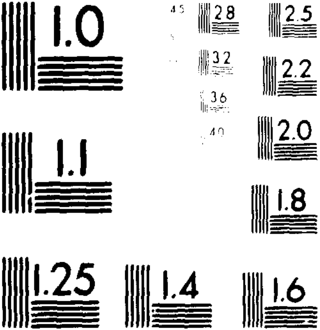

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SIMULATION OF THE MOTION OF CHARGED PARTICLES IN THE DRIFT TUBE OF THE "TRAP" EXPERIMENT (MEASUREMENT OF THE GRAVITATIONAL ACCELERATION OF THE ANTIPROTON)

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Simulation of the motion of charged particles in the drift tube of the TRAP experiment

(measurement of the antiproton gravitational acceleration)

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Abstract

The drift of a bunch of charged particles has been simulated in a scenario based on the set-up for the proposed experiment on the gravitational acceleration of antimatter. The effect of the mutual electrostatic repulsion of the particles and of the external magnetic field on the Time Of Flight distribution has been studied.

1. INTRODUCTION

We present here some preliminary results obtained by computer simulation for the vertical drift in vacuum of a bunch of very low energy charged particles moving in the earth gravitational field. The assumed hypotheses for particle energy, drift length, confining magnetic field, and initial charge spatial density are nearly equal to those presented for the set-up of the antiproton gravitational mass experiment [1], where time of flight distributions for several particles bunches are measured.

The goal of this simulation is to get informations on the time of flight distribution taking into account the effect of the coulombian repulsion in the bunch and to evaluate the relevance of some experimental parameters (particle energy, confining magnetic field intensity, initial charge density and velocity distribution).

The simulation program, written in FORTRAN 77, is running now on our μVax II using external libraries (from CERNLIB) only for random number generation and histogram accumulation and/or manipulation.

The preliminary results presented in this note required a very large CPU time (about 400 hours) on our computer. The running time roughly increasing as the square of the number of particles in the bunch, the possibility to implement the program on a parallel architecture machine is envisaged to spare time and increase statistics.

SI units will be used in this note.

2. OUTLOOK TO THE PHYSICS AND THE GEOMETRY

The final part of the set-up described in ref. [1] is the ground of the physical situation in which the simulated drift motion is studied. A small number of particles ($N \approx 100$-1000) is catched in a small Penning trap [2], few centimeters in size and kept at very low temperature T of the order of a few °K (see fig. 1). At a given time t_0 the potential of one electrode is dropped at a lower value in few nanoseconds and the particles are vertically launched, through a hole, in a vacuum cylindrical drift tube, 1 meter long, with a detector on the top; a uniform magnetic field, about 1 Tesla in intensity, directed along the system axis and parallel to the gravity fied vector, confines radially the particles.

To set the initial conditions for the simulation we assume that the particles are uniformely distributed in the trap and that their velocity distribution is a Maxwell distribution with temperature T. We assume that trap and drift tube are connected via a constriction of length h = 50 mm and diameter Φ = 2 mm. The initial cloud is formed with particles whose position is picked-up at random in a uniform space distribution inside the trap and whose velocity is picked-up at random from a Maxwell distribution.

Once the potential of the upper trap electrode is dropped, the constriction acts as a collimator for the outgoing particles and at the bottom of the drift tube a sort of "needlike" cloud is formed.

Before entering the details of the calculation we would like to remember the orders of magnitude of some relevant quantities.

During the initial transient, the acceleration due to the coulombian repulsion can be estimated (for N=100 particles and a cloud size L=10mm) to be

$$a_e = [<\mathcal{E}^2>^{1/2}]/m \approx 1.25\ 10^5\ \mathrm{ms}^{-2} \qquad (1)$$

where m is the antiproton mass and $\mathcal{E}$ is the strength of the electrostatic force. We remark that

$$a_e >> g = 9.81\ \mathrm{ms}^{-2} \qquad (2)$$

However for transient times during which two or more charges approach very close each other, the acceleration can be much greater than a_e.

Since the simulation (as we will see in next section) is based on time step integration, it is interesting to get an idea of the of the order of magnitude of the time intervals in which this technique can be adequate to the solution of the problem. For this purpose, one has to evaluate some parameters of the particle motion in combined uniform electric and magnetic fields [3].

