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A REVIEW OF ANALYSIS METHODS ABOUT THERMAL BUCKLING

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ABSTRACT

This paper highlights the main items emerging from a large bibliographical survey carried out on strain-induced buckling analysis methods applicable in the building of fast neutron reactor structures [1], conducted within the framework of a study by the Codes and Standards working group of the E.C.C.

The work is centred on the practical analysis methods used in construction codes to account for the strain-buckling of thin and slender structures.

Methods proposed in the literature concerning past and present studies are rapidly described. Experimental, theoretical and numerical methods are considered.

Methods applicable to design and their degree of validation are indicated.

1. INTRODUCTION

The building of pool type fast breeder reactors involves the fabrication of large main vessels made with thin shells. The strong thermal stresses present during operation lead to the risk of thermal buckling. This behaviour covers buckling phenomena in which applied strains, constant or cyclic in time, play a part either alone or with other loads such as primary stresses for example.

The construction codes, commonly referred to in the nuclear industry, fail to supply practical analysis methods to estimate this risk, and need to be developed.

A good way to direct research programmes on the subject and possibly to write analysis methods into the codes, is to make a critical analysis of results available in the literature concerned.

2. EXAMINATION OF CONSTRUCTION CODES NORMALLY USED IN NUCLEAR INDUSTRY

The codes used in the nuclear industry give recommendations for the analysis of buckling, mainly when able to be caused by pressure. This is the case for instance with the ASME rules Section III [2] in articles NB, NC and ND 3130 and 3240 on the one hand and NE on the other.

For components thinner but still under pressure, code ASME CASE N284 [3], British Standards[4] or C.E.C.M. [5] give computing diagrams. There are no special developments in these codes to deal with strain induced buckling. This means that for practical applications designer is condamned to neglect thermal stresses or to consider them as primary ones.

Code ASME CASE N47 [6] is the first to offer some indications to treat strain-induced buckling differently from buckling under load. This code essentially gives a description of these two effects and the respective margins to be guaranteed, but no practical methods to carry out, the required analyses are provided explicitly.

3. REVIEW OF EARLIER WORK

It seems that the first large-scale studies on the thermal buckling of shells were carried out in the field of aeronautics. There a review of the situation was established by D. Bushnell in the context of the development and validation of the finite difference computing code BOSOR [7]. This author, considering the case of buckling due to local heat gradients of conical and cylindrical shells, compiles a survey of analytical and experimental studies on the elastic buckling of thin circular shells stiffened by rings and subjected to monotonic increasing local temperature gradients [8] and [9]. He points out that it is essential to account for pre-buckling behaviour (large displacements), the location of any defects with respect to the thermally affected zone and especially the clamping conditions, particularly difficult to master experimentally. This article mentions the use of knock-down factors to allow a "follow-up" adjustment between a given type of analysis and a type of configuration (shape and load). These factors consist of the ratio between the actual (experimental) critical temperature and the critical temperature calculated by the analysis method. They range from 0.2 to 1 with highly simplified methods (of the uniform thermal stress kind) and from 0.66 to 0.71 with the BOSOR 3 analysis, according to the slenderness of the structures.

It appears that studies devoted to this behavior are not numerous and that no really safe and generally applicable analysis methods emerge clearly for the treatment of thermal buckling. This explains the development of more recent works described below.

4. REVIEW OF RECENT WORKS

The most recent studies have been classified arbitrarily into three categories according to whether their development principle is experimental, analytical (for simple models) or based on computing programmes.

4.1. EXPERIMENTAL METHODS

A survey of earlier experimental work has shown that the tests are tremendously difficult if thermal loads are to be considered even on simple shell structures. Tests have been carried out on reduction of the critical mechanical load (primary axial compression) under the effect of applied strain, constant (similar to residual stresses) or variable with time. Fatigue-thermal buckling tests have also been performed.

4.1.1. Effect of constant applied strain

Residual stresses or thermal stresses can substantially lower the tolerable primary load. A test run on cylindrical tubes, with attempts to create a simple known field of residual stresses, has been carried out at the CEA.DEMT [10]. The test specimen geometry and test rig are shown in Figure 1. The specimen consists of a tube with an internal tie-rod.
To guarantee that both components buckle (and bend) together, spacer washers are placed regularly between the tie-rod and the tube. The axial membrane prestrain is imposed in compression by a screw/nut system in the tube. The beam is hinged at both ends.

A correlation was tried between the secant modulus Es obtained on the material stress-strain for the imposed prestrain and the effective modulus E_{eff} obtained from the experiments by the following formula :

$$P_{critical} \text{ (experiments)} = \frac{\pi^2 E_{eff} I}{L^2}$$

where I is the inertia modulus and L the length between the supports. It shows a scattering which reaches four orders of magnitude. An initial unknown defect together with a possible interaction between differents parts of the specimen (tie-rod, tube and washers) during bending deformation can explain partially the scattering.

The influence of constant thermal stresses on buckling has been tested in shell configurations close to those found in industry. These tests include the CEA/DEMT experiments reported in [11] concerning evaluation of the instability load on a thin shell ($e/r = 0.002$; e = thickness, r = radius) subjected to uniform outer pressure, uniform axial traction and an axial thermal gradient. The shell is mounted on two stiff rings (Figure 2).

