CONF-871105--4-Vugraphs

Study of Resistive Pressure-Gradient-Driven Turbulence.

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[Outline]

I. Overview

- Motivation

- Review of Previous Work
  [Ref. : B.A. Carreras, L. Garcia and P.H. Diamond,
  Phys. Fluids, 30, 1388 (1987)]

  * Saturation of RPGDT
  * Diffusivities

II. Study of Wavenumber Spectrum

- Two-Point Theory of RPGDT

- Wavenumber Spectrum

III. EM Model with Temperature Evolution

- Model Equations

- Renormalized Nonlinear Theory

IV. Discussion & Summary

- Self-consistent Evolution of EM Turbulence

I. Overview

* **Motivation :** Previous studies have shown the resistive pressure-gradient-driven turbulence (RPGDT) is a likely cause of observed turbulent fluctuations and anomalous transport in magnetically confined plasmas.
  More recent study of RPGDT found a true saturation criterion and predicted significantly larger pressure diffusivity over simple mixing-length estimate.

* **Purpose of this work :** In this study, we investigate wavenumber spectrum for more detailed characteristics of this driven turbulence and consider an electromagnetic model with elctron temperature evolution to study the effect of magnetic fluctuations on thermal transport.

* **Review of Previous Work :** The followings are a brief review of main results from previous work.

$$\Downarrow$$

[*Ref.* : B.A. Carreras, L. Garcia and P.H. Diamond, Phys. Fluids, **30**, 1388 (1987) "]

NONLINEAR ANALYTICAL MODEL

: Previous work

- Starting point: Electrostatic model reduces to two equations

$$\frac{dU}{dt} = -\frac{1}{\eta\rho_m}\nabla_{\parallel}^{(0)^2}\phi + \frac{1}{\rho_m}\mathbf{z}\cdot[\nabla\Omega\times\nabla\tilde{p}] + \mu\nabla_\perp^2 U \ ,$$

$$\frac{d\tilde{p}}{dt} = \chi_\perp\nabla_\perp^2\tilde{p} - \frac{1}{r}\frac{\partial\phi}{\partial\theta}\frac{dp_0}{dr} \ .$$

- To derive the renormalized response equations, we start from

$$\frac{\partial U_{\mathbf{k}}}{\partial t} + N_{1\mathbf{k}} - \mu\nabla_\perp^2 U_{\mathbf{k}} = \frac{1}{\eta\rho_m}k_\parallel^2\phi_{\mathbf{k}} - \frac{i}{\rho_m}\frac{d\Omega}{dr}k_y\tilde{p}_{\mathbf{k}} \ ,$$

$$\frac{\partial\tilde{p}_{\mathbf{k}}}{\partial t} + N_{2\mathbf{k}} - \chi_\perp\nabla_\perp^2\tilde{p}_{\mathbf{k}} = -i\frac{dp_0}{dr}k_y\phi_{\mathbf{k}} \ ,$$

where $k_y \equiv m/r$, $k_z \equiv n/R_0$, and $k_\parallel = mB_0x/(R_0qL_q)$.

- The nonlinear diffusivities are

$$D_{xx} \cong \gamma_{\langle m\rangle}^{(0)}\left(W_{\langle m\rangle}^{(0)}\right)^2\Lambda^2 \ ,$$

$$\mu_{xx} \cong \frac{m^2}{\langle m\rangle}\gamma_{\langle m\rangle}^{(0)}\left(W_{\langle m\rangle}^{(0)}\right)^2\Lambda \ ,$$

where

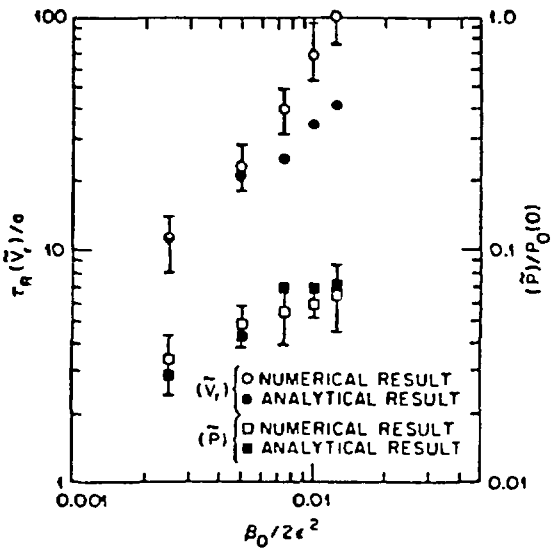
$$\Lambda \equiv \frac{2}{3\pi}\ln\left[\frac{\beta_0}{c^2}\left(a\frac{d\Omega}{dr}\right)\left(\frac{-a}{p_0(0)}\frac{dp_0}{dr}\right)\left(\frac{r}{am_0}\right)^4\frac{64a^4S^2}{\tau_R^2D_{xx}\mu_{xx}}\right] \ .$$

: Enhancement factor

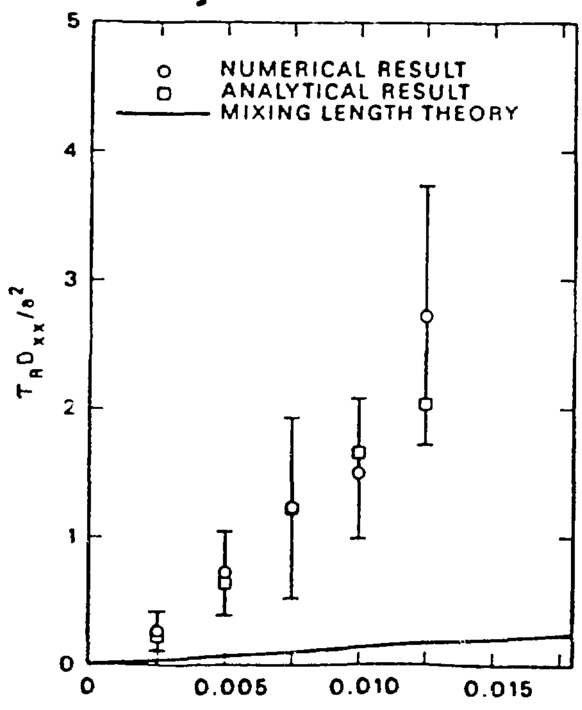
NONLINEAR RESISTIVE INTERCHANGE CALCULATIONS

* Main results : (Previous Work)

i) The numerical calculations agree well with the analytical results



ii) Mixing length results scale with β like the results of the nonlinear calculation but the value is lower by about a factor of 8



II. Study of Wavenumber Spectrum

* **Theoretical Study :** Renormalized Two-Point Theory.
From previous work of RPGDT ; See previous page

- Two-point Correlation functions (Quadratic Quantities)

$\langle \tilde{u}\ \tilde{u} \rangle$: Enstrophy-like
$\langle \tilde{p}\ \tilde{p} \rangle$: Internal Energy
$\langle \tilde{\phi}\ \tilde{u} \rangle$: Kinetic Energy
& Cross Correlations $\langle \tilde{p}\tilde{\phi} \rangle$, $\langle \tilde{u}\tilde{p} \rangle$

- Time-Evolution of Two-point correlations

Vorticity-Vorticity Correlation (Enstrophy-like)

$$\frac{\partial}{\partial t}\langle \tilde{u}\tilde{u} \rangle - \mu_\perp (\nabla_{\perp 1}^2 + \nabla_{\perp 2}^2)\langle \tilde{u}\tilde{u} \rangle + T_u$$
$$= -\frac{1}{\eta \rho_m}\left[\langle \tilde{u}\ \nabla_{\parallel 2}^2 \tilde{\phi} \rangle + \langle \tilde{u}\ \nabla_{\parallel 2}^2 \tilde{\phi} \rangle \right]$$
$$+ \frac{1}{\rho_m}\frac{d\Omega_0}{dr}\left[\langle \tilde{u}\ \nabla_{y_1} \tilde{p} \rangle + \langle \tilde{u}\ \nabla_{y_2} \tilde{p} \rangle \right]$$

Pressure-Pressure Correlation (Energy-like)

$$\frac{\partial}{\partial t}\langle \tilde{p}\tilde{p} \rangle - \chi_\perp (\nabla_{\perp 1}^2 + \nabla_{\perp 2}^2)\langle \tilde{p}\tilde{p} \rangle + T_p$$
$$= -\frac{dP_0}{dr}\left[\langle \tilde{p}\ \nabla_{y_1} \tilde{\phi} \rangle + \langle \tilde{p}\ \nabla_{y_2} \tilde{\phi} \rangle \right]$$

Triplets :

$$T_u = \langle \nabla_{\perp 1} \tilde{\phi}(1) \times \hat{z} \cdot \nabla_{\perp 1} \tilde{u}(1)\tilde{u}(2) \rangle + \langle 1 \leftrightarrow 2 \rangle$$
$$T_p = \langle \nabla_{\perp 1} \tilde{\phi}(1) \times \hat{z} \cdot \nabla_{\perp 1} \tilde{p}(1)\tilde{p}(2) \rangle + \langle 1 \leftrightarrow 2 \rangle$$

* **Renormalized Theory & Predictions :**

• Renormalization of Triplets in Relative Coordinate.