In the conditions above described for the magnetic field **B** and the coulcmbian electric field **E** = $\mathcal{E}$/e, the following values for the drift velocity v_d of the guiding centers of the Larmor helices described by the particles and the cyclotron frequency v_c can be obtained:

$$v_d = | \mathbf{E} \times \mathbf{B} | / B^2 \approx 1 \text{ mm s}^{-1} \quad (3)$$

and

$$v_c = eB/2\pi m \approx 15 \text{ MHz} \quad (4)$$

In these conditions and for the particles at temperature T=10°K, i.e. kinetic energy of about 10^{-3} eV, the Larmor radius R_L is

$$R_L = v_\perp / 2\pi v_c << 1 \ \mu m \quad (5)$$

where $v_\perp$ is the component of the particle velocity perpendicular to the vertical axis.

To properly choose the time integration step Δt for the motion equations of a bunch of N particles, one must evaluate also the distribution of the quantities

$$\Delta t_{ij} = | \Delta z_{ij} / \Delta v_{ji} | \quad (6)$$

where Δz_{ij} and Δv_{ij} are the differences between the vertical positions and velocities of the i-th and j-th particle respectively. At the very beginning of the drift, Δz_{ij} is of the order of the initial average distance between two near particles. If we suppose to freeze the velocities of the charges at certain time t, and we calculate the crossing times under this hypothesis, from the obtained distributionof the Δt_{ij} we can define a quantity $t_{0.99}$ by the equation

$$\int_{t_{0.99}}^{+\infty} \Phi(\Delta t)\, dt \Big/ \int_{-\infty}^{+\infty} \Phi(\Delta t)\, dt = 0.99 \quad (7)$$

$t_{0.99}$ (essentilly the lower limit of the 99% of the crossing times in the bunch) is plotted versus t in fig. 2 for the first 10^{-3} seconds of drift time in a bunch of N=100 particles initially Maxwell distributed at temperature T = 10 °K, doped with a 5% at 1°K (the simulation has been executed with time steps of 10^{-8} s). From this plot it comes out that the lower limit in the charges average crossing time is less than 10^{-5} s after the first 700 μs of drift.

Finally, since in the simulation the electric field is calculated for any charge distribution and kept constant during any time step in the integration of the particle motion, from the previous discussion it can be deduced that a time step of the order of 10^{-6} seconds is surely adequate to reproduce carefully the charge effects acting on the bunch, excluding a short initial transient which "reshuffles" the particles.

Apart from the mutual electric repulsion in the bunch, in ref. [1] problems related to other electrostatic effects were raised.

We remark here that the charge image effect can be neglected at least in the region away from the top and the bottom of the drift tube. In fact the effect due to a conducting cylinder surface is surely less than the one due to a conducting spherical surface of the same radius surrounding the bunch. The calculations for this last configuration are easier, and one can conclude

$$<a^2>^{/2}_{im.ch.,\ cyl} < <a^2>^{1/2}_{im.ch.,\ sphere} \approx 1.7\ 10^2\ ms^{-2} << a_e \qquad (8)$$

(calculated for N=100 particles with 10 mm RMS radius).

All other electrostatic effects like the perturbation in the charges motion due to the residual trap electric field in the collimator have been neglected for the moment.

3. METHOD AND LIMITS OF THE SIMULATION

In this section we briefly describe the methods used for the time step integration; the procedure to solve the motion equations and get the particles time of flight (TOF) distribution can be resumed as follows :

1. After having generated the space and velocity coordinates inside the trap for a bunch of N charges, we select the particles that can pass through the collimator.
2. For a time step of Δt we calculate the average electric field experimented by each particle in this way: starting from the

actual position and velocity vectors $\mathbf{r}_j$ and $\mathbf{v}_j$ of the j-th particle, we calculate the quantity

$$\mathbf{r}'_j = \mathbf{r}_j + \mathbf{v}_j \ (\Delta t/2) \qquad (9)$$

for j=1,....,N and we assume that during the interval Δt the electric field for each charge has the average constant value $<\mathbf{E}_j>$ due to a charge distribution where the $\mathbf{r}'_j$ are the charge positions so that

$$<\mathbf{E}_j> = \Sigma_{k\neq j}\ e^2\ (\mathbf{r}'_j-\mathbf{r}'_k)\ /4\pi\varepsilon_o\ |\mathbf{r}'_j-\mathbf{r}'_k|^3 \qquad (10)$$

