for $SU(2) \subset SU(3)$ are $\underline{3} = \underline{3}$, $\underline{8} = \underline{3} + \underline{5}$. The only difference will be that one obtains one additional fermion multiplet in $\underline{16}(-1)$ due to the fact that $\underline{3} = \underline{3}^*$ for the SU(2). ($SU(2) \times U(1)$ embedding into S correspond to the line 3 of the Table 1, embedding of the line 4 do not give Higgses and useful fermions).

3.2.3. $S/R = Sp(4)/SU(2) \times U(1)$ with $H = SO(10) \times U(1) \times U(1)$.

Now we will proceed as in the case 3.2.2 with the main difference that we embedd now: $SU(2) \times U(1) \subset SU(3)$. This gives

$$\underline{248} = (\underline{2},\underline{1})(3,0) + (\underline{1},\underline{16})(0,-3) + (\underline{2},\underline{1})(1,4) +$$

$$(\underline{1},\underline{1})(-2,4) + (\underline{2},\underline{10})(1,-2) + (\underline{1},\underline{10})(-2,-2)$$

$$+ (\underline{2},\underline{16}^*)(1,1) + (\underline{1},\underline{16}^*)(-2,1) + \text{h.c.} +$$

$$(\underline{3},\underline{1})(0,0) + (\underline{1},\underline{1})(0,0) + (\underline{1},\underline{1})(0,0) + (\underline{1},\underline{45})(0,0).$$

Content of the four-dimensional theory crucially depends upon embedding $U(1) \subset R$ into $U(1) \times U(1) \subset SU(2) \times U(1) \times U(1) \times SO(10)$. One may obtain scalar and fermion fields in different

combinations from 1,10,16. We find that the interesting theory arise when U(1) is identified with the second U(1) factor(identifying with the first one bring us exactly to the case 3.1.2). We have for the Higgses $\underline{10}(2,2) + \underline{16}^*(1,1) +$ h.c. and $f_L = \underline{10}(2,2) + \underline{16}(-1,-1)$ (embedding $R \subset S$ correspond to the line 4 of the Table 1).

3.3. SU(5) Grand Unified Models

3.3.1. $S/R = SO(7)/SO(6)$ with $H = SU(5) \times U(1)$

Identifying SU(5) with the same factor which appear in the decomposition $SU(5) \times SU(5) \subset E_8$ and then taking the $SO(6) \times U(1)$ maximal subalgebra of the another SU(5) factor we obtain the following branching rules for $SO(6) \times SU(5) \times U(1) \subset E_8$

$$\underline{248} = (\underline{4},\underline{1})(-5)+(\underline{1},\underline{10})(4)+(\underline{4},\underline{10})(-1)+(\underline{4},\underline{5})(-3)+$$

$$(\underline{6},\underline{5})(-2)+ \text{h.c.}+(\underline{15},\underline{1})(0)+(\underline{1},\underline{24})(0)+(\underline{1},\underline{1})(0).$$

We have $\underline{5}^*(2) + \underline{5}(-2)$ for the Higgs fields and the fermion sector coincides with the phenomenologically acceptable one $f_L = \underline{5}^*(3) + \underline{10}(-1) + \underline{1}(-5)$.

3.3.2. $S/R = Sp(4)/SU(2) \times U(1)$ with $H = SU(3) \times SU(5) \times U(1)$

Using the $U(1) \times SU(2) \times SU(3)$ subalgebra of SU(5) we have for $SU(2) \times SU(3) \times SU(5) \times U(1) \subset E_8$:

$$\underline{248} = (\underline{1},\underline{1},\underline{5})(6)+(\underline{1},\underline{3}^*,\underline{5})(-4)+(\underline{2},\underline{3},\underline{5})(1)+(\underline{2},\underline{1},\underline{10}^*)(3)+$$

$$(\underline{1},\underline{3},\underline{10}^*)(-2)+(\underline{2},\underline{3},\underline{1})(-5)+\text{h.c.}+(\underline{1},\underline{1},\underline{1})(0)+$$

$$(\underline{3},\underline{1},\underline{1})(0) + (\underline{1},\underline{8},\underline{1})(0) + (\underline{1},\underline{1},\underline{24})(0) .$$

Interesting content in four dimensions gives the line 4 of the Table 1. We obtain the fermion sector of the minimal SU(5) model

$f_L = (\underline{3}^*,\underline{10})(2) + (\underline{3}^*,\underline{5}^*)(-1)$ and for the Higgs particles one gets $(\underline{3}^*,\underline{10})(2) + (\underline{3}^*,\underline{5}^*)(-1)$ + h.c. We would obtain three fermion generations if it would be possible to identify SU(3) with the flavour symmetry with subsequent appropriate symmetry breaking.