⇒ Using weak-coupling closure with directly beated driven solutions and ensemble average.

• Renormalized Two-point Correlation Evolution Eq.

Pressure-Pressure Correlation

$$\frac{\partial}{\partial t}\langle \tilde{P}\tilde{P}\rangle_- - \frac{\partial}{\partial x_-}\left(D_-^{xx} + 2\chi_{10}\right)\frac{\partial}{\partial x_-}\langle \tilde{P}\tilde{P}\rangle_- - \frac{\partial}{\partial y_-}\left(D_-^{yy} + 2\chi_{10}\right)\frac{\partial}{\partial y_-}\langle \tilde{P}\tilde{P}\rangle_-$$

$$= \langle S_\varepsilon \rangle_-$$

with

$$D_-^{xx} = 2D^{xx} - \left(D_{(1,2)}^{xx} + D_{(2,1)}^{xx}\right)$$

$$D_-^{yy} = 2D^{yy} - \left(D_{(1,2)}^{yy} + D_{(2,1)}^{yy}\right)$$

Vorticity-Vorticity Correlation

$$\frac{\partial}{\partial t}\langle \tilde{u}\tilde{u}\rangle_- - \frac{\partial}{\partial x_-}\left(\mu_-^{xx} + 2\mu_{10}\right)\frac{\partial}{\partial x_-}\langle \tilde{u}\tilde{u}\rangle_-$$

$$- \frac{\partial}{\partial y_-}\left(\mu_-^{yy} + 2\mu_{10}\right)\frac{\partial}{\partial y_-}\langle \tilde{u}\tilde{u}\rangle_-$$

$$+ \frac{\partial}{\partial x_-} C_-^{xx}\frac{\partial}{\partial x_-}\langle \tilde{u}\tilde{\phi}\rangle_- + \frac{\partial}{\partial y_-} C_-^{yy}\frac{\partial}{\partial y_-}\langle \tilde{u}\tilde{\phi}\rangle_-$$

$$= \langle S_v \rangle_- - \langle R_v \rangle_-$$

with

$$\mu_-^{xx} = 2\mu^{xx} - \left(\mu_{(1,2)}^{xx} + \mu_{(2,1)}^{xx}\right)$$

$$C_-^{xx} = 2C^{xx} - \left(C_{(1,2)}^{xx} + C_{(2,1)}^{xx}\right)$$

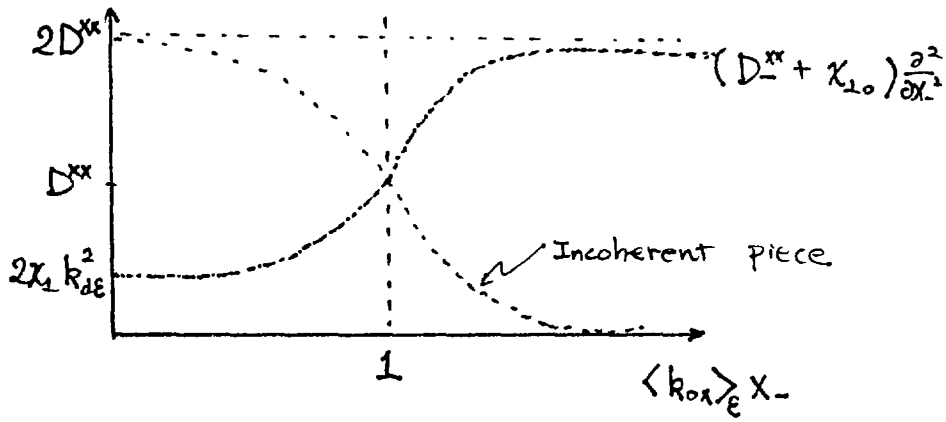
- Spatial Structure of Renormalized Triplets

⇒ Asymptotic limits in $\mathbf{x}_-$ show very different evolution characteristics of $<\tilde{U}\hat{U}>$ and $<\tilde{p}\tilde{p}>$ correlations.

⇒ We are mainly interested in a wavenumber spectrum of energy-like correlation function.

$$* \quad \mathcal{L}_-^{\varepsilon} \langle \tilde{P}\tilde{P} \rangle_- = \langle S_\varepsilon \rangle_-$$

$[\mathcal{L}_-^{\varepsilon}]$



$\langle k_{ox} \rangle_\varepsilon$: rms radial Correlationlength over energy-like spectrum (with $<\tilde{P}\tilde{P}>_{\vec{k}}$).

$k_{d\varepsilon}$: dissipation - range wavenumber for energy-like correlation (with χ_{1o} & D^{xx}).

$$\Rightarrow \quad \langle \tilde{P}\tilde{P} \rangle_{\vec{k}} = \int d\vec{x}_- \; \tau_{cl}^{\varepsilon}(\vec{x}_-) \langle S_\varepsilon \rangle_- \, e^{-i\vec{k}\cdot\vec{x}_-}$$

where two-point correlation time

$$\tau_{cl}^{\varepsilon}(\vec{x}_-) = [\mathcal{L}_-^{\varepsilon}]^{-1}$$

⇒ For $\langle \tilde{U}\tilde{U} \rangle$ - evolution, the spatially inhomogeneous characteristics of triplets in $\mathbf{x}_- \approx 0$ and consevation properties of nonlinearity in $\mathbf{x}_- \geq 1$ show :

$$* \quad \underline{\mathcal{L}_-^{V} \langle \tilde{u}\tilde{u} \rangle_- = \langle S_V \rangle_- - \langle R_V \rangle_-}$$

$|\vec{k}_{0\varepsilon} \cdot \vec{X}_-| \ll 1$: $\mu_-^{xx}, \mu_-^{yy}, C_-^{xx}, C_-^{yy} \to 0$.
(due to incoherent contribution)

$|\vec{k}_{0\varepsilon} \cdot \vec{X}_-| > 1$: μ_-^{xx}, μ_-^{yy} terms are cancelled by C_-^{xx}, C_-^{yy} terms.
(Due to Conservation property of N_E

⇒ Because C_-^{xx}, C_-^{yy} terms act as nonlinear Source of $\langle \tilde{u}\tilde{u} \rangle$ correlation in $\mathcal{L}_-^{V}$ -operator, and by noting ($k_{0\varepsilon} < k_{0V}$: two different evolution proce at steady-state , the spatial dependence of $\mathcal{L}_-^{V}$ are shown to be very weak compare to $\mathcal{L}_-^{\varepsilon}$ in $\underline{|\vec{k}_{0\varepsilon} \cdot \vec{x}_-| \simeq 1}$. ($\langle \tilde{u}\tilde{u} \rangle_k \sim k^0$ expected)

⇒ It gives the steady-state condition of Spectrum in the range of $|\vec{k}_{0\varepsilon} \cdot \vec{x}_-| \simeq 1$ by

$$\underline{\langle R_V \rangle_- \simeq \langle S_V \rangle_-} .$$

: Saturation Criteria which derived in 'CGD' is applicable in this balance through two correlation length-Scale (Δ_m and δ_m).