3. To derive the new coordinates at the time $t+\ \Delta t$ of the j-th particle moved by a total electromagnetic and gravitational force

$$\mathbf{F}_j = e\ [\ <\mathbf{E}_j> + (\mathbf{v}_j \times \mathbf{B})\ ] + m\mathbf{g} \qquad (11)$$

one could use the equations

$$\mathbf{r}_j\ |_{t+\ \Delta t/2} = 2\ \mathbf{r}_j\,|_{t+\ \Delta t/2} - \mathbf{r}_j\,|_t + (\mathbf{F}_j/m)\ |_{\ t+\ \Delta t/2}\ (\Delta t/2)^2 \qquad (12)$$

$$\mathbf{v}_j\ |_{t+\ \Delta t/2} = \mathbf{v}_j\,|_t + (\mathbf{F}_j/m)\ |_{\ t+\ \Delta t/2}\ (\Delta t/2) \qquad (13)$$

which are approximated at the third and second order in $\Delta t/2$ respectively, as can be deduced by combining the Taylor series development of the quantities $\mathbf{r}_j(t)$, $\mathbf{r}_j(t+\ \Delta t)$, $\mathbf{v}_j\,(t)$, and $\mathbf{v}_j\,(t+\ \Delta t)$ around the point $t+\ \Delta t/2$.

However, in order to achieve the requested accuracy with a reasonable time step Δt, it is easier to split the problem in 3 parts :

i. The motion due to the electrostatic and gravitational fields along the vertical axis ;

ii. The drift of the guiding centers of the Larmor helices ;

iii. The Larmor rotation.

We have seen that a time step $\Delta t \approx 10^{-6}$ s is adequate to solve the problem i. under the conditions stated above.

The effect ii. can be linearly superimposed calculating at the time t the Larmor radius and the drift velocity from equation (5).

On the contrary, it comes out from the simulation that the effect iii. can be neglected with respect to the TOF distribution.

4. The process is repeated for each particle. When a particle hits the tube walls or fails on the bottom, it is removed from the sum; when a particle hits the top of the drift tube, it is removed and its actual TOF is stored.
5. The process ends when there are no more particles in the drift tube.

The method is affected by a main incertitude source, related to the approximation in which we take constant the electric field during the the time step Δt. The effect of this approximation is that the method does not describe exactly in deterministic way the motion of each single particle in the bunch. It rather samples the effects of the field at time intervals Δt solving statistically the problem of the smearing of the initial distribution during the drift, provided that Δt is less of the proper crossing times.

Unless explicitely stated, the results presented in this note have been obtained using a time step $\Delta t = 10^{-6}$ s in the simulation.

4. TEST ON THE STRENGTH OF THE MAGNETIC FIELD

As seen in section 2), the effect of the combination of the electric and magnetic fields gives to the guiding centers of the narrow Larmor helices described by the particles a radial drift speed

$$v_d = E_\perp / B \qquad (14)$$

where $E_\perp$ is the radial electrostatic field.

The containement of the bunch in the drift tube becomes then more efficient for increasing values of the axial magnetic field, fighting against the spreading effect of the mutual electric repulsion between the particles.

In order to determine an optimal choice for the value of B, a set of runs has been made with a bunch of N = 500 particles varying B between 0.02 and 1T (clearly the confinement is ensured in a safer way for N<500).

The dependence from B of the RMS spread σ_x in the particles position at the top of the drift tube has been studied (fig. 3).

From the initial condition described in section 1., the value σ_x = 6.1 mm would be expected if no electric repulsion and no magnetic field are taken into account. Looking at fig. 3, one can see that, with electric repulsion, the quantity σ_x saturates for B > 0.2 T, reaching a value a little larger than the one expected without electric repulsion. This magnetic field value is therefore to be considered safe for the containment of the bunch in a region of the plane orthogonal to the axial motion of area compatible with the sensitive surface of the detector at the top of the dirft tube. For completeness we observe that the value of B must be also compatible with the stability conditions in the launching trap depending on the potential applied to the electrodes.

5. ESTIMATORS OF g FROM THE TOF DISTRIBUTION

As estimator of the value of g, in ref. [1] the "cutoff time" t_c is proposed ; t_c is the TOF of the slowest particle arriving to the top of the drift tube. Neglecting the effects related to the mutual repulsion in the bunch, one has

$$t_c = (2L/g)^{1/2} \qquad (15)$$

where L is the length of the drift tube (t_c = 0.452 s for L = 1m).