Tests under thermal stress alone (shell simply posed) showed no sign of buckling in spite of quite large gradients (880°C over 15 mm and 1100°C maximum temperature). It has nonetheless been shown that if the thermal load is cyclic a progressive locally concentrated deformation appears.

Tests with combinations of different loads show that for the chosen traction and thermal stresses, the instability load is not significantly affected, remaining much the same as that obtained under external pressure only. Tests are in progress to establish the effect of larger heat gradients.

4.1.2. Effect of applied cyclic strain

Cyclic thermal strains, even more than "constant" ones, can reduce the tolerable primary load by a kind of ratchet effect. For the purpose of studying this load reduction experimentally a series of progressive buckling tests with a 316L austenitic steel at room temperature has been under way at the CEA/DEMT for some years [12] and the first have given rise to a detailed report [13] by Lebey and al.

The aim of the tests is to determine the couples (primary compression load P - secondary cyclic twist load ΔQ) which cause buckling after a certain number of cycles. These simple tests have shown experimentally that buckling, though not produced by a compression load P below the critical load P_{cr} of the sample, could occur with this load P if combined with an applied cyclic strain sufficiently high. Figure 3 plots axial shortening versus number of cycles for several test-pieces. Shortening is seen to accelerate, very sharply when the samples buckle after a certain number of cycles, but is gradual and slow for samples which withstand a very large number of cycles without buckling.

The specimens dimensions are differenciated by their stiffness ratio $\alpha = \sigma_E/S_Y$ = Euler stress/Yield stress which ranges between 0.6 and 1.6.

The possible initial defect which is very low (less than one tenth of the thickness) is not measured.

An attempt was made to establish a practical rule from these experimental results by placing the points corresponding to each test in an efficiency diagram, the axes of which are :

- for the abscissa, the ratio of the elastically calculated secondary stress to the applied primary stress,

- for the ordinate, the ratio of this primary load to the critical primary load (obtained without secondary stress).

The experimental points placed in this diagram are marked differently according to whether or not progressive unstable buckling has occurred for the number of cycles applied (see Figure 4). The numbers of cycles ranges from 0.25 to 35000.

The curve corresponding to the efficiency diagram chosen to estimate progressive strain by Clement and al [14] conservatively separates the two sets of points identified. This conservatism is greater when the stiffness ratio is high i.e. when buckling behavior involves more plasticity.

Additionnal tests worth noting are those carried by Waeckel [15] on the thermal buckling of circular plates singly or doubly supported at the ends and submitted to a central cold temperature shock. These tests on a ferritic steel indicate, in particular, the influence of unknown initial defects and the occurrence of local progressive deformation when shocks are repeated.

Biaxial thermal fatigue tests were performed by Spandick [16]. In these tests, buckling behavior reduced the fatigue life and concentrated deformation in a region close to the clamped extremities of the specimens.

Simmons [17] tested various curved plates fixed at both ends on a rigid concrete support and submitted to a temperature rise. He gives a description of a snap through behavior that leads the central part of the plate to move away from its concrete back support.

These tests show that thermal buckling occurs very rapidly when the clamping conditions are too rigid, and strain concentration zones, detrimental where fatigue is concerned, may appear.

Thermal stresses can increase the effect of primary compressive charge and thus accelerate buckling behavior.

The results underline the need to consider thermal buckling risks if large thermal stresses are present by ad'hoc analysis. Some practical methods coming from experiments are proposed to treat theses problems.

4.2. ANALYTICAL METHODS

These methods generally lead to practical conclusions in certain very limited mechanical configurations. They have been adapted for inclusion in more general computing programmes such as those based on the finite elements method. In this latter case they will be presented in the next section.

4.2.1. Asymptotic theories

Returning to the presentation made by D. Bushnell [18], these theories rest on Koiter's general theory of elastic post-bifurcation [19] allowing development of the variations in the loading parameter λ versus the amplitude of the modal buckling displacement Wb. This formulation is only strictly valid around the bifurcation point in the plane defined by these two parameters. Originally these methods only allowed the ultimate loads to be calculated for simple structures in the elastic domain and they do not differenciate the effect of stress controlled and strain controlled situations.

4.2.1.1. Simplified methods. Progressive buckling of a beam

Gontier and Hoffmann, [20] and [21], calculate the cyclic elasto-plastic buckling of a beam clamped at both ends and subjected to the combined action of a constant primary compression load and a cyclic heat gradient through the thickness. The displacement increments produced by the cyclic temperature variations are calculated at each cycle by a conventional plastic computing method (Prandt-Reuss, Von Mises criterion, asymptotic defect increase, use of tangent modulus for the plastic bifurcation). In this simple case, the convergence conditions, which guarantee that the displacement reaches a value below the instability displacement, can be established. These conditions are given in practical form in Figure 5 for a 316 L steel traction curve at room temperature. This figure gives, for four types of relative rigidity measured by the ratio $\sigma_E/\sigma_{0.2\ \%}$ (Euler's critical load σ_E over the 0.2 % yield strength of the material), the loading conditions $Q/\sigma_{e\ 0.2\ \%}$, $\sigma_m/\sigma_{e\ 0.2\ \%}$ and the initial defect η_0 for which this progressive strain process stops at a stable state.