⇒ For steady-state spectrum of the energy-like two-point functions, it is needed to invert evolution operator $\mathcal{L}^{\varepsilon}_{-}$ by Green's function method on it's moments.

$$* \quad \langle \tilde{p}\tilde{p} \rangle_{-} = \tau^{\varepsilon}_{cl}(\vec{X}_{-}) \langle S_{\varepsilon} \rangle_{-}$$

$$\tau^{\varepsilon}_{cl}(\vec{X}_{-}) = -\frac{\tau^{\varepsilon}_{c}}{4} \ln\left[\frac{1}{(1+R_{\varepsilon})} + \frac{(k_{ox}^{2}X_{-}^{2} + k_{oy}^{2}y_{-}^{2} + k_{oz}^{2}z_{-}^{2})}{(1+R_{\varepsilon}^{-1})}\right]$$

where $\tau^{\varepsilon}_{c} = (D^{xx}k_{ox}^{2} + D^{yy}k_{oy}^{2})$

: Coherence time (one-point) for energy-like

$$R_{\varepsilon} = \left[\frac{D^{xx}k_{ox}^{2} + D^{yy}k_{oy}^{2}}{\chi_{\perp 0}(k_{ox}^{2} + k_{oy}^{2})}\right]$$: Effective Reynolds # for energy-like.

* Fourier Transform

$$\langle \tilde{p}\tilde{p} \rangle_{\vec{k}} = \frac{\tau^{\varepsilon}_{c}}{2k_{oy}k_{oz}} \langle S_{\varepsilon} \rangle_{0} \int_{0}^{1} \rho\, d\rho\, [J_{0}(\beta\rho)] \times \left\{\sqrt{1-\rho^{2}} - \sqrt{R_{\varepsilon}^{-1}+\rho^{2}}\tan^{-1}\left(\frac{\sqrt{1-\rho^{2}}}{\sqrt{R_{\varepsilon}^{-1}+\rho^{2}}}\right)\right\}$$

* Spectrum-decay index (Inertial Range, Summation over k_z)

$$\langle \tilde{p}\tilde{p} \rangle_{k_y} = k_y^{-2}\left(1 - J_{0}(k_y/k_{oy})\right)\left[\pi k_{oy} \tau^{\varepsilon}_{c} \langle S_{\varepsilon} \rangle_{0}\right]$$

* **Theoretical Predictions :**

Examining $\langle \tilde{p}\tilde{p} \rangle_{k_y}$ at three different regions of k_y and steady-state condition for spectrum balance

$$* \quad \langle \tilde{p}\tilde{p} \rangle_{k_y} \sim \begin{cases} k_y^{0} & : k_y < k_{oy} \\ k_y^{-2\sim-2.5} & : k_y \simeq k_{oy} \\ k_y^{-1.5} & : k_y \simeq k_d \end{cases}$$

$$* \quad \langle \tilde{\phi}\tilde{\phi} \rangle_{k_y} \sim k_y^{-\frac{19}{6}} \sim k_y^{-4.} \quad : k_y \simeq k_{oy}$$

* **Numerical Study :** Using Initial Value code 'KITE'.

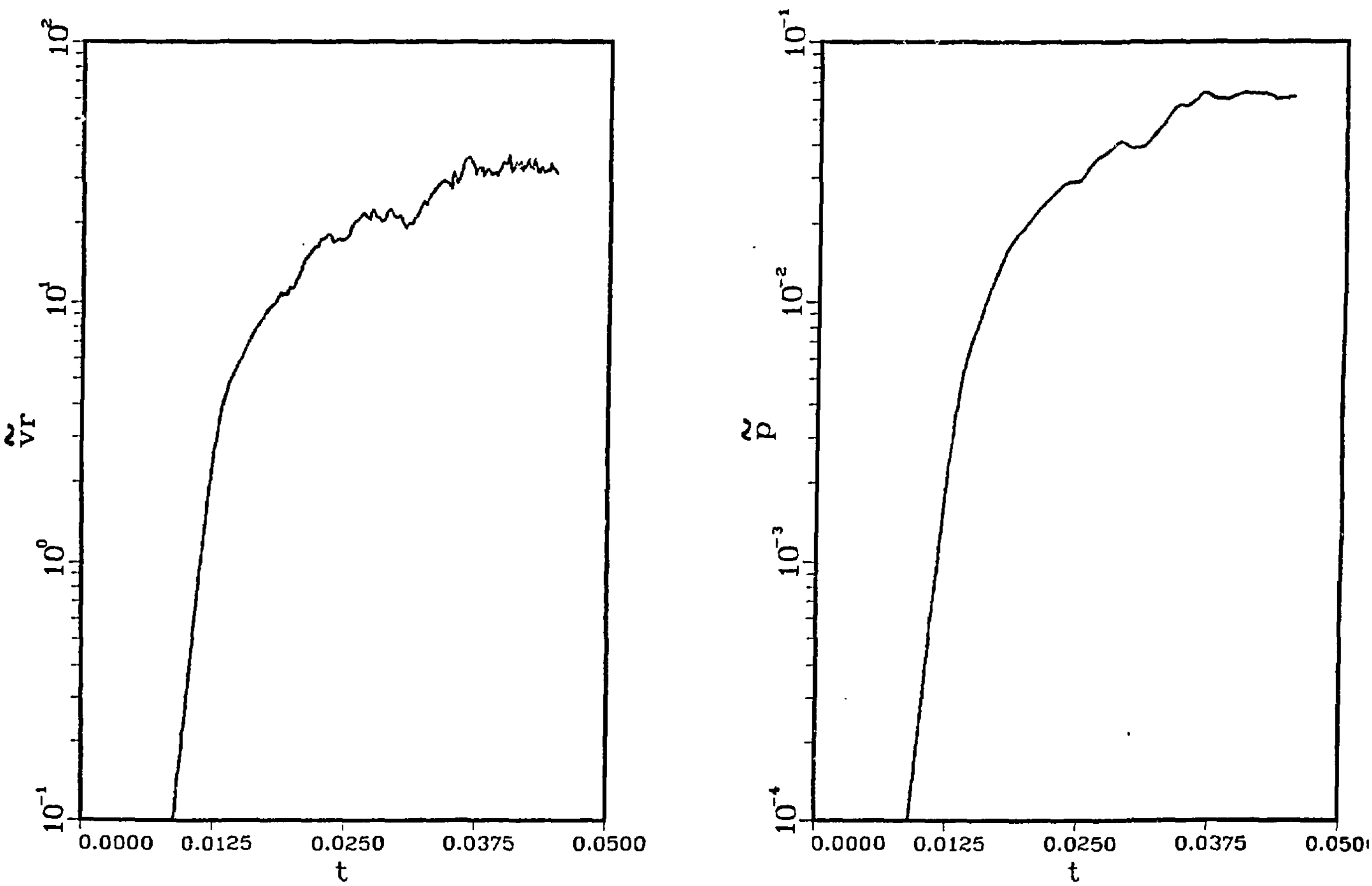
Cases Presented : $S = 10^5$

- $\beta_0/2\epsilon^2$: 1.0×10^{-2} and 1.25×10^{-2}
- # of modes (m, n) : 550.
- # of grids in radial : 440.
- # of different helicity : 300.