The estimator t_c is very sensitive to g, since, from (15)

$$|dt_c/t_c| = 0.5\ |dg/g| \qquad (16)$$

Then, in order to evaluate g at 1%, one has to determine t_c at 0.5 % (i.e. , for a drift length of 1m, with an error of ± 2 ms). With a good data reduction technique, a determination even an order of magnitude -or more- better seems to be feasable, neglecting the effect of the mutual electric repulsion (see ref. [1] and [4]).

However, the precision in the determination of g with this method depends on the number of particles in the bunch with speed near to the minimum value v_c needed to reach the top of the drift tube

$$v_c = (2gL)^{1/2} \tag{17}$$

and then on the initial distribution of velocities assumed for the bunch.

For instance, in the hypotheses outlined in para 3. (selection of particles by a collimator in a Maxwell distributed gas), from the v_z distribution at the entrance of the collimator at T = 0.1 °K (fig. 4a) one can estimate that ≈ 10^{-4} times the total number of particles are expected to have a TOF larger than 300 ms (fig. 4b), even with no electric repulsion.

The distribution of v_z at the entrance of the drift tube assumed in ref. [1] is more favourable to determine t_c, but not completely realistic. Nevertheless, we used it to study the order of magnitude of the losses in the late TOF region (i.e. the TOF region near the cutoff time t_c).

In order to determine the smearing superimposed by the electric interaction to the TOF distribution, a set of 1000 runs with N = 50 particles at T = 0.3 °K has been made with the distribution of v_z plotted in fig. 5a (this corresponds to a sample of 10^5 particles according to the hypotheses of ref. 4).

The late TOF distribution is strongly affected by the electric field effect (fig. 5b). Even in the optimistic hypothesis that this effect does not change the line shape, one has to face the effect of a reduction of the statistics by a factor R for which we can give a preliminary estimate

$$R = 0.16 \pm 0.03 \tag{18}$$

in the TOF region beyond 300 msec.

The technique of the Poisson fitting to the late TOF distribution, proposed in [1], has then to be carefully reconsidered, making a detailed study of possible variations in the line shape. We have not enough statistics to be conclusive on this last point, that requires the implementation of the program on more powerful computers.

On the other hand, the possibility of using a "global" estimator of the TOF distribution -like the mean- seems to be strongly depressed due to the low sensitivity to variations of g of this kind of "averaging" parameters.

In the case of the mean TOF $<t>$, at T = 10°K, one has

$$|d<t>/<t>| \approx 10^{-5} \, |dg/g| \qquad (19)$$

This result has to be compared with the intrinsic RMS of the mean TOF. For a set of bunches of 50 particles, we calculated with the simulation

$$\sigma_{<t>}/<t> \approx 5 \; 10^{-2} \qquad (20)$$

In order to determine g at 1% using the estimator $<t>$, one should therefore analyze TOF distributions of a number M of bunches

$$M > 10^{11} \qquad (21)$$

as can be deduced from the comparison of (19) and (20).

6. CONCLUSIONS

The conclusions of our studies can be summarized as follows :

1. The containment of the bunch in the drift tube is efficiently done even by an axial magnetic field of strength 0.2 T (20% of the value proposed in [1]).

2. The mutual electrostatic repulsion of the particles in the bunch strongly affects the late TOF distribution. This effect has then major consequencies on the cutoff time t_c, proposed in [1] as estimator of g. These consequencies have to be fully understood before approaching sophisticated data reduction techniques minimizing the statistical error on t_c. Moreover, even if more accurate studies will demonstrate that the line shape of the late TOF distribution is not appreciably changed, the statistics in this region will be in any case strongly depressed.

ACKNOWLEDGEMENT

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[2] G.H. Demhelt, Ad. At. and Mol. Phys. **3** (1967) 53.

[3] see for instance L. Landau and E. Lifchitz, "Theory of the Fields", Pergamon Press, ch. 22.

[4] M.V. Hynes, Memo P15-87-U-387 (Los Alamos 19.8.87).

FIGURE CAPTIONS

(1) Layout of the set-up of the drift tube used in the simulation.

(2) σ_x on the top of the drift tube (mm) vs. the strength of the confining magnetic field (Tesla).

(3) $t_{0.99}$ (s), defined in sect. 2, vs. the time drift (ms).

(4) Distribution of velocities at the entrance of the drift tube (a) and TOF distribution for t>100 ms (b) under the hypotheses of sect. 2, for T = 0.1°K. Units of probability.