These results can be compared with CEA experiments concerning progressive buckling of tubes. They show the same general influence of rigidity ratio and offer a way to refine the practical method based on the efficiency diagram concept exposed upper.

4.3. METHODS USING COMPUTING CODES

Incremental methods are distinguished here from special methods combining pre-buckling calculations and bifurcation calculations for particular and simple structures.

4.3.1. Incremental methods

This category of methods includes finite elements or difference methods which determine a maximum instability load by predicting essentially the pre-buckling trajectory (OBAC in Figure 6) by increasing monotonically the loading or the displacement.

These calculations account for the presence of initial defects by ad'hoc meshing and non-linear behaviours, geometrical (large displacements) and material (traction curve). Many examples of such calculations may by found in the literature. The article of Dhalla [22] refers to the MARC code and to the codes ABAQUS, ADINA and ANSYS. In this context it is also worth mentioning the CASTEM system computing code TRICO, presented by Hoffmann [23], or NOVNL by Carnoy [24]. These codes may be used to calculate the limit load λ_L for a perfect structure or $\lambda_{L'}$ for a flawed structure, but not a bifurcation load proper which can differ significantly from the limit one. In addition two main difficulties of these methods are the need for accurate modelling of the defect, and in some cases the use of costly three-dimensional meshing and the inaccuracy on the limit load determination since the calculation aims at an extreme value not easily attained unless the structure and the loading are simple. When, for example, the computation is piloted by a load this value is determined by the non-convergence of the calculation and may then depend closely on numerical problems.

4.3.2. Special methods

Bifurcation analysis codes have been developed to analyse the buckling of simply shaped structures but frequently used in nuclear industry such as axisymmetrical elements. These codes compute the pre-buckling behaviour and the bifurcation point (λ_B or $\lambda_{B'}$) separately. The bifurcation calculation (B or B' in Figure 6) is possible for these simple shapes because the modelization of the structure can be one-dimensional. Only one meridian is considered for this analysis.

The code BOSOR 5 developed by D. Bushnell [23] is a finite difference computing code adapted to axisymmetrical structures. The pre-buckling behaviour is calculated by an incremental plasticity method. Various options are available to determine the tangential rigidity matrix for the bifurcation load, using indifferently either the finite or the incremental plasticity theory. Many comparisons of this code with experimental results or otner codes are presented in [25] .

The CASTEM system computing code INCA [26] was developed recently for finite elements bifurcation calculations on quasi-axisymmetrical structures by Combescure and al. [27] and [24] . The structures may contain initial non-axisymmetrical defects. They are modelled by decomposition into Fourier series. Comparison with experiments are described by Brochard in [11] .

Computing programmes using the finite elements or differences method are found to treat thermal loads of applied displacement as they do primary loads. No special development is involved for strain buckling and experimental validations in this field are still under way.

5. DEVELOPMENT OF DESIGN RULES

The RCC-MR [28] contains rules to account for strain-induced buckling in the case of thin shell structures. The aim is to consider increases in the risk of other damages occurring through a possible tendency towards buckling caused by thermal stresses.

Practical methods of checking these rules are given in RCC-MR appendix A7. These are divided into two groups according to whether the loads involved are monotonic or cyclic.

Rules intended for monotonic loads include a method based on load reduction diagrams and elastic calculations, and also an elastoplastic method combining large displacement incremental calculations and elastoplastic bifurcation calculations carried out by the use of computing codes such as

BOSOR 5 or INCA mentioned above. According to the stability criteria given explicitly in the elastoplastic analysis the instability load is not reached if three conditions are fulfilled. Firstly the structure is in stable equilibrium, secondly no bifurcation is encountered, and thirdly the change in shape of the structure and the deformations of the material are not excessive.

The rules accounting for cyclic loads are based on elastic analysis in which the primary and secondary stress values are amplified to allow for geometrical non-linearities due to possible buckling. They include in particular the progressive buckling rule, based on the efficiency diagram developed to some extent above and some indications about fatigue amplification.

6. GENERAL CONCLUSIONS

Thermal buckling may occurs in the thin structures of fast breeder reactors if large thermal stresses are present. It may enhance the effect of primary loads, if any. In pure strain buckling it can lead to local excessive deformation or fatigue. This effect is still larger if these thermal stresses are applied cyclically.

Thermal buckling experiments show a decisive influence of sample clamping conditions, temperature distributions, even local, and initial defects. These tests lead to practical conclusions only in cases where the conditions are very simple and where local strains can be measured when fatigue is under consideration. The relative influence of slenderness and plasticity is addressed.

Analysis methods are being developed to treat differently the stresses due to displacements and those due to loads. Simplified methods give some interesting practical applications. That of general computing codes, which seems applicable in most cases, is in the process of experimental validation.

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LIST OF CAPTIONS

FIG. 1 : Test specimen configuration. Prestraining device Displacement controlled buckling.