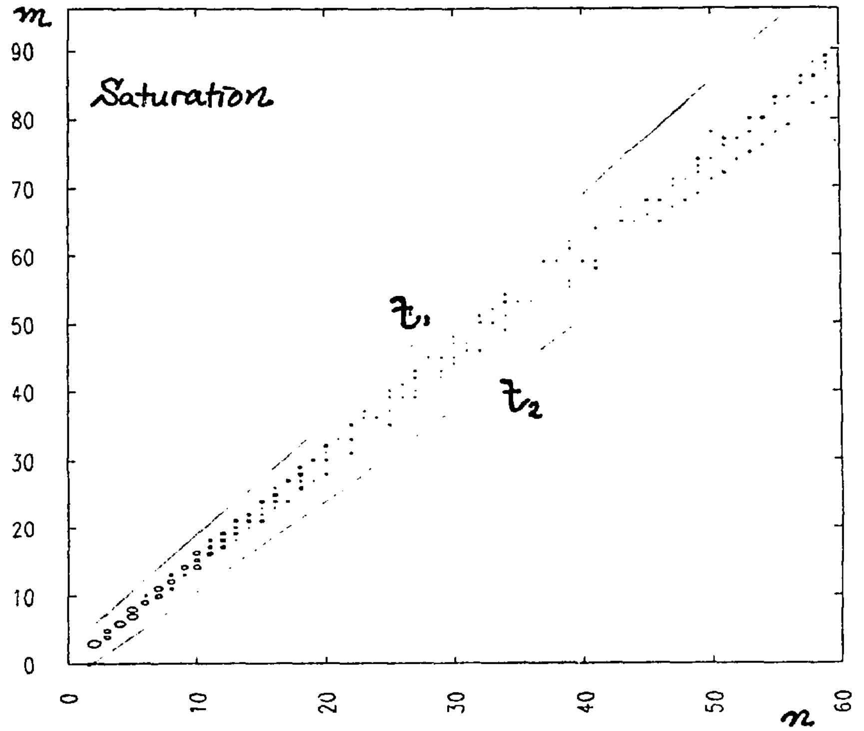
$\Rightarrow$ Spectrum was taken $t = 0.034 \sim 0.036$
(Well after Saturation)

- Time average over fraction of τ_c
- Local in radial position & Integrated in r.

At saturation,



- Energy in Modes



- main helicity

  $m/n = 3/2$

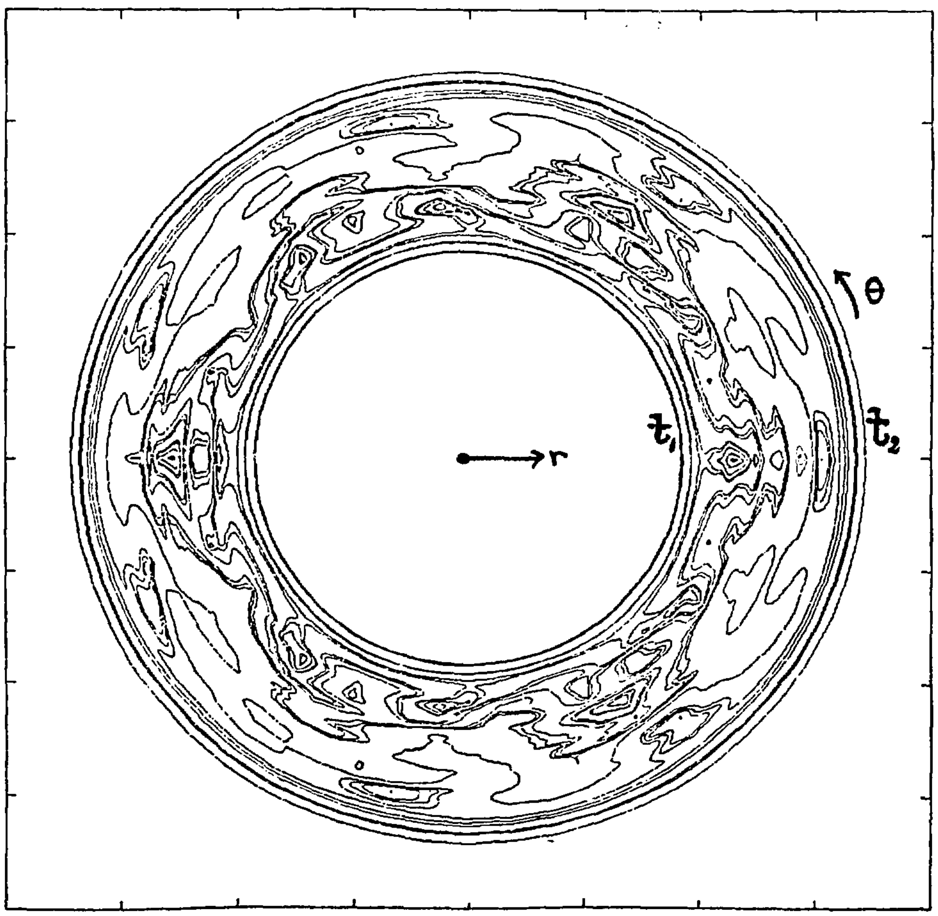
- $r = r_s(3/2)$

  0.5228 a

- 550 (m,n)'s

  with ~ 300 helicities

- Spatial Structure at Saturation



- ***m*** **- wavenumber spectrum :**
  (summed over ***n*** -number)

At radial position r = 0.523 (near q =3/2),
energy-like two-point correlation $<\tilde{p}\tilde{p}>$ exhibits,

Case :

$$S = 10^5$$

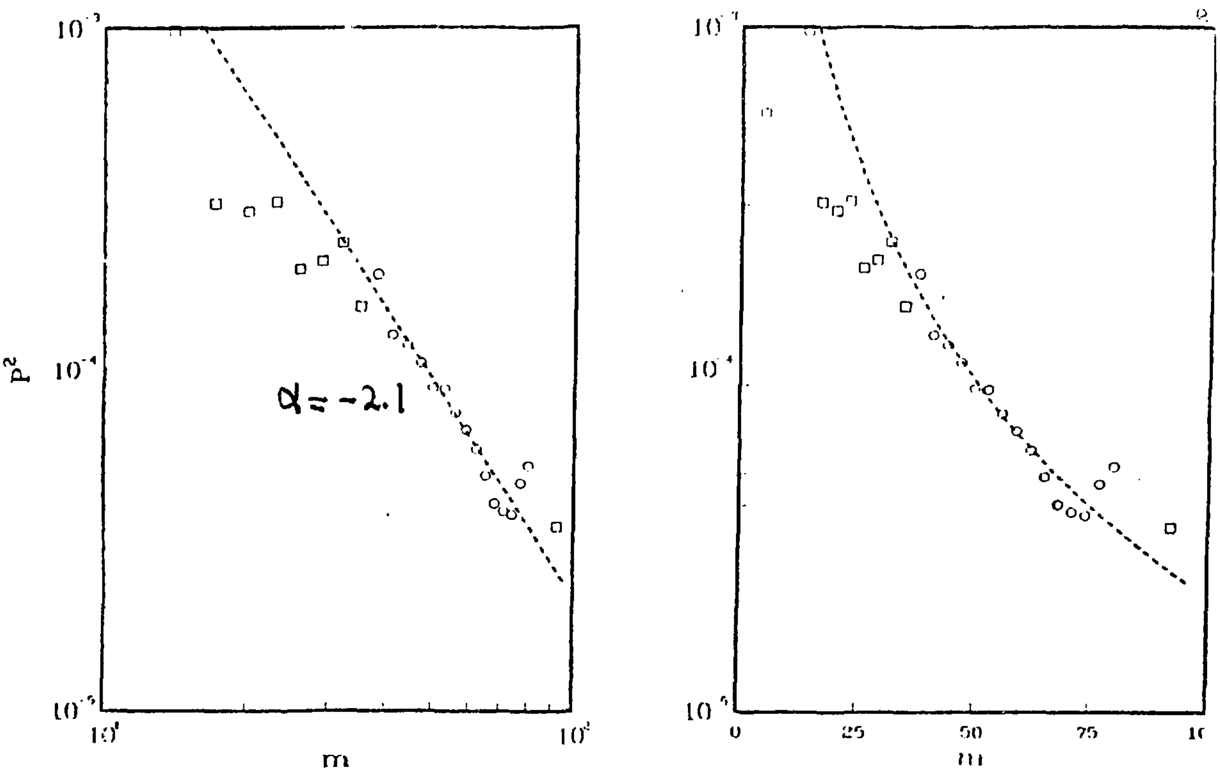
$$\beta_0/2\epsilon^2 = 1.25 \times 10^{-2}$$