3.3.3. S/R = SU(4)/SU(3)xU(1) with H = SU(5)xU(1)xU(1)

We have for $SU(3)\times SU(5)\times U(1)\times U(1) \subset E_8$:

$$\underline{248} = (\underline{3},\underline{1})(0,-4)+(\underline{1},\underline{10}^*)(4,0)+(\underline{3},\underline{10})(1,-1)+(\underline{1},\underline{10})(1,3)+$$
$$(\underline{3}^*,\underline{5})(-2,-2)+(\underline{3},\underline{5})(-2,2)+(\underline{3}^*,\underline{1})(-5,1)+(\underline{1},\underline{1})(-5,-3)+$$
$$\text{h.c.}+(\underline{8},\underline{1})(0,0)+(\underline{1},\underline{24})(0,0)+(\underline{1},\underline{1})(0,0)+(\underline{1},\underline{1})(0,0) \ .$$

As in the case 3.2.3 there is different embeddings of $U(1) \subset R$ in $U(1)\times U(1) \subset G$ here as well. If we choose embedding $Y = (3/8)Y_1 + (7/8)Y_2$, where Y is the generator of $U(1) \subset R$ and Y_1 and Y_2 are generators of $U(1)\times U(1) \subset E_8$ then we get fermions in $\underline{1}(5,-1) + \underline{5}^*(2,-2) + \underline{10}(1,3)$ and no one scalar field.

The results of this chapter are summarised in Table 2.

4. CONCLUSIONS

In this paper we have analysed particle content obtained after coset space dimensional reduction of pure Yang-Mills E_8 theory in 10 dimensions searching for the standard grand unified models after geometrical symmetry breaking. We find that fermions with correct SU(3) x SU(2) x U(1) quantum numbers appear more or less in the regular way, however, the number of generations n is less than is required by phenomenology (with the

only exclusion of 3.3.2). One may avoid this difficulty assuming more generations in 10-dimensions. For example one may think about N = 4 supersymmetric E_8model [6] which will lead then to the 4n generations in four dimensions. Such a model could help to resolve another difficulty of this approach. We do not find complete sequence of Higgs fields which are needed for the Grand unification symmetry breaking. Usually appear only Higgses which could be responsible for the electroweak breaking (this require to identify inverse radius of coset space with M_W). The ways out could be as follows i) Some "primordial" Higgses are responsible for the symmetry breaking. (for example its role could be played by scalar particles of N = 4 SUSY E_8[6]). ii) One should investigate coset spaces with more than one characteristic curvature [2,5]. However, in oder to resolve hierarchy problem one should obtain (geometrical) SUSY braking on the electoweak scale. In principle such C.S.D.R. procedure may exist. iii) C.S.D.R. should be combined with other geometrical mechanism of symmetry breaking, say of the type of "Wilson loops".

In any case one might hope that the models discussed here may have some connection with the true theory.

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Table 1

Homogeneous spaces S/R of dimension 6 (S is simple) and branching rules for the vector and spinor of SO(6) into representations of R are listed (see [5]). We do not consider the space SU(3)/U(1)xU(1) and omit this one from the Table.

S/R	Vector	Spinor
1. SO(7)/SO(6)	$\underline{6}$	$\underline{4}$
2. SU(4)/SU(3)xU(1)	$\underline{3}(-2) + \underline{3}^*(2)$	$\underline{1}(3) + \underline{3}(-1)$
3. Sp(4)/SU(2)xU(1)	$\underline{3}(-2) + \underline{3}(2)$	$\underline{1}(3) + \underline{3}(-1)$
4. Sp(4)/SU(2)xU(1)	$\underline{1}(2)+\underline{1}(-2)+\underline{2}(1)+\underline{2}(-1)$	$\underline{1}(2)+\underline{1}(0)+\underline{2}(-1)$
5. G_2/SU(3)	$\underline{3} + \underline{3}^*$	$\underline{1} + \underline{3}$

Table 2

Content of physical fields in reduced theory

S/R	gauge group	scalar fields	fermion fields
G /SU(3)	E_6	$\underline{27}$	$\underline{27}+\underline{78}$
SO(7)/SO(6)	SO(10)	$\underline{10}$	$\underline{16}$
	SU(5)xU(1)	$\underline{5}(-2)+\overset{*}{\underline{5}}(2)$	$\overset{*}{\underline{5}}(3)+\underline{10}(-1)+\underline{1}(-5)$
SU(4)/SU(3)xU(1)	SO(10)xU(1)	$\underline{10}(-2)+\underline{10}(2)$	$\underline{16}(3)$
	SU(5)xU(1)xU(1)	-	$\underline{1}(5,-1)+\overset{*}{\underline{5}}(2,-2)+\underline{10}(1,3)$
Sp(4)/SU(2)xU(1)	E_6 xU(1)	$\underline{27}(+1)+\underline{27}(-2)$	$\underline{1}(0)+\overset{*}{\underline{27}}(2)+\overset{*}{\underline{27}}(-1)+\underline{78}(0)$
	SO(10)xU(1)	$\underline{10}(-2)+\underline{10}(2)$	$\underline{16}(-1)+\underline{16}(3)$
	SO(10)xU(1)xU(1)	$\underline{10}(2,2)+\underline{16}(1,1)$	$\underline{10}(2,2)+\underline{16}(-1,1)$
	SU(3)xSU(5)xU(1)	$(\overset{*}{\underline{3}},\underline{10})(2)$ $+(\overset{*}{\underline{3}},\overset{*}{\underline{5}})(-1)$	$(\overset{*}{\underline{3}},\underline{10})(2)+(\overset{*}{\underline{3}},\overset{*}{\underline{5}})(-1)$