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NEUTRON STARS WITH ORBITING LIGHT?

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ABSTRACT

We show that the space-time of nonsingular final states of collapse is not necessarily asymptotically empty and simple (although presently favoured nuclear equations of state seem to lead to this class at least in the spherical case).

Б. Лукач: Нейтронные звезды с вращающимся светом? KFKI-1987-74/B

АННОТАЦИЯ

Показывается, что пространство-время несингулярных конечных состояний коллапса не обязательно должно быть асимптотически пустым и простым, хотя выглядит так, что наиболее современные ядернофизические уравнения состояния (по крайней мере, в сферическом случае) приводят к этому классу.

Lukács B.: Neutroncsillagok keringő fénnyel? KFKI-1987-74/B

KIVONAT

Megmutatjuk, hogy a nemszinguláris kollapszusvégállapotok térideje nem feltétlenül aszimptotikusan üres és egyszerű, habár a pillanatnyilag legjobbnak tartott magfizikai állapotegyenletek - legalábbis gömbszimmetrikus esetben - ezen osztályhoz tűnnek vezetni.

1. INTRODUCTION

General Relativity tends to predict the absence of regular final states of gravitational collapse for too massive stars. This phenomenon is, e.g., indicated by a finite maximal mass for equilibrium spherical configurations in any specific calculation up to now. Then the final state of a star above this limit cannot be one cold, compact object as e.g. a neutron star or a white dwarf. Of course, after the exhaustion of the nuclear fuel of the star, a rapid contraction will start, therefore one might expect that the deliberation of "gravitational energy" would eject the outer mass shells of the star so that the final state is one compact object with sufficiently small mass, plus matter dispersed at spatial infinity. However, supernova simulations indicate that the ejection cannot be sufficiently efficient above cca. 10 solar masses (Arnett, 1967); then the central part of the matter cannot stop in any regular state of the final collapse, and in the spherically symmetric case a black hole will develop, with an unobservable central singularity.

Now, the inevitability of such singular final states of stellar evolution seems to be a genuine relativistic effect. Namely, it is the common consequence of two facts: the self-amplifying nature of gravity and the limiting nature of the velocity of light. The first fact suggests that with increasing mass somewhere gravity starts to dominate all the other effects, while the second prohibits the incompressible matter even as a limiting case (Curtis, 1950). So, indeed, one may expect a finite maximal mass, although the particular value obviously depends on the details of the equation of state. In fact, some singularity theorems (Hawking & Ellis, 1973) predict the inevitability of singular final states under quite general circumstances.

Obviously, it is an appealing task for astronomy to find and investigate such singular final states. Up to now the efforts have not been too successful, partly because of the tremendous technical difficulties involved. However, there are at least two fundamental problems as well. First, the signals for such states are often defined by the absence of something; second, it is not yet sufficiently clear, which are the common features of all the possible "irregular" final states of collapse.

The first point does not need too much discussion. Consider a spherical black hole as an obvious candidate for the final state of a too massive spherical star. Then there is a singularity at $r=0$, whose neighbourhood might produce very characteristic signals. However, this singularity is surrounded by a horizon (at $r=2m=2GM/c^2$, where G is the Cavendish-constant), preventing any signal to pass outward. Therefore a lonely black hole seems to represent the ideal *Ding an sich.* Of course, even a black hole will disturb the neighbouring stars via gravity, so in principle one might look for stars possessing orbits as if around an unseen companion, but this is obviously a negative signal, and one may list various reasons not to see a companion. Another possible signal is X-ray radiation, coming from infalling matter from $r=$(several times m); but this is possible also for neutron stars, therefore again one has to look for X-ray sources without characteristic neutron star features (e.g. pulsar effects) and above the predicted maximal equilibrium mass. The absence of observed pulsar behaviour may have various other explanations, as e.g. our particular location, while the predicted upper mass bound is not necessarily an onthologic fact.

One may expect help by investigating some global features of space-time. The idea is that the emergence of a singularity or a horizon leads to some changes in the global structure of space-time, which may have observable consequences, although very probably not too practical ones at the present status of art. (In fact, we will not discuss here the possible such signals.) And there we have arrived at the second point, the problem of common features of "irregular" final states of collapse. This will be the object of our discussion in this paper.

2. FINAL STATES WITH AND WITHOUT HORIZON

It is well known that all singular final states of a spherical collapse must possess horizon as well. Here we only recapitulate the main points. The most general spherical line element in General Relativity is as follows:

$$ds^2 = -e^{a(r,t)}dt^2 + e^{b(r,t)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.1)$$

Now, accepting the standard form of the Einstein equation, without cosmologic constant,

$$R_{ik} - (R/2)g_{ik} = -(8\pi G/c^4)T_{ik}, \quad (2.2)$$

where R_{ik} is the Ricci tensor, g_{ik} is the metric tensor, and T_{ik} is the energy-momentum tensor, one obtains equations among the functions a, b and the characteristic quantities of the matter. Now, outside the matter $T_{ik}=0$, and then Birkhoff's theorem (Misner & al., 1973) states that the solution of eq. (2.2) is unique:

$$e^a = e^{-b} = 1 - 2m/r, \quad (2.3)$$

where m is an arbitrary constant, connected to the mass M observable for very distant observers as

$$m = GM/c^2 . \quad (2.4)$$

Hence $m>0$ is expected for space-times around a star, and then the line element (2.1), (2.3) has a horizon indeed at $r=2m$ (Misner & al., 1973).

Of course, this particular result is valid only outside the matter, so the horizon is not necessary if the matter exceeds $r=2m$. In some such cases there still is a horizon, as e.g. for a Reissner-Nordström metric (Misner & al., 1973) containing only Maxwell field, in some other cases there is not, as for a perfect fluid. However, if there is a collapse, and the outermost shell of the matter has just passed inward $r=2m$, the horizon appears, and afterwards the matter cannot stop because g_{tt} becomes positive below 2m, so $dr=0$ ceases to be a timelike path.