Spectrum Decay-Index (m : 35 ~ 80) ooo

* $\langle \tilde{P}\tilde{P} \rangle_{k_y} \simeq k_y^{-2.1}$

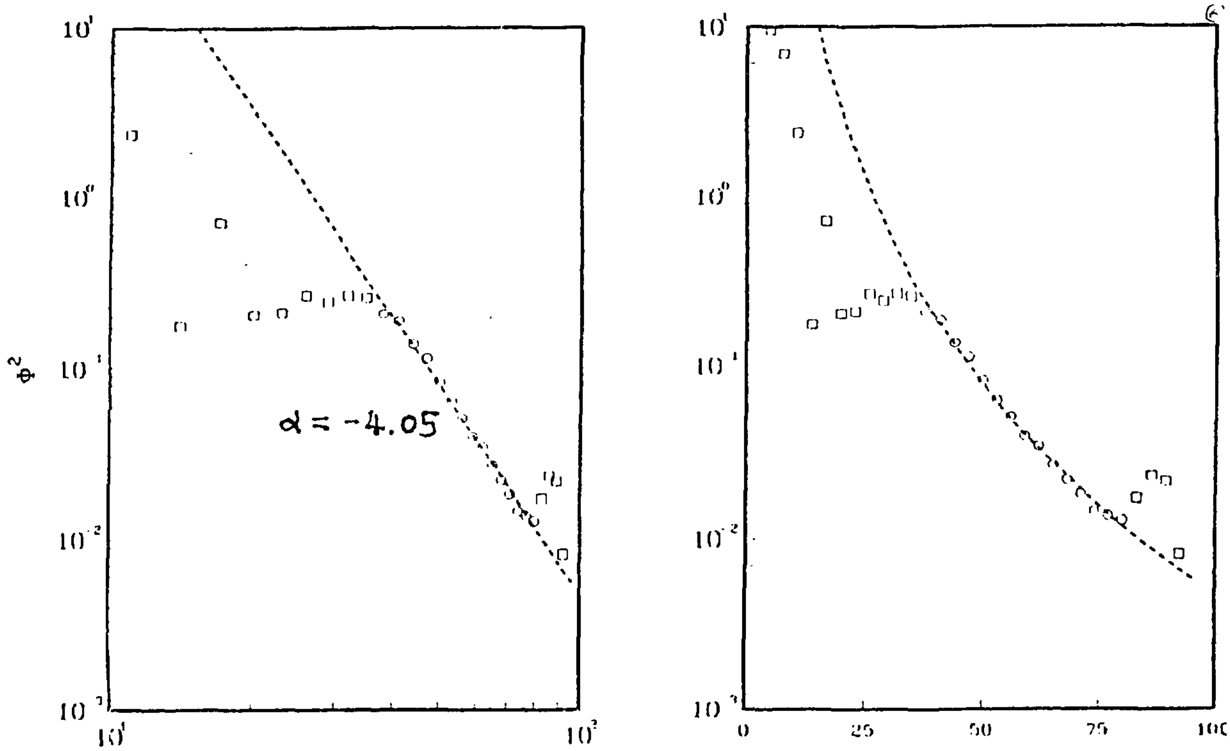
* $\langle \tilde{\phi}\tilde{\phi} \rangle_{k_y} \simeq k_y^{-4.05}$

* $\langle \tilde{U}\tilde{U} \rangle_{k_y} \simeq k_y^{-0.1}$ (flat)

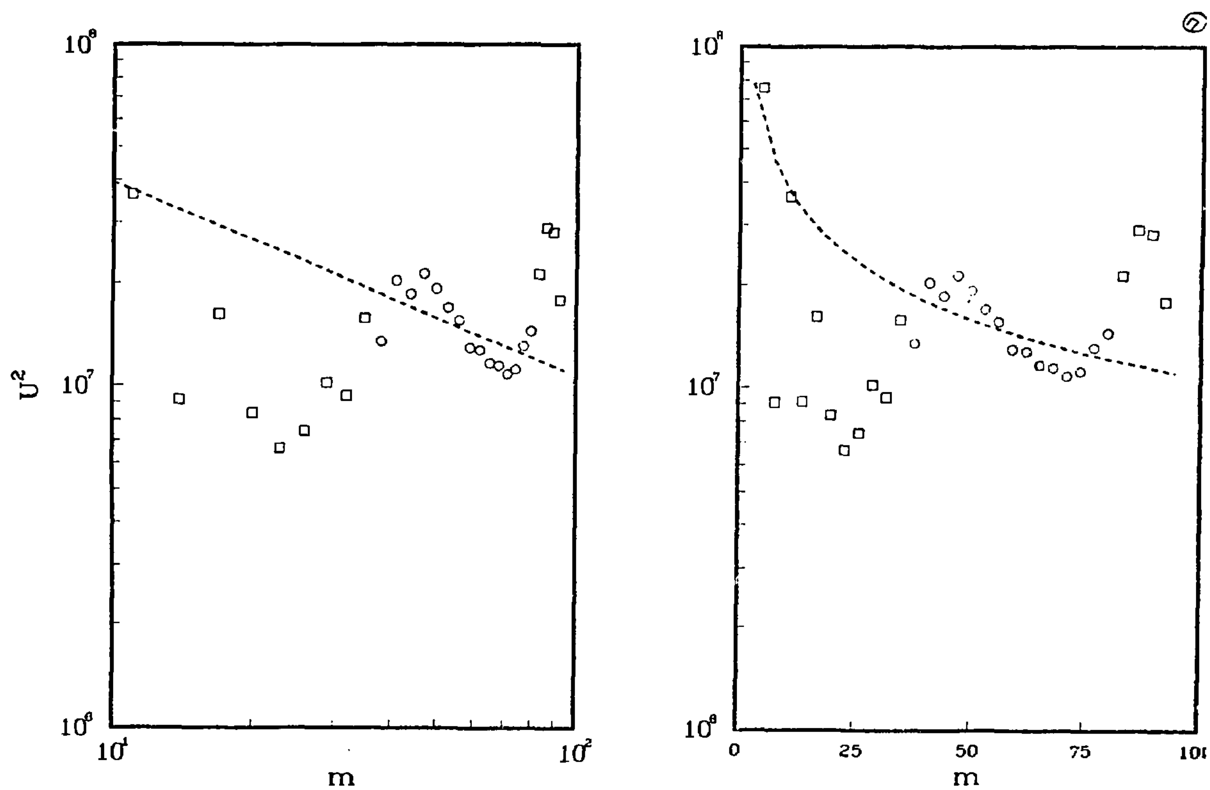


• *m* - wavenumber spectrum :