FIG. 2 : CEA-DEMT Thermo-mechanical buckling on circular cylindrical shell. First experimental results.

FIG. 3 : Axial shortening versus number of cycles. Experimental results for 4(15/16) tube specimens. Stable and unstable behaviors.

FIG. 4 : Verification of applicability of efficiency diagram for progressive buckling assesment.

FIG. 5 : Hoffmann and Gontier's diagrams for progressive buckling.

FIG. 6 : Schematic of load - displacement curves - Limit loads and bifurcation loads.

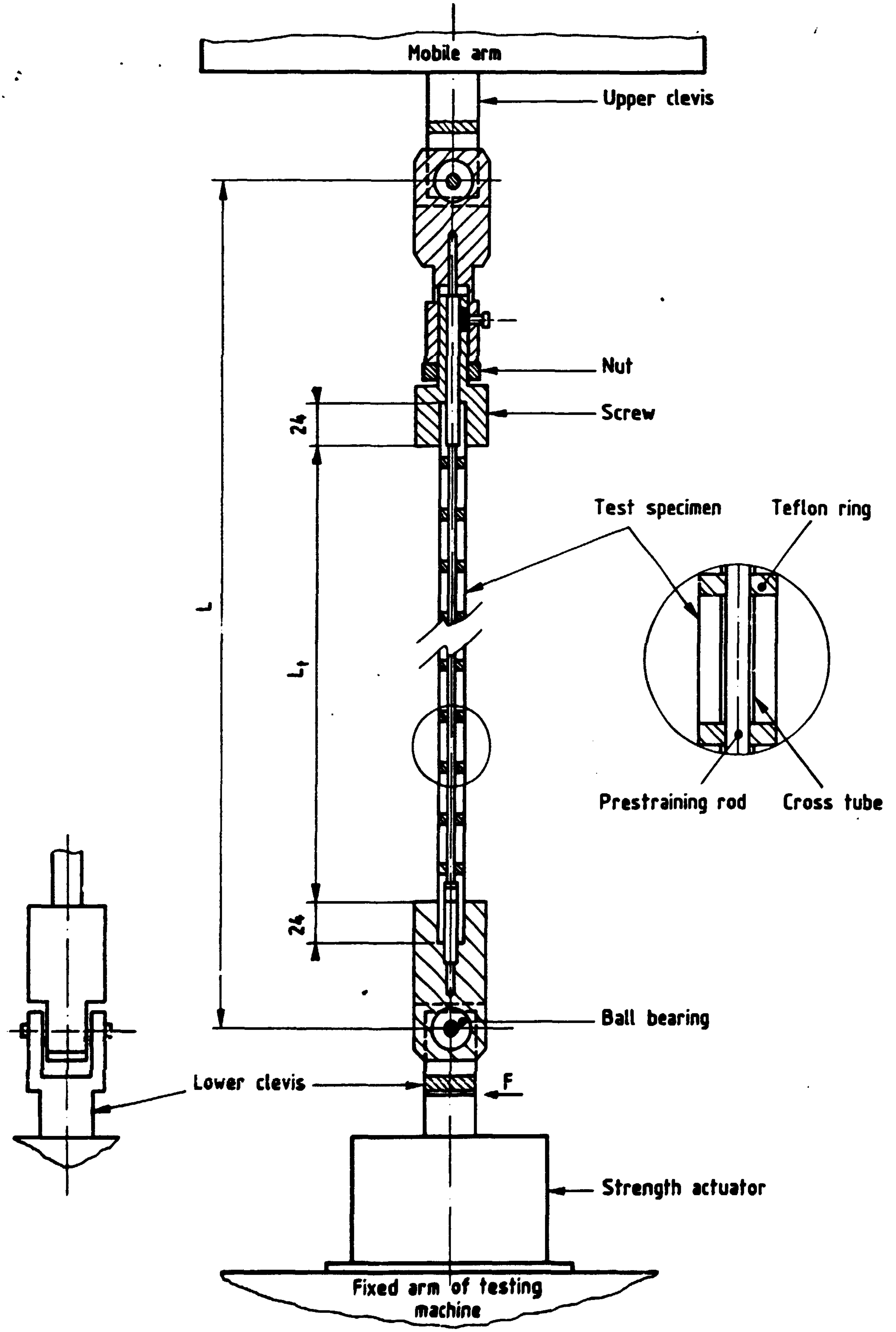
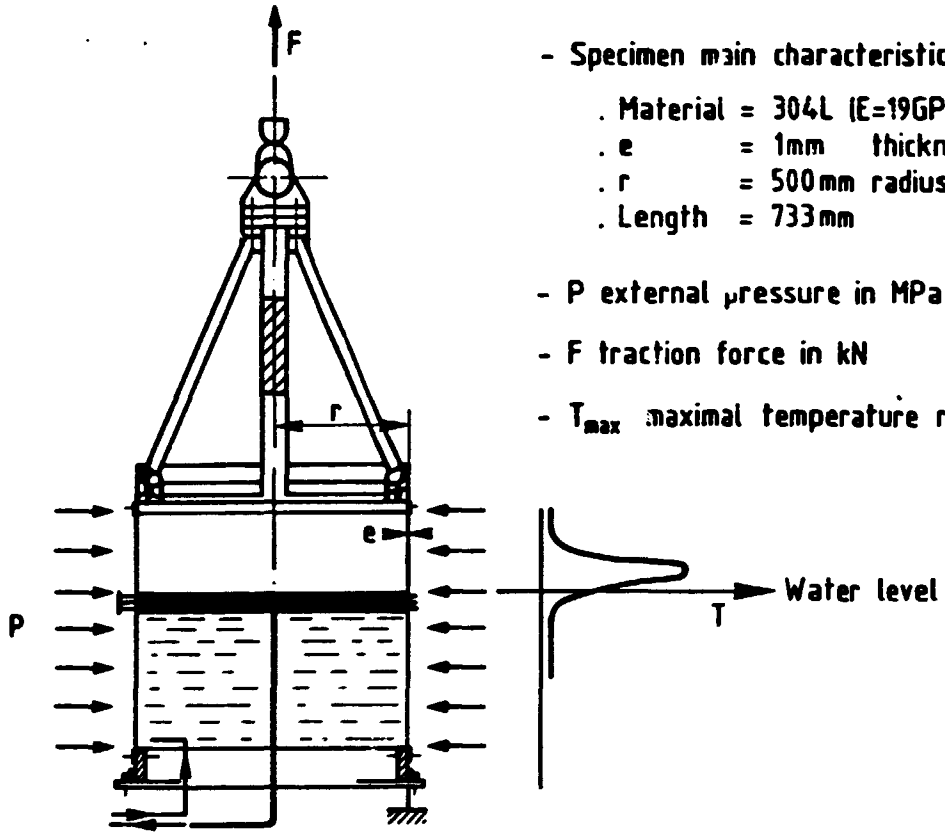