Now consider a star with angular momentum. Then the line element (2.1) is no more valid, and without spherical symmetry Birkhoff's theorem does not hold, therefore there is no unique solution even outside the matter. However, it is proven (Robinson, 1975) that the Kerr solution (Misner & al., 1973)

$$ds^2 = \alpha^2(\partial^{-1}dr^2 + d\theta^2) + (r^2 + a^2)\sin^2\theta d\phi^2 + $$
$$+ 2mr\alpha^{-2}(a\sin^2\theta d\phi - dt)^2 - dt^2 \qquad (2.5)$$
$$\alpha^2(r,t) \equiv r^2 + a^2\cos^2\theta$$
$$\partial(r) \equiv r^2 - 2mr + a^2$$

is the unique vacuum solution with regular horizon. Therefore it is the only black hole solution without uncompensated electric charge and magnetic field.

Here we are not going to discuss the details of the Kerr solution; for this see Boyer & Lindqvist (1967). However, it is worthwhile to note that the new parameter a is connected to the total angular momentum I of the source as

$$a = I/Mc. \qquad (2.6)$$

Now, if $a<m$, the solution (2.5) possesses two horizons at

$$r_{\pm} = m \pm (m^2 - a^2)^{1/2}. \qquad (2.7)$$

However, for $a>m$, there is no horizon, the (ring) singularity is naked.

Since the ratio a/m is simply connected to the ratio I/M^2, one could predict which type of final state is expected for a particular star. For realizing this possibility one meets serious problems; first the ejection of rotating shells is far from sufficiently cleared up, second there is no unicity proof for $a>m$ Kerr solutions. Nevertheless, one may perform rough guesses with the results that the majority of the rapidly rotating A and B stars will not develop horizons in the collapse (see Appendix A). Therefore, definitely various types of global structures may be expected as "irregular" final states of collapse. The details of the possible structures are still partly unknown, because the complete list of solutions with angular momentum is not yet at reach.

Of course, the singularity itself must be common for all the irregular final states. But in some cases it is curtained off by the horizon, and in the horizonless cases generally we do not know the details of the collapse. For the a>m Kerr solution the appearance of the naked singularity would generate causality anomalies, often eliminated by a principle that such a solution cannot be realized (Penrose, 1969). Now, one may guess that matter will counteract very firmly when the change of the geometry tries to put it on the verge of temporal paradoxes, and this response may lead to a result conform to the above principle. So it is not sure that the external observer could see the singularity before $t=\infty$.

The immediate conclusion of the above arguments is that a common distinctive signal of *all* the irregular final states of collapse can be based neither on the horizon nor on the singularity.

3. ASYMPTOTICALLY EMPTY AND SIMPLE SPACE-TIMES

Now, there is defined in the literature a class of space-times, which is the obvious candidate to contain only regular final states and all of them. This is the class of asymptotically empty and simple space-times (Hawking & Ellis, 1973) (henceforth *AES*). For the precise definition see Appendix B or Hawking & Ellis (1973). Here we only list the properties directly used in further argumentations. These space-times

- go to the Minkowski metric in the space-like infinity and do this sufficiently rapidly;
- become matterfree in the same limit sufficiently rapidly; and
- cannot contain lightlike geodesics along which photons could escape to infinity.

Therefore for a very distant observer these space-times can be interpreted as containing only localized bodies in the internal region, leaving the infinity undisturbed; and, in addition, they cannot trap the light, only scatter. Such solutions seem to coincide with the intuitively felt characteristics of regular matter configurations; in fact, permitting light captation one could be confronted with the black hole solutions as well, absorbing light. Therefore up to now the AES class is the most fret-sawn class for solutions *globally* similar to Minkowski, excluding the minimal number of internally "strange" ones. Hawking & Ellis (1973), therefore, state that this class contains the solutions whose matter content has not undergone collapse (in their terminology "collapse" means "collapse without regular final state"). If so, then we have at least one clear distinctive difference between regular and irregular final states of collapse: the first ones belong to the AES class, the second not. Hence later one may find distinctive observables.

However, unfortunately, the situation is not so simple. In the remaining of this paper we show counterexamples: regular final states which do *not* belong to the AES class. Therefore the question of distinctive classification is again open. *For these counterexamples we restrict ourselves* to spherical solutions; since no constructive method is known for *exhausting* the nonspherical solutions *with matter*, one simply cannot go further in this moment. However, a list of counterexamples does not have to be complete.

4. THE TOLMAN-OPPENHEIMER-VOLKOV EQUATION

Consider spherical final states with fluid matter. Then the metric has the structure (2.1) with a and b independent of t, while the energy-momentum tensor has the form

$$T^{ik} = \epsilon u^i u^k + p(g^{ik}+u^i u^k) \qquad (4.1)$$

where ϵ is the energy density, p is the pressure and u^i is the velocity of the fluid. The latter has obviously only time component, and this only component is completely determined, being u^i a unit vector. Both p and ϵ depend only on the coordinate r. Furthermore, a final state cannot be hot, therefore both of them depend only on the particle density, so (Harrison & al, 1964)

$$p = p(\epsilon). \qquad (4.2)$$

The actual form of this equation of state depends on the actual matter content, and in principle is fully determined.

However, above several normal nuclear density (at or above usual neutron star densities) the $p(\epsilon)$ function is very poorly known, mainly because of the technical problems in the theories of strong interaction.

If eq. (4.2) is specified, the unknown quantities are 3, namely ϵ, a and b. Similarly, we have 3 nontrivial components of the Einstein eq. (2.2), say, the $_{00}$, $_{11}$ and $_{22}$ ones. Therefore the solutions are completely determined up to constants of integration. In order to see the number and rôle of these constants, here we shortly recapitulate, how to generate the solutions. For details see Hawking & Ellis, 1973; Harrison al., 1964).