At radial position $r = 0.523$ (near $q = 3/2$),

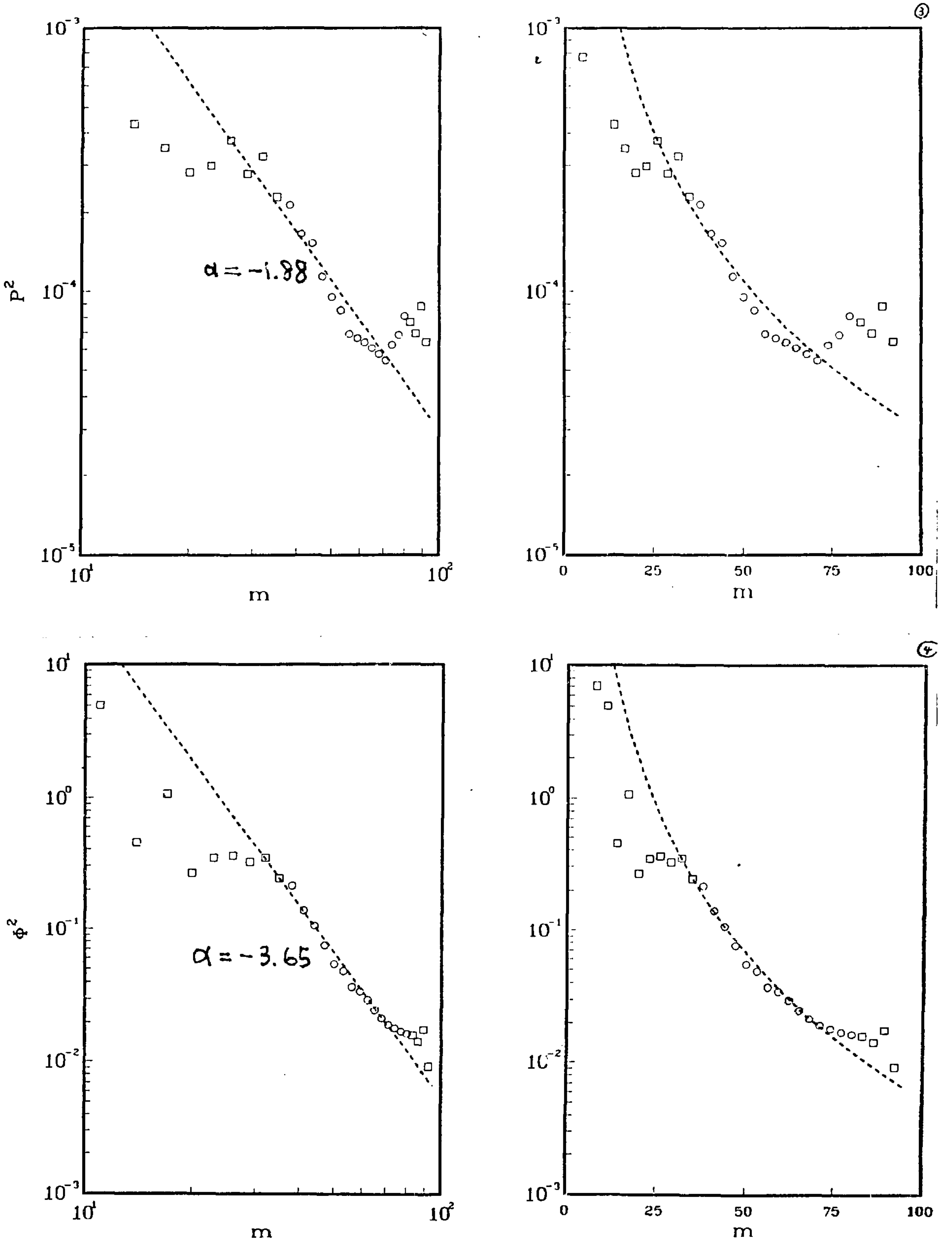


* Flat Spectrum $\langle \tilde{u}\tilde{u} \rangle_{ky}$: $m \approx \langle m \rangle_\varepsilon$



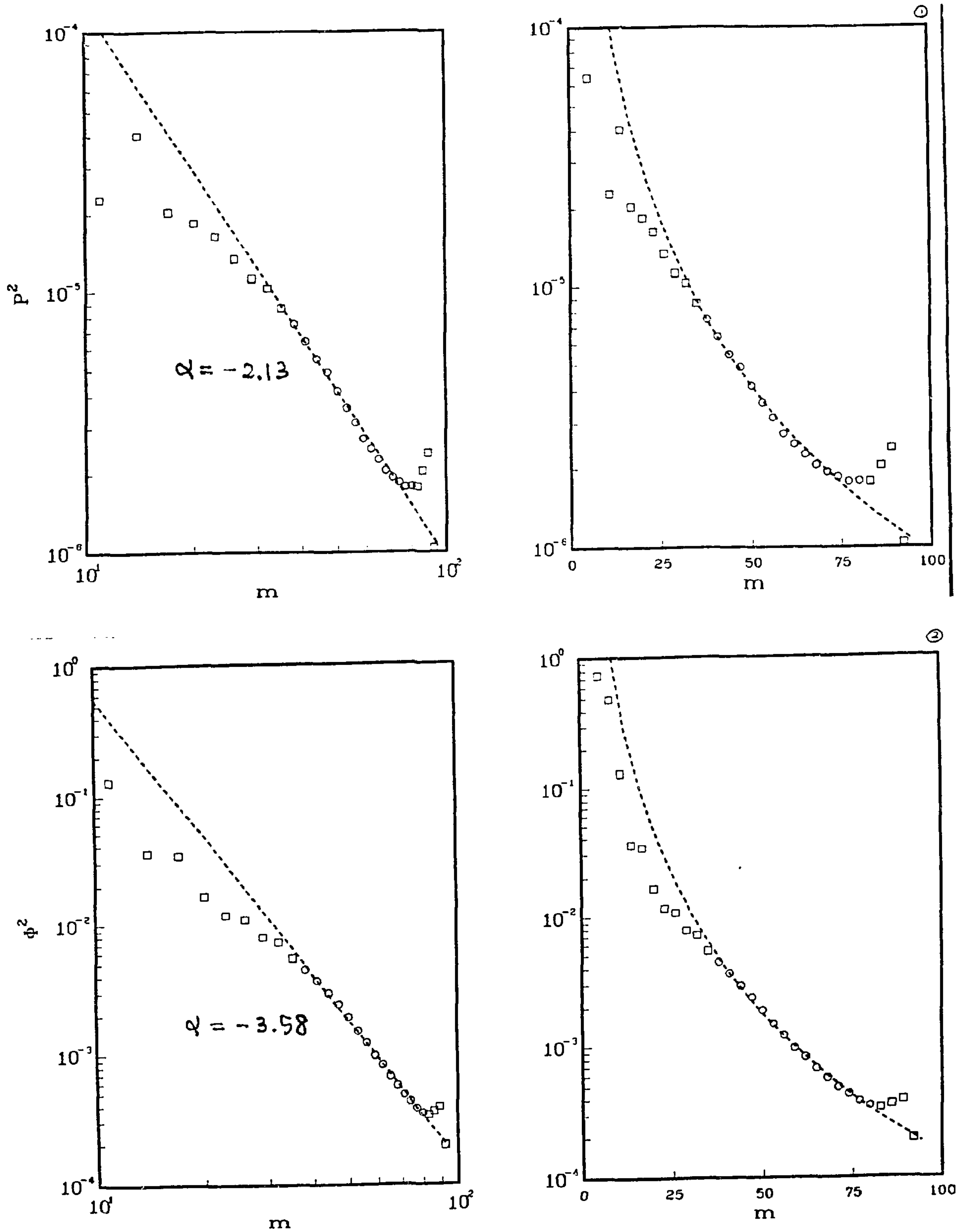
• ***m* - wavenumber spectrum :** Similar Decay-Index

At radial position r = 0.511 (q = 1.52),



• *m* - wavenumber spectrum (radially integrated) :