Fig. 1 - TEST SPECIMEN CONFIGURATION. PRESTRAINING DEVICE.



- Specimen main characteristics:
  . Material = 304L (E=19GPa, σ_y=210MPa)
  . e = 1mm thickness
  . r = 500mm radius
  . Length = 733mm
- P external pressure in MPa
- F traction force in kN
- T_{max} maximal temperature rise in °C

CYLINDER NUMBER	TEST NUMBER	THERMAL LOAD		HEATING PERIOD	BUCKLING
		T_{max}	G		
0	1	240°C	180°C/15mm	2 s	no
	2	370°C	260°C/15mm	7 s	no
	3	490°C	350°C/15mm	10 s	no
	4	580°C	350°C/15mm	11 s	no
	5	630°C	460°C/15mm	15 s	no
	6	760°C	580°C/15mm	17 s	no
	7	880°C	680°C/15mm	20s	no
	8	980°C	860°C/15mm	22s	no
	9	1100°C	880°C/15mm	45s	no
1	1 to 20	980°C	760°C/15mm	30s	no

CYLINDER NUMBER	TEST NUMBER	MECH. LOADING		THERMAL LOAD		BUCKLING
		Pressure MPa	Traction kN	T_{max}	G	
2	1	0,0032	348	415°C	280°C/15mm	no
	2	0,0160	348	415°C	280°C/15mm	no
	3	0,0160	116	415°C	280°C/15mm	no
	4	0,0300	0	0	0	yes
3	1	0,0280	280	0	0	yes
4	1	0,0320	280	0	0	yes