(5) a. Distribution of v_z at the entrance of the drift tube for the simulation described in sect. 5 (bins of 5 ms^{-1}).

b. Late TOF distribution (drift time > 100 ms) from the simulation run described in sect. 5, expected without electrostatic repulsion (continuous line) and actual (dashed line). On the x axis, the TOF in bins of 10 ms each. The arrow indicates the cutoff time t_c for L=1m.

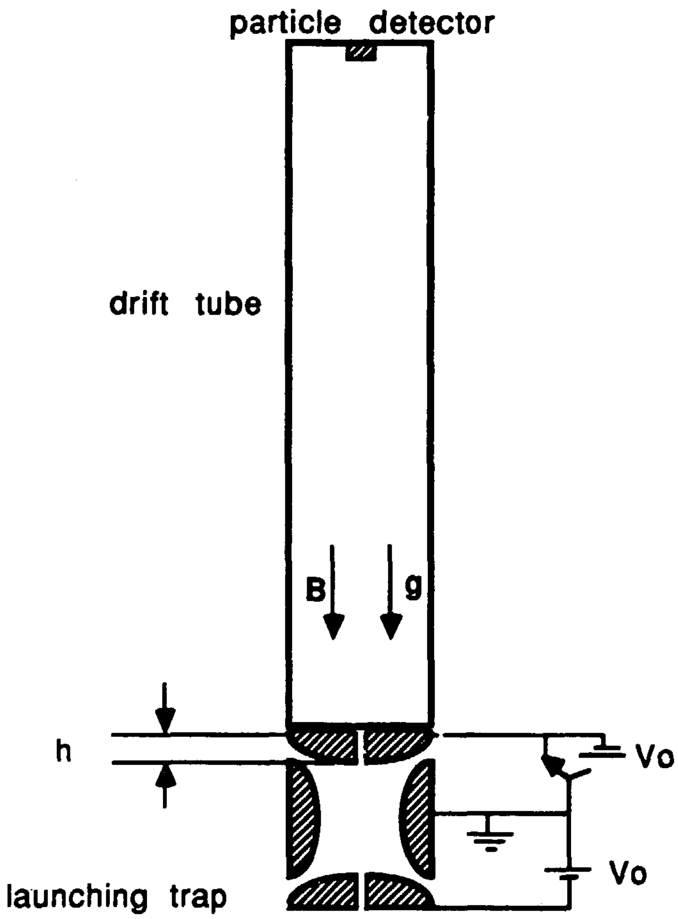
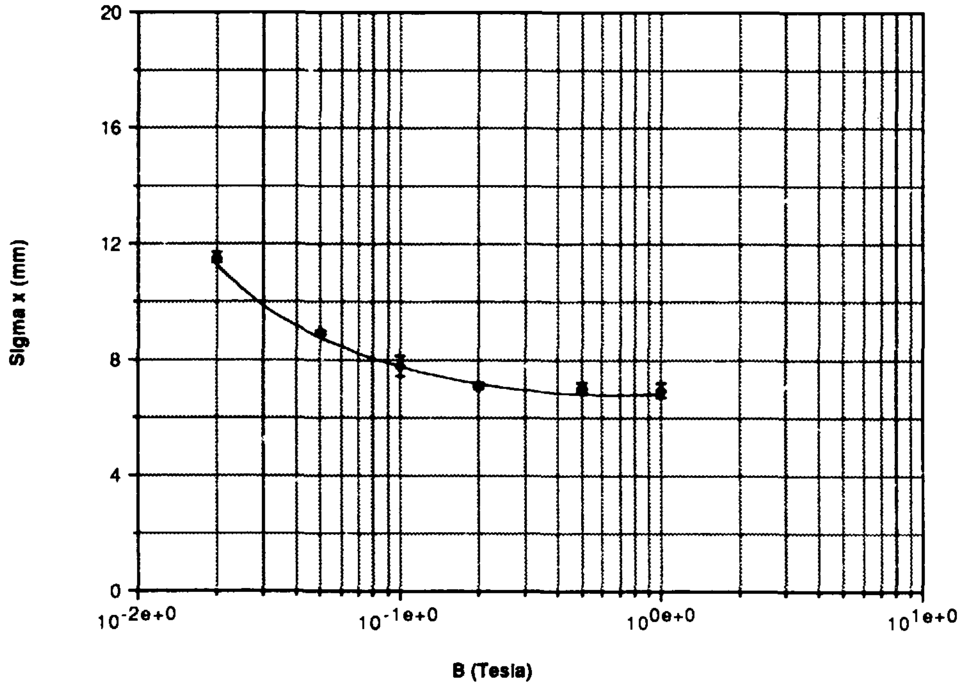
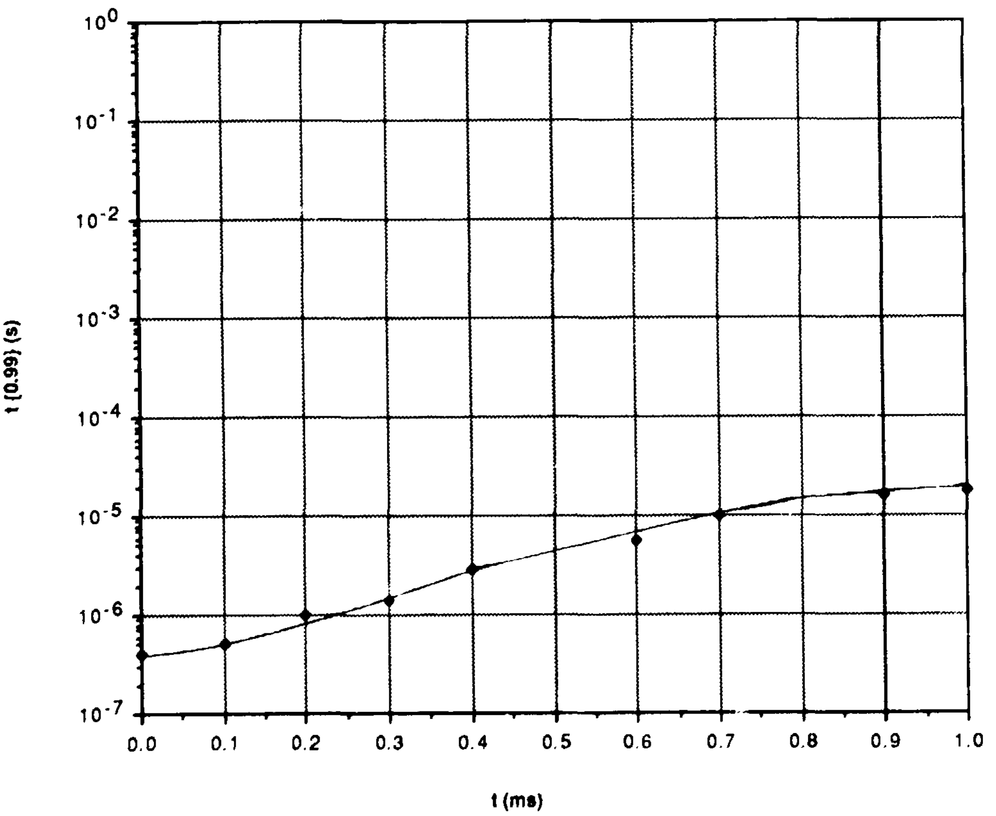