First we introduce a mass function m(r) by the *definition*

$$e^{-b(r)} = 1-2m(r)/r. \qquad (4.3)$$

Then the Einstein equation gives

$$m' = 4\pi Gc^{-2}\epsilon r^2, \qquad (4.4)$$

$$p' = -Gc^{-2}(\epsilon+p)(m+4\pi pc^{-2}r^3)/[r(r-2m)], \qquad (4.5)$$

$$a' = -(1/2)p'/(\epsilon+p), \qquad (4.6)$$

where the prime denotes r derivative. Now, eq. (4.4) can be integrated as

$$m(r) = 4\pi Gc^{-2}\int_0^r \epsilon(x)x^2\,dx + m_0. \qquad (4.7)$$

Then, substituting this into eq. (4.5), for given $p(\epsilon)$, $p(r)$, $\epsilon(r)$ and $m(r)$ can be obtained, containing m_0 and $\epsilon_0=\epsilon(0)$ as parameters. Finally, by integrating eq. (4.6), a multiplicative factor appears in $g_{tt}=e^a$. However, this factor can be removed by a proper redefinition of the time coordinate, needed when matching to exterior Schwarzschild.

Localised configurations must possess a boundary, now at some $r=R$. There p and e^b must be continuous (Lichnerowicz, 1955). But outside eq. (2.3) holds. Therefore

$$m(R) \equiv m = GM/c^2 \qquad (4.8)$$

So m(r) is indeed the mass within radius r in geometric units.

But then m_0 is the mass in singular state at r=0. Indeed, if at r=0 ϵ and p are positive and finite, and $m_0>0$, then eq. (4.5) leads to singularities in the p(r) and ϵ(r) functions. Hence for all regular configurations $m_0=0$, and

$$m(r) = 4\pi Gc^{-2}\int_0^r \epsilon(x)x^2\, dx. \qquad (4.9)$$

Eqs. (4.5) and (4.9) constitute an integro-differential equation, called the *Tolman-Oppenheimer-Volkov* (henceforth TOV) equation, whose only constant of integration, as seen above, is ϵ_0, which will be the only free constant of the whole metric. Static, cold spherical regular configurations of *specified* matter are completely determined by the central density (Harrison & al., 1964).

Now we are able to explicitly formulate the conditions for having a counterexample. The Schwarzschild solution is not AES, because it possesses a lightlike orbit at r=3m (Robertson & Noonan, 1969). Therefore the space-time of a regular spherical static fluid configuration is not AES if at the surface m(R)/R>1/3. (The other possibility for not being AES is to possess a lightlike circular geodesic *inside* the matter, but we will not discuss here this rather exotic case.) Our question is: is it possible to build up such a regular sphere?

There is a well-known such example, the so called *interior Schwarzschild* solution (Tolman, 1934) of constant density. This density can be chosen such that the surface be located below 3m. However, the corresponding equation of state is acausal, permitting infinitely fast "sound" signals (Curtis, 1950), therefore we should rather exclude it from the present discussion. Here we require that the equation of state be causal

$$dp/d\epsilon \le 1 \qquad (4.10)$$

and the energy positivity conditions (Hawking & Ellis, 1973) hold:

$$\epsilon \ge 0, \qquad (4.11)$$

$$(\epsilon+p) \ge 0.$$

Furthermore, we restrict ourselves to positive pressures, although this would not follow from any fundamental principle (Lukács & Martinás, 1984). Our reason is that negative pressures are *hydrodynamically* unstable (Danielewicz, 1979), not expected in final states, and, anyway, the surface is at $p=0$.

Even a superficial observation at the TOV equation reveals that there are serious troubles when $2m(r)\to r$. Since at small r's $m(r)\sim r^3 \ll r$, at the first such crossing $p'\to -\infty$. By assuming that $m(r)-2r$ changes its sign from - to + with finite first and second derivatives at a regular r_0, one obtains that there $dp/d\epsilon \to +\infty$, prohibited by causality. So one cannot expect $R<2m(R)$ from the TOV equation. (Of course, there is a stronger *global* argument that inside the horizon a velocity pointing into ^{t} direction is forbidden. But now we see how this is reflected in the equilibrium equation.)

However, there is no such problem when $3m(r)\to r$; there are quite regular approximate solutions of the TOV equation there. So the TOV equation in itself does not guarantee that all its regular solutions be AES. Therefore in the subsequent Sections we will investigate if $R<3m(R)$ is possible for equations of state fulfilling Conds. (4.10-11).

5. EINSTEIN'S DUST

A classical example for $R<3m$ as limiting behaviour is the Einstein dust (Einstein, 1939). Consider non-interacting particles revolving around the common center of mass in each direction. Then spherical symmetry is preserved and the configuration remains static. (Of course, now the TOV equation is not valid, the matter not being a fluid.) We again have the 3 independent components of the Einstein eq. (2.1); the unknown quantities are *a*, *b*, $\epsilon = m_0 c^2 n$ and the tangential component of u^i. Therefore one quantity is free, e.g. the density. (For details see Appendix C.) It is easy to evaluate the equations when ϵ is confined to a thin shell; not surprisingly then the minimal possible value of R/m is 3 (Einstein, 1939). This remains valid for a general regular $\epsilon(r)$ function (see Appendix C).

6. ASYMPTOTIC SOLUTIONS WITH τ-LAW

Sometimes it is assumed that at very high densities

$$p = (\tau-1)\epsilon \qquad (6.1)$$

where τ is a number constant between 1 and 2 in order to get $p>0$ and satisfy (4.10). It is, of course, impossible to prove or disprove such a belief, based on the assumption of some kind of scale laws. However, two cases do deserve specific attention: $\tau=4/3$ for free relativistic Fermi gas, while $\tau=2$ for high densities of a matter containing repulsive pair interactions of finite range, and is believed to be realized in a very dense nuclear matter. In any case, for τ-laws this latter is the lest compressible matter.