Fig. 2 - CEA - DEMT THERMO-MECHANICAL BUCKLING ON CIRCULAR CYLINDRICAL SHELL. FIRST EXPERIMENTAL RESULTS

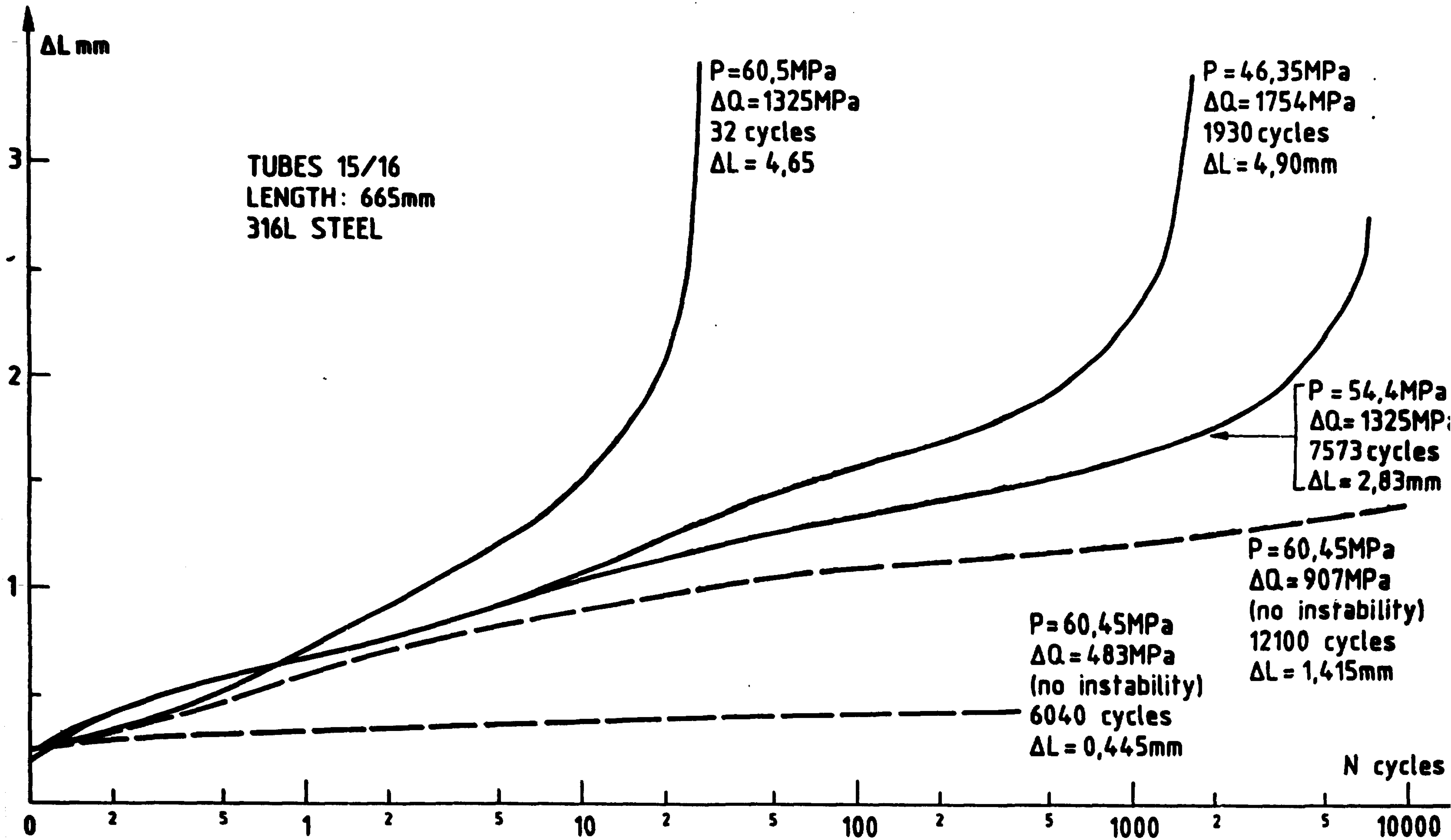


Fig. 3 - AXIAL SHORTENING VERSUS NUMBER OF CYCLES - EXPERIMENTAL RESULTS FOR 4(15/16) TUBE SPECIMENS - STABLE AND UNSTABLE BEHAVIORS