Fig. 1



20
16
12
8
4
0
Sigma x (mm)
10-2e+0
10-1e+0
100e+0
101e+0
B (Tesla)

Fig. 2



10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
t (0.99) (s)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
t (ms)

Fig. 3

Fig. 4a

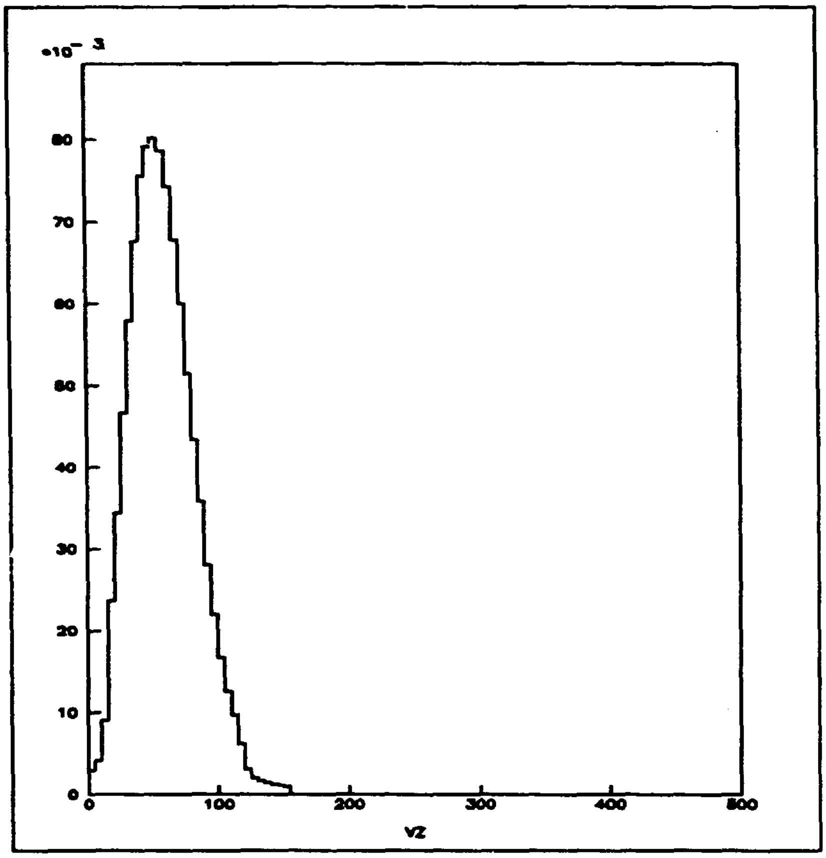

*10^{-3}
80
70
60
50
40
30
20
10
0
0
100
200
300
400
500
VZ

Fig. 4b

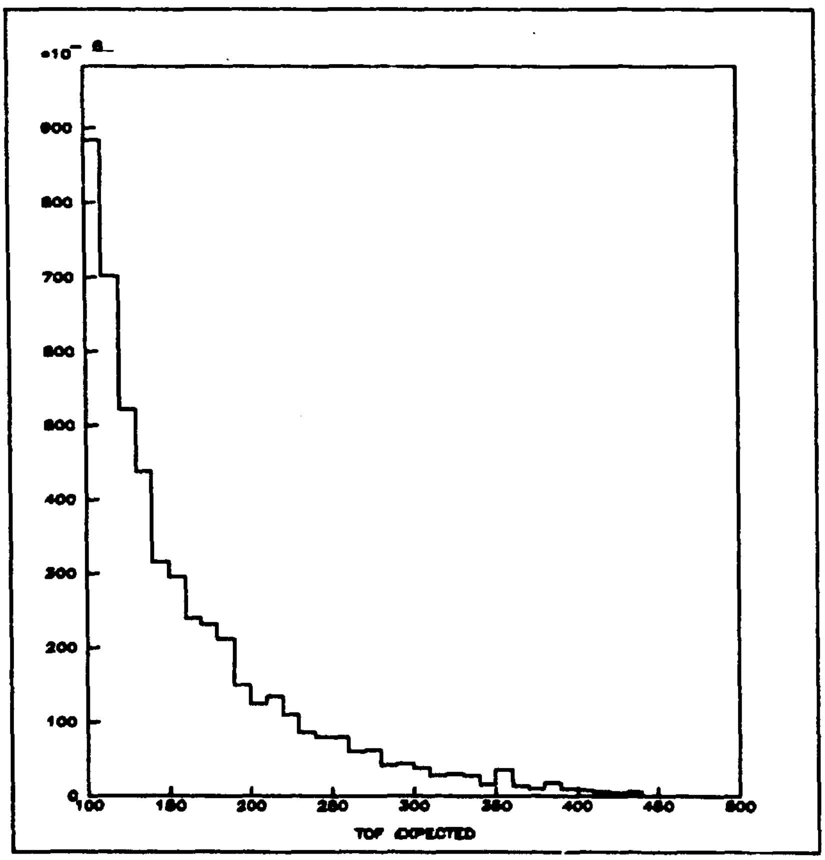

*10^{-6}