Now, the TOV equation is too complicated for being solvable analytically even for such a simple law, excepting the limiting case $\epsilon_0=\infty$. Then $\epsilon\sim 1/r^2$. Probably this limiting case would give the most compact object, but the (6.1) law does not permit really localisable bodies. One can see this in the limiting case: ϵ does not vanish at any *finite* r, and $p\sim\epsilon$. Therefore the boundary condition cannot hold. However, one can be roughly orientated by observing that anywhere inside the matter

$$m(r) \sim \beta r. \qquad (6.2)$$

where β is a constant and (Harrison & al., 1964)

$$\beta = 2(\tau-1)/(\tau^2+4\tau-4). \qquad (6.3)$$

So, for $\tau=4/3$, $\beta=3/14$, while for $\tau=2$, $\beta=1/4$, which latter thus seems to be the upper limit for $m(R)/R$ with a pure τ-law. Therefore in this case no lightlike orbit would appear.

7. THE SIMPLEST DEVIATION FROM τ-LAW

As mentioned above, a pure τ-law does not lead to finite objects. In fact, it is an everyday experience that at low densities $p\ll\epsilon$, and there is an ϵ_s so that $p(\epsilon_s)=0$. Then the sphere may have a surface. Now, the simplest equation of state with this property is as follows:

$$p = (\tau-1)\epsilon - p_0, \qquad (7.1)$$
$$p_0 = (\tau-1)\epsilon_s.$$

With such an equation of state the TOV equation is already impossible to be analytically solved. However, a factor

$$Gp_0/c^4 \equiv 1/r_0^2 \qquad (7.2)$$

giving the scale of the radius, can be removed from the equation, introducing dimensionless density and radius. So there is a scaling in p_0. Therefore the dimensionless mass and radius, together with R/m, will depend also only on the dimensionless ϵ_0. For $\tau=4/3$ and $\tau=2$ these functions are displayed on Figs. 1 and 2, respectively, while Fig. 3 is the density profile of a particular solution for $\tau=2$. One can see that for $\tau=4/3$ the minimum of R/m is cca. 3.5, so all these solutions belong to the AES class. However, for $\tau=2$ R/m goes below 3 in a wide region of the central density, having a minimum of cca. 2.75. Therefore the quite harmless equation of state

$$p = \epsilon - p_0 \qquad (7.3)$$

leads to regular static configurations, whose space-times do not belong to the AES class!

Still we are not ready. At least two other objections are possible. These solutions may be unstable, or the above equation of state may violate general principles other than discussed here.

8. ON THE STABILITY OF THE SOLUTIONS OF TOV EQUATION

Here we discuss the stability against radial oscillations. Nonradial oscillations are out of the scope of the present formalism, violating the spherical symmetry, but, anyway, it seems that for *small* oscillation the tangential component causes secondary effects; other instabilities may exist too, but they are handled by other methods. Here the only other instability is a thermodynamic one, but it does not appear, because $dp/d\epsilon$ is never negative for the present equation of state.

The stability conditions against radial oscillations are completely known, and in the absence of phase transition