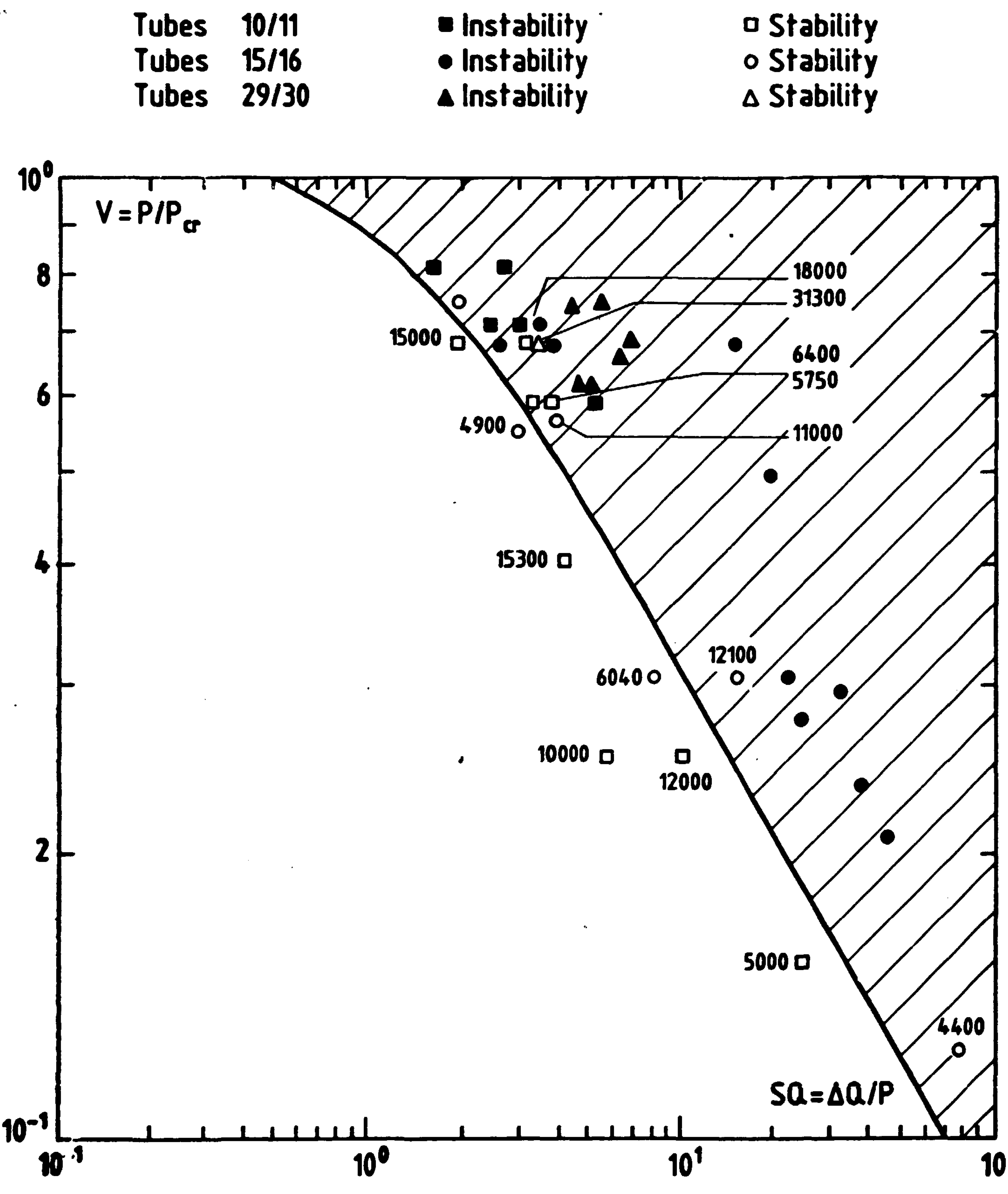
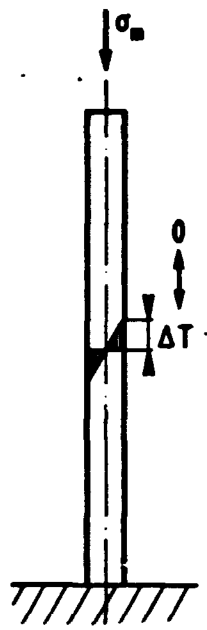
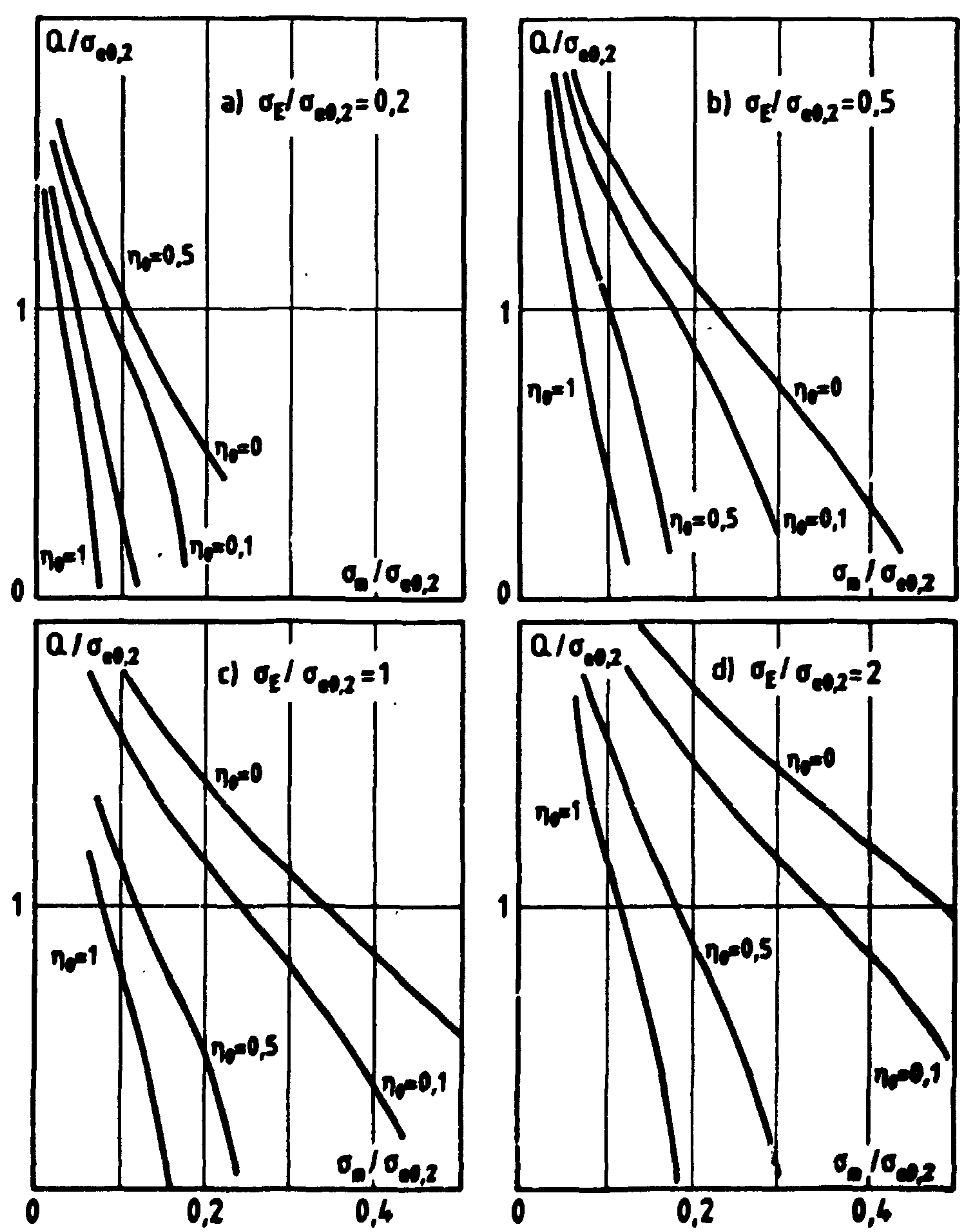