900
800
700
600
500
400
300
200
100
0
100
150
200
250
300
350
400
450
500
TOF EXPECTED

Fig. 5a

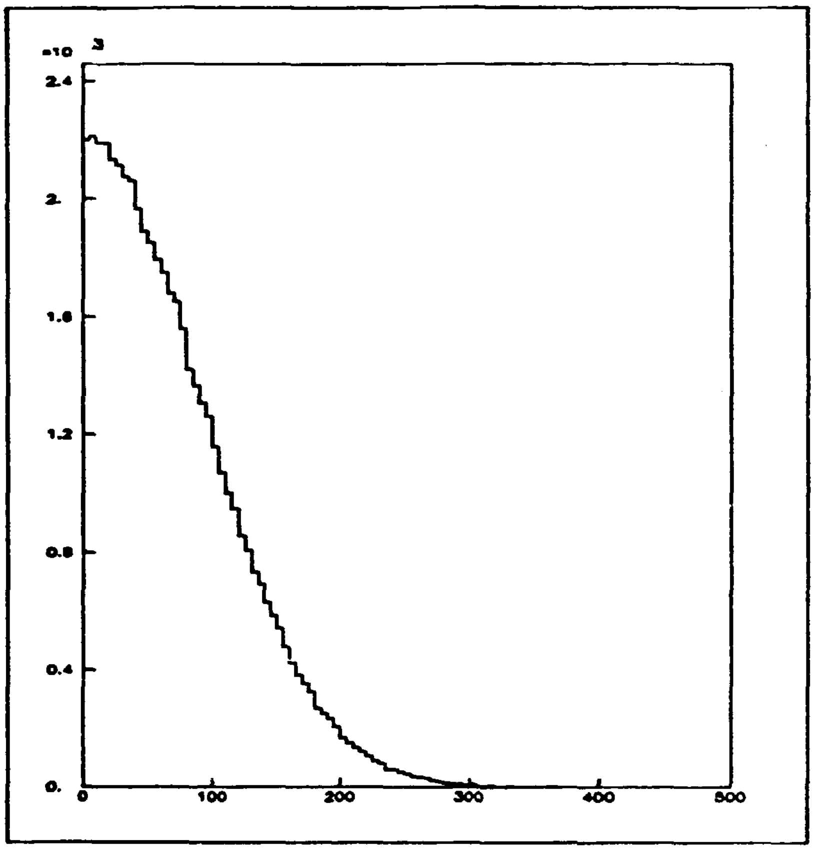
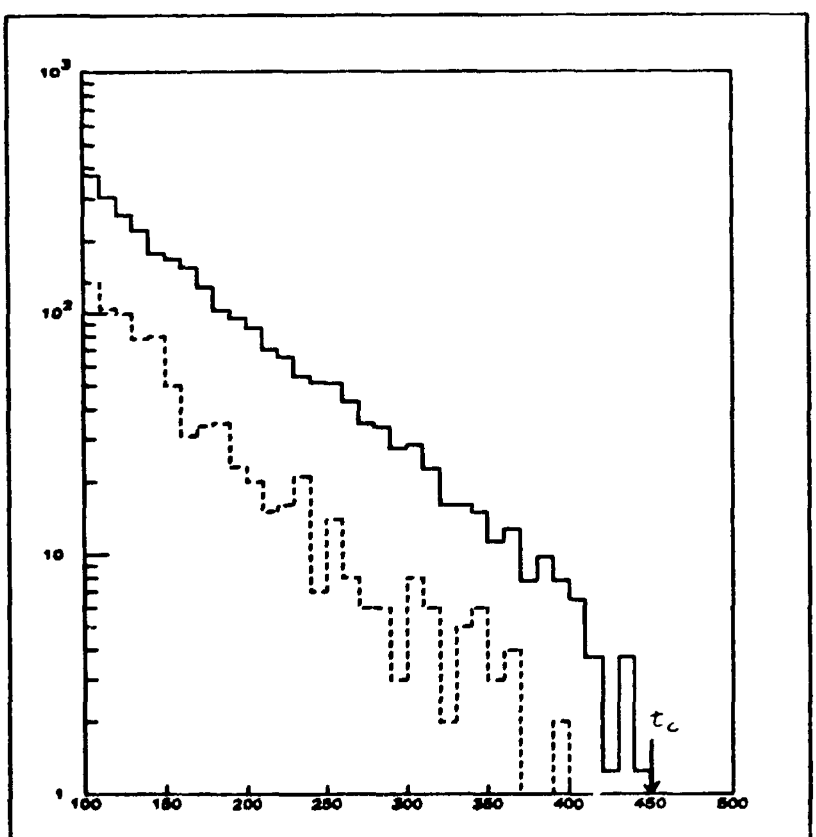

*10 3
2.4
2
1.6
1.2
0.8
0.4
0.
0
100
200
300
400
500

Fig. 5b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10²
10
1
100
150
200
250
300
350
400
450
500
t_c

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