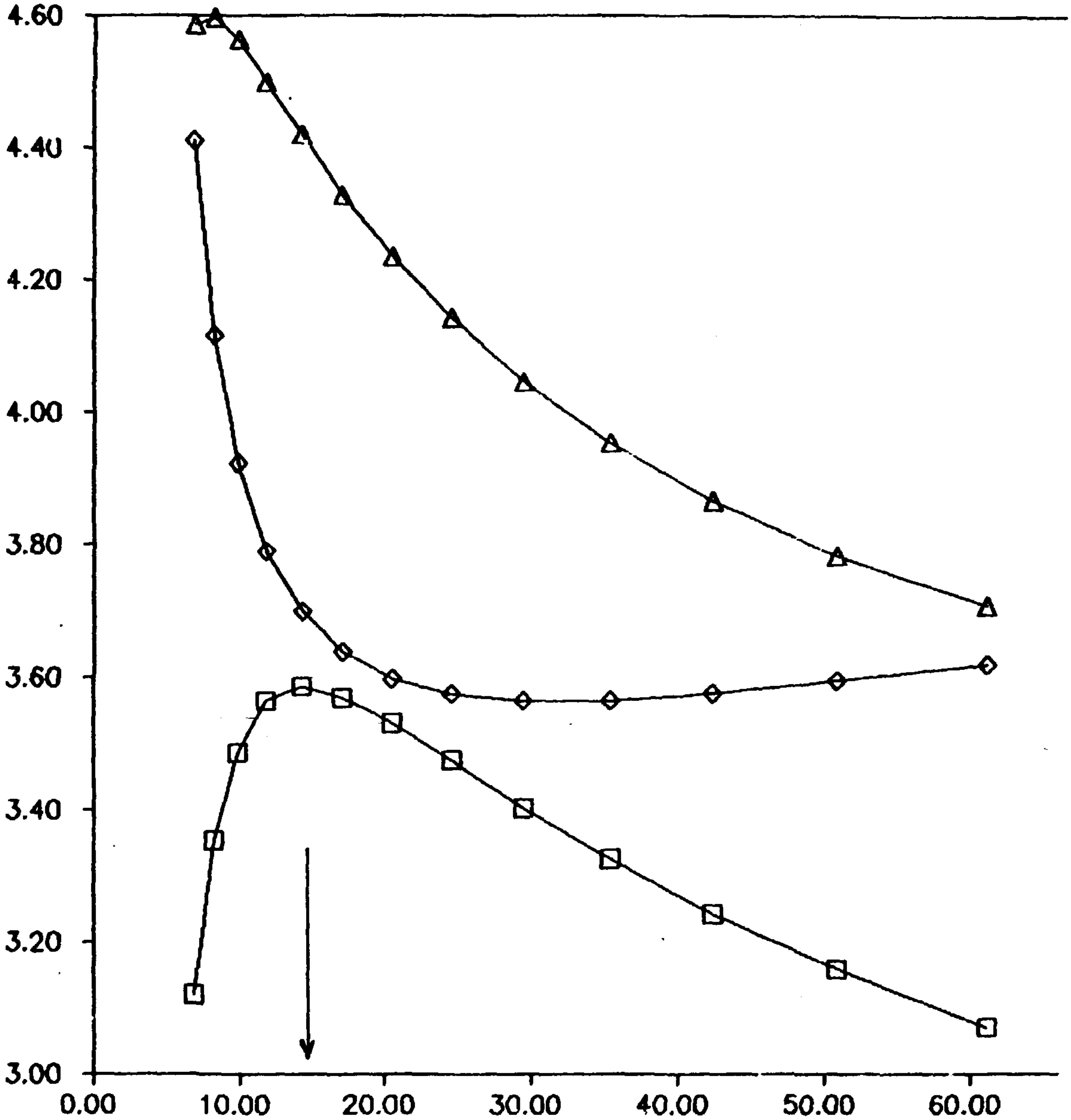

Fig. 1: 120*total mass, 40*radius and R/m(R) ratio versus central mass density for equation of state (7.1) with $\gamma=4/3$. Each quantity is dimensionless, the removed scale factors are: p_0/c^2 for mass density ϵ_0/c^2; $c^4/(G^3p_0)^{1/2}$ for total mass and $(c^4/Gp_0)^{1/2}$ for radius. Symbols: squares for mass, triangles for radius and rhomboids for R/m(R). The vertical arrow indicates the central density beyond which the configurations are unstable against radial oscillation.

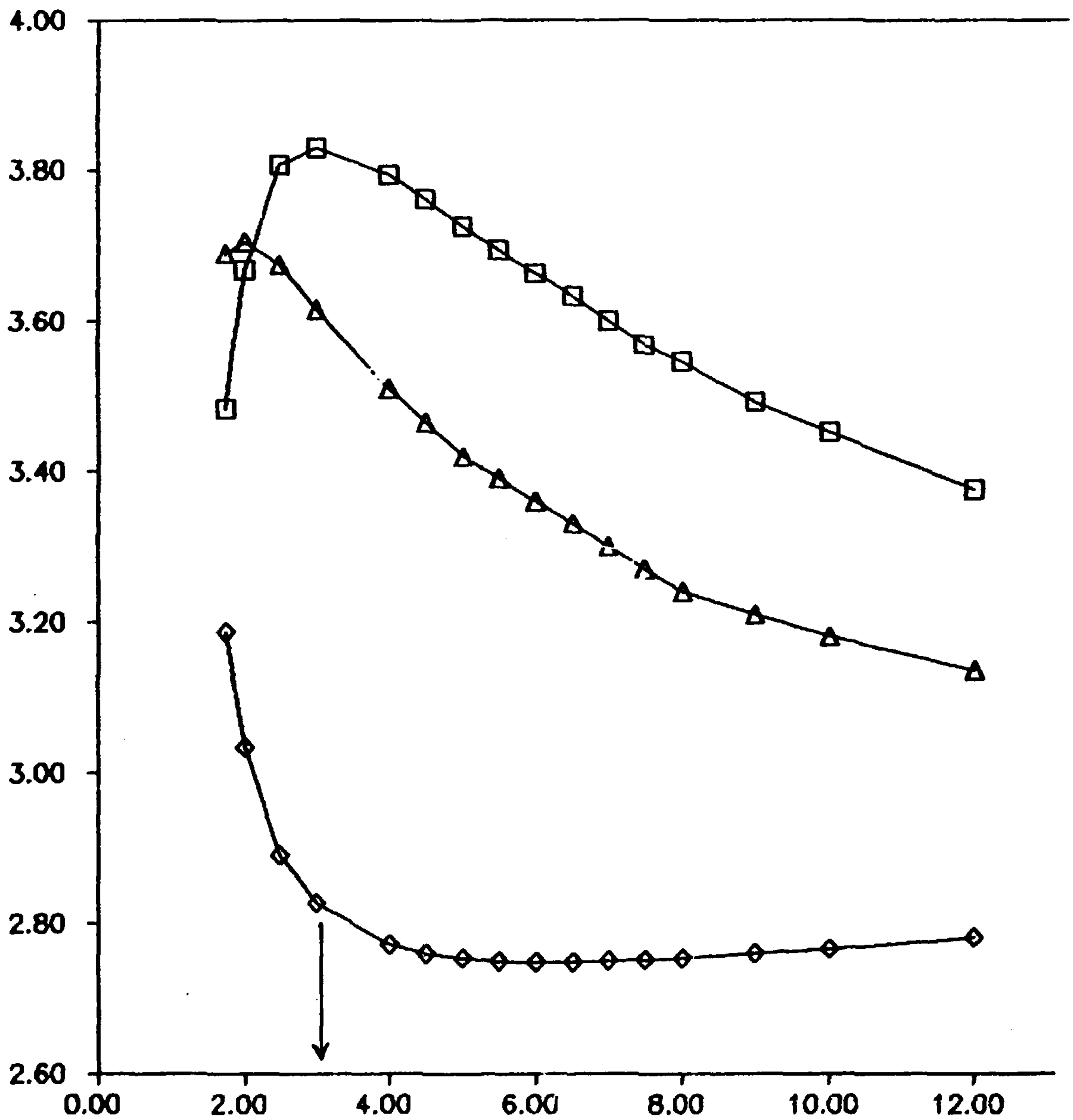

Fig. 2: 45*total mass, 15*radius and R/m(R) for $\gamma=2$. Scales and symbols as on Fig. 1.