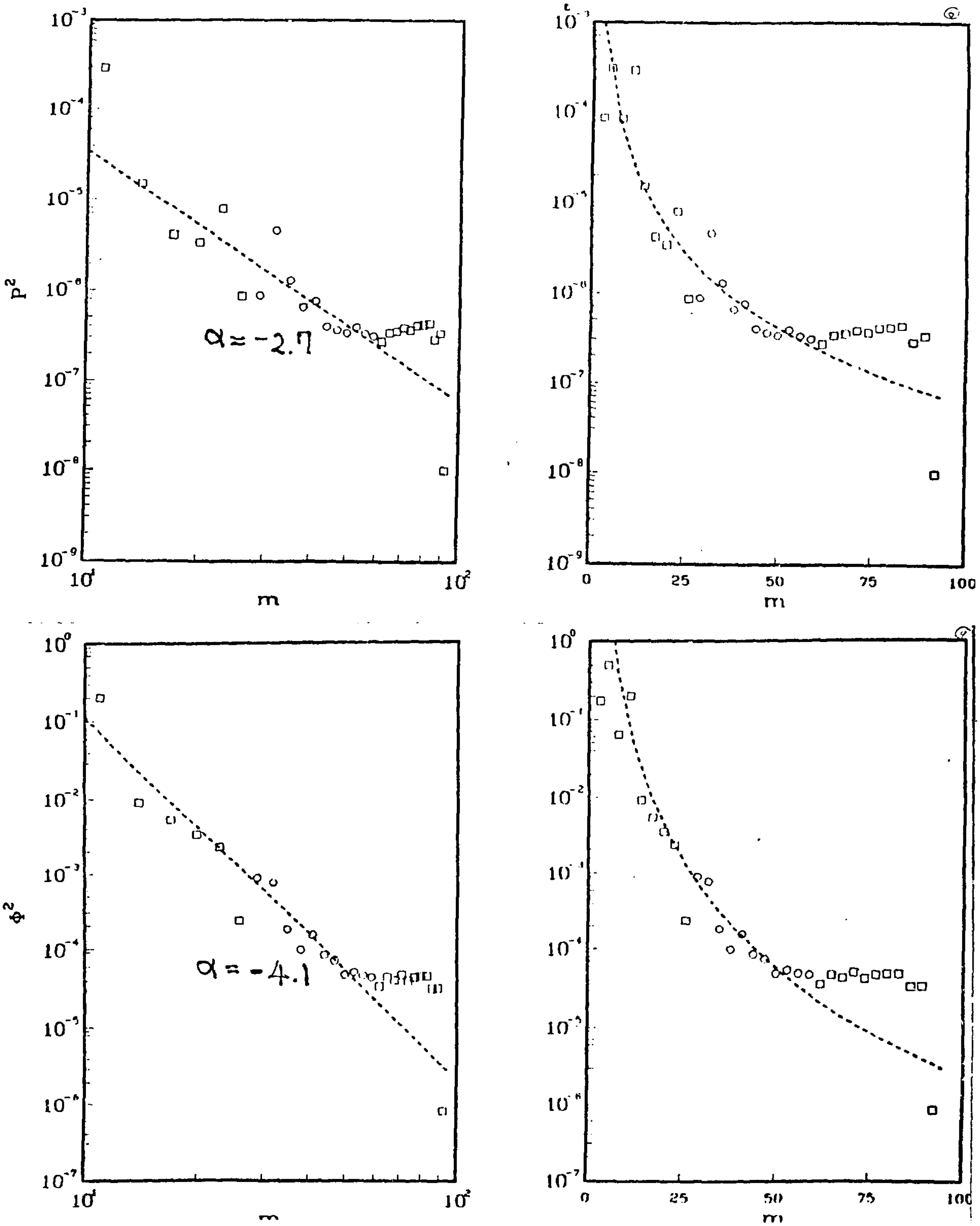
Same Case: $\beta_0/2\epsilon^2 = 1.25 \times 10^{-2}$



• *m* - wavenumber spectrum (radially integrated) :

Case : $S = 10^5$, $\beta_0 / 2\epsilon^2 = 1.0 \times 10^{-2}$

ooo m - range (25 – 60) for decay - Index



III. EM - Model with T_e Evolution

* **Basic Equations :** Extended version of reduced set of resistive MHD equations with temperature evolution in cylindrical geometry (r, θ, ζ).

$$\partial_t \tilde{\Psi} = -\mathbf{v}_\perp \cdot \nabla_\perp \Psi - \partial_\zeta \Phi + S^{-1}(\eta J_\zeta - E^W_\zeta)$$

$$\partial_t \tilde{U} = -\mathbf{v}_\perp \cdot \nabla_\perp \tilde{U} - \nabla_{||} J_\zeta + \chi_U \nabla_\perp^2 \tilde{U} + \underline{\hat{\zeta} \bullet (\nabla_\perp \Omega_0 \times \nabla_\perp \tilde{\mathbf{p}})}$$

$$\partial_t \tilde{\rho} = \underline{-\mathbf{v}_\perp \cdot \nabla_\perp \rho} + \chi_\rho \nabla_\perp^2 (\rho - \rho_0)$$

$$\partial_t \tilde{T}_e = -\mathbf{v}_\perp \cdot \nabla_\perp T_e + \rho^{-1} \nabla_{||} \underline{(\chi_{||} \rho \nabla_{||} T_e)} + \chi_\perp \nabla_\perp^2 (T_e - T_{e0})$$

where $\mathbf{v}_\perp = \nabla_\perp \tilde{\Phi} \times \zeta$ and $\underline{\nabla_{||} = \partial_\zeta - (\nabla \Psi_T \times \zeta) \cdot \nabla}$

$$J_\zeta = \nabla_\perp^2 \Psi \text{ and } \tilde{U} = \nabla_\perp \cdot (\nabla_\perp \tilde{\Phi})$$

$$S = \tau_R / \tau_H \text{ with } \tau_R = \mu_0 a^2 / \eta(0)$$

$$\tau_H = R / V_A$$

* Here, the couplings between $\tilde{\Psi}$ and $\tilde{T}_e$ are kept minimally, because of computational constraints. It will be nessesary to keep symmetry-breaking terms for self-consistent evolution of EM - turbulence.
* <u>In this study, we investigate the effect of magnetic fluctuations on thermal transport due to RPGDT.</u>

* Nonlinear Theory :

- Emphasis is on $\rho^{-1}\nabla_{\|}(\chi_{\|}\rho\,\nabla_{\|}T_e)$ - nonlinearity.
- Weak coupling closure.

$$* \ \frac{\partial}{\partial t}\tilde{\psi}_{\vec{k}} + N_{\psi\vec{k}} = -ik_{\|}\tilde{\phi}_{\vec{k}} + \hat{\eta}(\nabla_{\perp}^2\tilde{\psi})_{\vec{k}}$$

$$* \ \frac{\partial}{\partial t}\tilde{U}_{\vec{k}} + N_{\phi\vec{k}} = -ik_{\|}(\nabla_{\perp}^2\tilde{\psi})_{\vec{k}} + N_{J\vec{k}} + \hat{\mu}\,\nabla_{\perp}^2(\nabla_{\perp}^2\tilde{\phi})_{\vec{k}} + ik_y(\tilde{n}_{\vec{k}} + \tilde{T}_{\vec{k}})\frac{d\Omega_0}{dr}$$

$$* \ \frac{\partial}{\partial t}\tilde{n}_{\vec{k}} + N_{n\vec{k}} + ik_y\frac{dn_0}{dr}\tilde{\phi}_{\vec{k}} = 0$$

$$* \ \frac{\partial}{\partial t}\tilde{T}_{\vec{k}} + N_{t\vec{k}} + ik_y\frac{dT_0}{dr}\tilde{\phi}_{\vec{k}} = \hat{\chi}_{\perp}(\nabla_{\perp}^2\tilde{T})_{\vec{k}} + \hat{\chi}_{\|}\left\{\left(k_y k_{\|}\frac{dT_0}{dr}\right)\tilde{\psi}_{\vec{k}} - k_{\|}^2\tilde{T}_{\vec{k}} + \frac{dT_0}{dr}H_{1\vec{k}} - ik_{\|}H_{2\vec{k}} - H_{3\vec{k}} + \mathbb{C}_{\vec{k}}\right\}$$

where

- $N_{\vec{k}}$: Convective Nonlinearities except
- $N_{J\vec{k}} = \sum_{\vec{k}'}\left[(\nabla\tilde{\psi}\times\hat{z})\cdot\nabla_{\perp}\tilde{J}\right]$
- $$\begin{cases} H_{1\vec{k}} = \sum_{\vec{k}'}\left[(\nabla\tilde{\psi}\times\hat{z})\cdot\nabla_{\perp}(\nabla_y\tilde{\psi})\right] \\ H_{2\vec{k}} = \sum_{\vec{k}'}\left[(\nabla\tilde{\psi}\times\hat{z})\cdot\nabla_{\perp}\tilde{T}\right] \\ H_{3\vec{k}} = \sum_{\vec{k}'}\left[\nabla\tilde{\psi}\times\hat{z}\cdot\nabla_{\perp}(\nabla_{\|}^{s}\tilde{T})\right] \end{cases}$$
- $\mathbb{C}_{\vec{k}} = \overset{\Delta}{\sum\sum}\left\{\left[(\nabla\tilde{\psi}\times\hat{z})\cdot\nabla_{\perp}\right](\nabla_{\perp}\tilde{\psi}\times\hat{z}\cdot\nabla_{\perp}\tilde{T})\right\}$