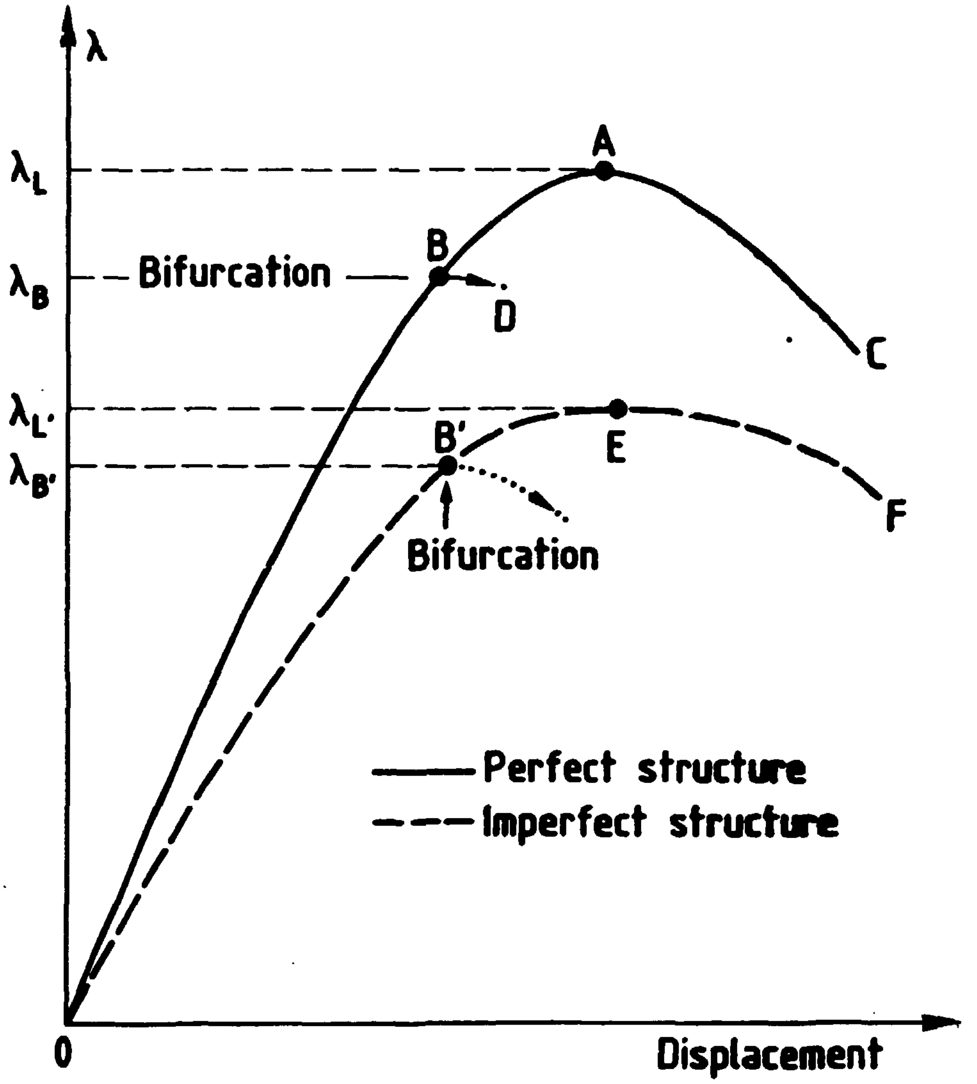
Fig. 4 - VERIFICATION OF APPLICABILITY OF EFFICIENCY DIAGRAM FOR PROGRESSIVE BUCKLING ASSESMENT



- σ_m membrane primary stress in MPa
- $Q = E\alpha\Delta T/2(1-\nu)$ bending elastic stress in MPa
- η_0 initial modal defect normalized to thickness
- σ_E Euler's membrane stress in MPa
- $\sigma_{e0,2}$ = yield stress (0,2% offset) in MPa
- material $\sigma-\varepsilon$ curve of a 316 steel at R.T.

(Q, σ_m) Conditions for stable and limited buckling





λ_L - Limit load for incremental method on perfect structure

λ_B - Buckling load on perfect structure. Bifurcation method

$\lambda_{L'}$ - Limit load for incremental method on imperfect structure

$\lambda_{B'}$ - Buckling load on imperfect st. ucture. Bifurcation method

Fig. 6 - SCHEMATIC OF LOAD - DISPLACEMENT CURVES - LIMIT LOADS AND EIFURCATION LOADS