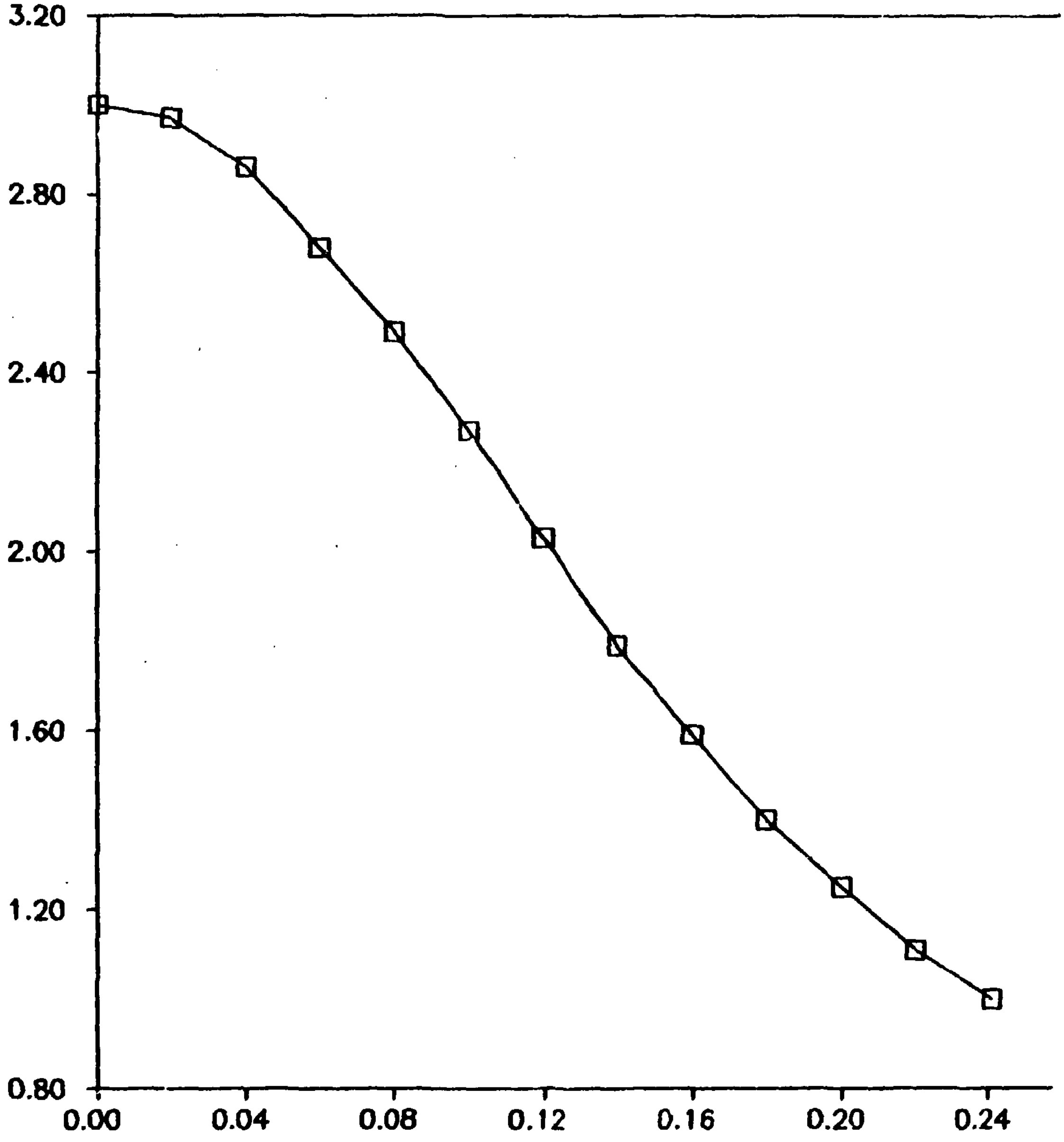

Fig. 3: Dimensionless mass density versus dimensionless radius within a characteristic static configuration for $\tau=2$. The boundary is located at unit density.

are listed in (Harrison & al., 1964). (The above equations of state do not contain phase transition.) The final results can be recapitulated here shortly as follows:

The possible modes of oscillations can be arranged according to increasing frequency square Ω^2. The mode is unstable if $\Omega^2 < 0$. Now, modes can switch from stable to unstable or backward only at extrema of $m(\epsilon_0)$, and

for *max.* with $R(\epsilon_0)$ *increasing*: the last unstable one becomes stable;

decreasing: the first stable one becomes unstable;

for *min.* with $R(\epsilon_0)$ *increasing*: the first stable one becomes unstable;

decreasing: the last unstable one becomes stable.

For very low masses one can use Newtonian methods and then these configurations must be stable. Hence one gets the stability regions indicated on Figs 1 and 2: in the second case there remains stability down to $R/m=2.87$.

The physical meaning of the specific equations of state will be discussed later.

9. THE WALECKA MEAN FIELD EQUATION OF STATE

The equation of state (7.3) was able to produce non-AES solutions, but it was a handmade simple equation of state. In order to partially reveal its physical meaning, let us introduce the particle number density n. For a cold fluid of one component $p=p(n)$, $\epsilon=\epsilon(n)$, and, from thermodynamics (Harrison & al., 1964):

$$\epsilon = \mu n - p, \qquad (9.1)$$

$$d\epsilon = \mu dn,$$

where μ is the chemical potential. So

$$p = n d\epsilon/dn - \epsilon. \qquad (9.2)$$

Then eq. (7.3) can be integrated as

$$\epsilon = (C^2/2)n^2 + p_0/2, \qquad (9.3)$$

where C is some coupling constant. Now, the constant term $p_0/2$ does not seem too physical, representing a positive zero point for the energy density. Later we will return to this question; here we only note that it may be regarded as first approximation for terms slower than linear. Such a slower than linear term, if dominates, leads to negative compressibility, causing a first order phase transition. Now, very dilute neutron matter possesses such an instability at $\epsilon/c^2 \approx 10^{12}$ g/cm^3, where β-decay starts (Harrison & al., 1964). There is another (liquid-gas) phase transition between cca. 0.1 and 0.5 normal nuclear densities (Walecka, 1974). Therefore the form of eq. (7.3) in itself is not aphysical. However, in the true equation of state there must be terms between n^2 and n as well, e.g. the relativistic Fermi contribution $n^{4/3}$.