  : Cubic Nonlinearity

* Driven - Mode Solution & Renormalization :

With g-mode parity and nonlinear source terms,

$$\left\{\begin{aligned} \tilde{\psi}^{(2)}_{\vec{k}''} &\simeq \frac{S_\psi}{\Gamma_{\psi\vec{k}''}} - i\,\frac{\chi\,\ln(1+K_\phi K_\psi \chi''^2)}{2K_\phi}\left(\frac{S_u}{\Gamma_{\phi\vec{k}''}}\right) \Rightarrow \frac{S_\psi}{\Gamma_{\psi\vec{k}''}} \\ \tilde{J}^{(2)}_{\vec{k}''} &\simeq -\frac{iK_\psi\chi''}{(1+K_\phi K_\psi \chi''^2)}\left(\frac{S_u}{\Gamma_{\phi\vec{k}''}}\right) \longrightarrow -iK_\psi\chi''\,\frac{S_u}{\Gamma_{\phi\vec{k}''}} \end{aligned}\right.$$

$$\left\{\begin{aligned} \tilde{\Phi}^{(2)}_{\vec{k}''} &\simeq \frac{\ln(1+K_\phi K_\psi \chi''^2)}{2K_\phi K_\psi}\left(\frac{S_u}{\Gamma_{\phi\vec{k}''}}\right) \Rightarrow 0 \\ \tilde{U}^{(2)}_{\vec{k}''} &\simeq \frac{1}{(1+K_\phi K_\psi \chi''^2)}\left(\frac{S_u}{\Gamma_{\phi\vec{k}''}}\right) \longrightarrow \frac{S_u}{\Gamma_{\phi\vec{k}''}} \end{aligned}\right.$$

- $$\tilde{n}^{(2)}_{\vec{k}''} \simeq \frac{S_n}{\Gamma_{n\vec{k}''}}$$

- $$\tilde{T}^{(2)}_{\vec{k}''} \simeq \frac{S_T}{\Gamma_{T\vec{k}''}} + \underline{\hat{\chi}_{\parallel}\,\frac{k_{\parallel}''\,k_y''\,\frac{dT_0}{dr}}{\Gamma_{T\vec{k}''}}\left(\frac{S_\psi}{\Gamma_{\psi\vec{k}''}}\right)}$$

<u>where</u>

S_ψ, S_u, S_n, S_T : Nonlinear Source from direct beating of $\vec{k}'$ with $\vec{k}$

$\Gamma_{\psi\vec{k}''}, \Gamma_{\phi\vec{k}''}, \Gamma_{n\vec{k}''}, \Gamma_{T\vec{k}''}$: Propagators (including decorrelation rate)

with

$$\left.\begin{aligned} K_\phi &\equiv \frac{k_y''}{\Gamma_{\phi\vec{k}''} L_s} \\ K_\psi &\equiv \frac{k_y''}{\Gamma_{\psi\vec{k}''} L_s} \end{aligned}\right\} \text{related with inverse of inertial layer width}$$

$\Rightarrow$: $\left|\frac{\chi''}{\chi''_\phi}\right| \ll 1$ with finite $\left|\frac{\chi''}{\chi''_\psi}\right|$ limit.

* **Renormalized Eq. & Saturation :**

Because of electrostatic characteristic of RPGDT, all nonlinear terms except heat flux term $Q_{\parallel}$ and fluid - flux interaction terms ($\tilde{\Psi}$, $\tilde{J}_{\zeta}$ terms) have similar effect and interpretation as ES case (CGD).

$$N_{\phi\vec{k}}\ ,\ N_{n\vec{k}}\ ,\ N_{T\vec{k}} \quad \propto \quad |\tilde{v}_r|^2 \quad \text{and} \quad |\tilde{v}_\theta|^2$$

- Fluid - Flux Interaction Terms : $N_{J\vec{k}}$, $N_{T\vec{k}}$
  - Nonlinear Flux Diffusivity (resistivity) : $\tilde{v}_r$, $\tilde{v}_\theta$

$$\sim \frac{\partial}{\partial r}\eta^{xx}_{\vec{k}}\frac{\partial}{\partial r}\tilde{\psi}_{\vec{k}} - k_y^2\,\eta^{yy}_{\vec{k}}\,\tilde{\psi}_{\vec{k}}$$

$$+\frac{\partial}{\partial r}\Sigma^{xx}_{\vec{k}}\frac{\partial}{\partial r}\tilde{J}_{\vec{k}} - k_y^2\,\Sigma^{yy}_{\vec{k}}\,\tilde{J}_{\vec{k}}$$

: Due to convection of flux by turbulent fluids

  - Nonlinear Viscosity : $\tilde{b}_r$, $\tilde{b}_\theta$

$$\sim \frac{\partial}{\partial r}\alpha^{xx}_{\vec{k}}\frac{\partial}{\partial r}\tilde{\phi}_{\vec{k}} - k_y^2\,\alpha^{yy}_{\vec{k}}\,\tilde{\phi}_{\vec{k}}$$

: Due to Alfvènic effect on fluid vorticity

- Saturation Condition

For $\tilde{V}_r$ and $\tilde{n}$, same as ES case. (CGD paper)
By using steady-state condition, $\tilde{T}_e$, $\tilde{\Psi}$ and $\tilde{J}_{\zeta}$ can be related.

- Fluid Form of Heat Diffusivity due to Parallel Heat Conduction of Electron with $\tilde{B}$.

: Using saturation condition,

$$Q_{\parallel} = (Q_{\parallel})_{\phi} + (Q_{\parallel})_{\psi} + (Q_{\parallel})_{t}$$

$$\Rightarrow (Q_{\parallel})_{t} = -\hat{\chi}_{\parallel} k_{\parallel}^{2} T_{\vec{k}} - \hat{\chi}_{\parallel} \left\{ i k_{\parallel} (H_{2\vec{k}})_{t} + (H_{3\vec{k}})_{t} \right\} + \hat{\chi}_{\parallel} \left[\mathbb{C}_{\vec{k}} \right]_{t}$$

: Subscript - t denotes terms with $[\cdots]\tilde{T}_{\vec{k}}$

$\Rightarrow$ Quadratic Term : $\{ i k_{\parallel} (H_{2\vec{k}})_{t} + (H_{3\vec{k}})_{t} \} \simeq 0.$

$\Rightarrow$ Cubic Term (Following Cubic-DIA)

$$\left[\mathbb{C}_{\vec{k}} \right]_{t} \simeq + \frac{\partial}{\partial r} \left[X_{\vec{k}}^{xx} \right] \frac{\partial}{\partial r} \tilde{T}_{\vec{k}} - k_{y}^{2} X_{\vec{k}}^{yy} \tilde{T}_{\vec{k}}$$

where anomalous cross-field heat conductivity $X_{\vec{k}}$'s depend on $|\tilde{b}_{\theta}|^{2} |\tilde{v}_{r}|^{2}$ and $|\tilde{b}_{r}|^{2} |\tilde{v}_{\theta}|^{2}$.

IV. Discussion & Summary

- **Spectrum Study**

- In order to study difference between kinetic energy wavenumber spectrum $<\tilde{\Phi}\,\tilde{U}>$ and internal energy wavenumber spectrum $<\tilde{p}\,\tilde{p}>$, it is needed to invert $\nabla_{\perp}^2$- operator in driven potential. It is expected to give small effect on spectrum shape, in general.

  However, this will allow us better evaluation of the source function (so spectrum-integrated diffusivity), and saturation condition in terms of spectrum balance.

- Parallel wavenumber ($k_{\parallel}$) spectrum of RPGDT is also studied by correlation technique, because of radially localized mode structure and symmetric form w.r.t. rational surface, averaged wavenumber is found to be order of $(1/qR)$.

- **EM - Model Study**

- Numerical study of this model is in progress to check analytic theory predictions.

- The inclusion of symmetry breaking terms is also considered to reach the goal of self-consistently evolving electromagnetic turbulence and its effect on thermal transport.