As mentioned above, the true equation of state is still unknown well above normal nuclear density. However, skilled guesses do exist; maybe the best one is the Walecka equation of state (Walecka, 1974). It describes a dense baryonic matter, in our case neutron matter; correctly includes the Fermi behaviour of neutrons, while for the interaction introduces two *nonquantized* mesons, a scalar for long range attraction, and a vector for short range repulsion. The two coupling constants are fitted to the ground state density and energy of nuclear matter. The theory is thermodynamically consistent, and in the limit $n \to \infty$ the causality limit $dp/d\epsilon = 1$ is just reached. More discussion is unnecessary here, because the details can be found in (Walecka, 1974), and still no real evidences or counterevidences are known well above normal nuclear density.

Now, the TOV equation can be numerically integrated for the Walecka equation of state, and the curves $M(\epsilon_0)$, $R(\epsilon_0)$ and $(R/m)(\epsilon_0)$ are shown on Fig. 4. One can immediatly see that R/m remains everywhere above 3, but only with a narrow margin. And the fact is that R/m>3 does not come from the *structure* of the equation of state. In order to demonstrate this we have multiplied the repulsive coupling constant by 5 while keeping the *difference* of the repulsive and attractive

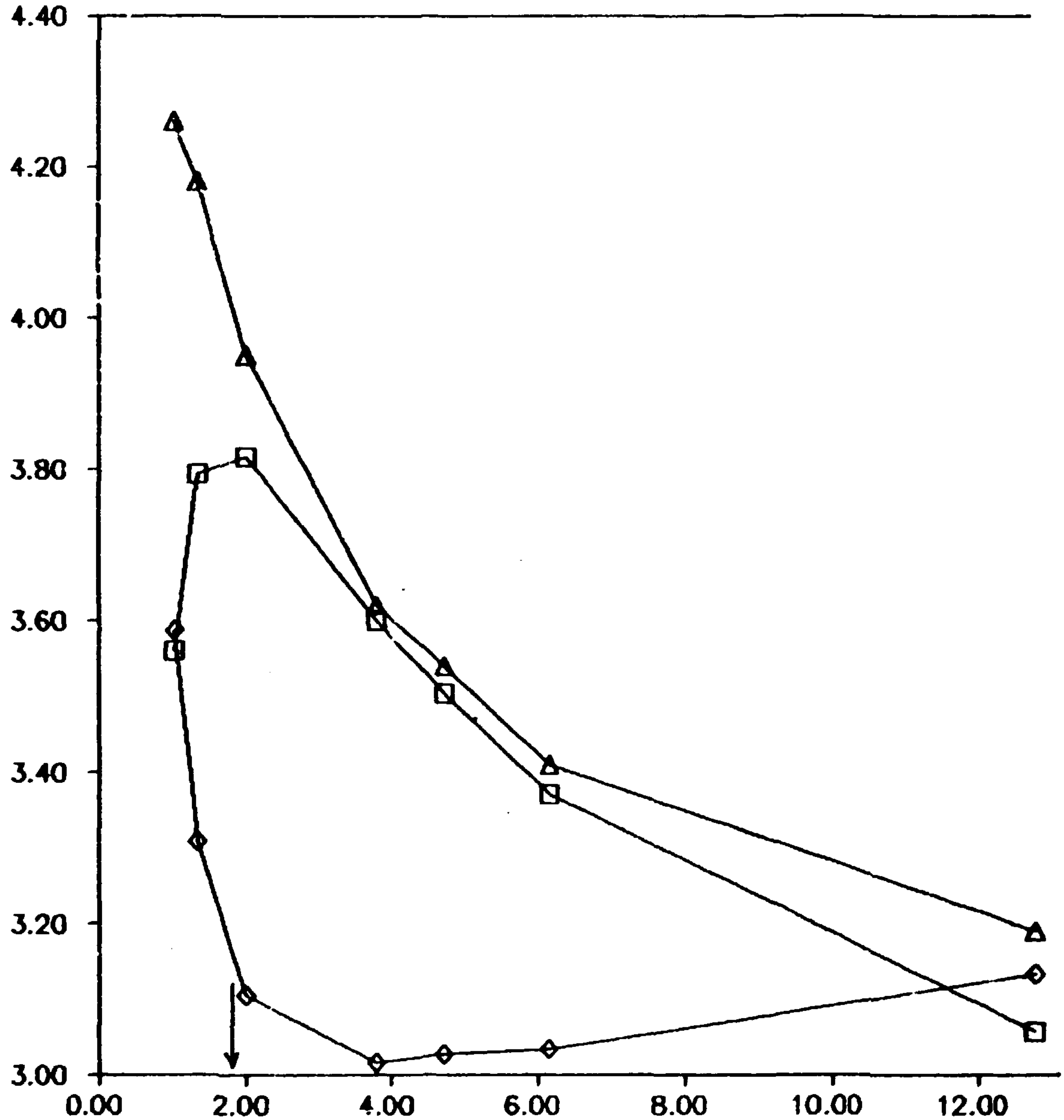

Fig. 4: 3*mass, radius and R/m(R) versus central mass density for the Walecka equation of state with canonical values of the coupling constants (Walecka, 1974). Mass and radius in kilometers, ϵ_0/c^2 in 10^{15} g/cm^3. Symbols as on Fig. 1.

coupling constants. Then R/m crosses 3 with the minimal value cca. 2.89. While such an equation of state does not reproduce the ground state of nuclei, it is still possible at higher densities (although, to be sure, we have no evidence for such higher coupling constants).

10. EXOTIC EQUATIONS OF STATE

Now we are going to return to the simple equations of state of Sect. 7. Both special equations used there possess some connection with hypothetical high density systems. This question deserves some further investigation. But first we show that if one wishes to get equations of states leading to $R/m<3$, and fo·· this pushes the *whole* problem well above normal nuclear densities, where indeed anything may be true *at the present status of art*, then he must be contented with "metastable" states.

At low densities the pressure is low but positive. Now, in a thermodynamically stable state the second derivative of ϵ with respect to the extensive densities must form a positive definite matrix (Kirschner, 1970). In our case the matrix reduces to one element, so

$$d^2\epsilon/dn^2 = (dp/d\epsilon)(\epsilon+p)/n^2 > 0. \qquad (10.1)$$

Now, on the middle side the second and third terms are non-negative, so the first one (the inverse compressibility) must remain positive in thermodynamically stable regions. So there p is monotonously increasing with ϵ. This means that there cannot be a second zero of p, needed for surface, at higher densities.

However, this argumentation strictly holds only if there is no first order phase transition between us and the hypothetical denser states. Namely, consider a first order transition. There will be a pair of ϵ values (on the two sides of an unstable region, outside) producing the same p and μ values, respectively: these two states mark the beginning and end of an *equilibrium* phase transition. If all the low density states are of positive pressure, then in equilibrium phase transition the matter evolves through positive pressures up to the exotic states and we cannot get a star consisting of purely exotic states. Nevertheless, let us start from the high density states. By some "overrarification" process one may go behind the startpoint of *equilibrium* transition; here still the compressibility is positive, so the matter is (thermodynamically) stable. Such states are sometimes called *metastable*; the actual state is stable, being a local energy minimum, but a mixture of proper ratio of two phases would be a lower energy state, so, if the second (low density) state is present as some nuclei of condensation, one can expect the transition. Nevertheless, in itself, the overrarified high density state is stable, and if the compressibility remains positive until $p=0$, the high density states can form localized stars completely beyond our present knowledge.

In a strict sense a metastable object is not a final state, because the phase transition will happen in finite time. However, this remark is not to be overemphasized. Perhaps the best example is the equation of state

$$p = \epsilon/3 - p_0. \quad (10.2)$$

This equation of state can be derived in the perturbative regime of QCD for a quark plasma; $p_0=4B/3$, where B is the so called "bag constant", the zero point of the energy density of the "perturbative vacuum" of QCD. Now, according to calculations, the QCD plasma at low densities (between cca. 15 and 6 normal nuclear densities) is transformed into nuclear matter (Kuti & al., 1980; Lukács, 1983) via formation of 3-quark groups; of course, this happens in equilibrium at positive pressures. However, it is sometimes claimed that Cyg X-3 may be a quark star (Baym et al., 1985). This can be in two ways. Either there the quark->nucleon transition has been delayed by a potential barrier for astronomical times, or, oppositely, its state may be of deeper energy, being a mixture of u, d *and s* quarks, and then the usual state of matter is metastable with a very long lifetime compared to any observation (De Rújula & Glashow, 1984).

Since Fig. 1 shows that a quark equation of state can lead only to AES solutions, the reference to Cyg X-3 only demonstrates that metastability may be extended to astronomical timescales. Now we turn to eq. (7.3). It is sometimes suggested that the energy density of an overcompressed nuclear matter may have a second minimum somewhere well above normal nuclear density. These states are called density isomers (Stöcker & al., 1979), and there is no evidence for them. The basis of the idea, however, is that mesons are also sources of the strong interaction, therefore one may imagine a nonlinear amplification of attraction (mediated by the mesons) with increasing density ("pion condensate"). If so, then in the neighbourhood of the second minimum the state is locally stable; there the energy density is quadratic in particle density, with a positive minimum, roughly as in eq. (9.1) corresponding to (6.3). And such an object may not be an AES solution.

Finally we note that if the exotic behaviour appears very far above nuclear densities, then the pressure of nuclear states can be regarded approximately as 0. Namely, the structure of the TOV equation leads to the suppression of the weight of the relatively low density states: the central core causes high gravity, therefore high pressure gradient outside, so the low density states constitute a thin layer negligible both in mass and in radius (Harrison & al., 1964). So one might construct an equation of state as follows: first a familiar nuclear matter behaviour, then above a first order phase transition with a huge density gap, finally an equation of state of type (7.3) on the high density branch. With such an equation of state probably R/m could go below 3 even with a nuclear matter surface. However the information is so poor about very high density states that this construction would be a mere play.

11. CONCLUSION

Our conclusion is as follows. One can choose such equations of state, with which some solutions of the Tolman-Oppenheimer-Volkov equation are static spherical configurations whose space time is *not* asymptotically simple and empty. These equations of state are harmless from thermodynamic viewpoint, causal, and obey energy positivity conditions. Therefore the Asymptotically Empty and Simple class does not include *all possible* nonsingular final states of stellar collapses; consequently this class is not the proper tool to distinguish between singular and regular final states.

The physical meaning of this result is that the AES class is slightly *too* restricted. Roughly speaking, the original goal was to exclude black holes and naked singularities. Both objects can absorb light (slower signals can be absorbed even by regular objects), so obviously only light *scattering* is to be permitted. Now, a "neutron star" with $R<3m$ and a lightlike orbit at $r=3m$ is just the borderline: this light *has not been* absorbed, but rather was there forever.

It is a different question whether one may expect the *real existence* of such "supercompact" final states. The answer of the present day nuclear physics is probably *no*, as far as the matter is predominantly neutrons. Although our knowledge about dense states is limited, one may believe this answer for *neutrons*, because from several nuclear densities not neutrons but quarks are expected. With increasing density the nonperturbative QCD regime and possible more exotic states are practically unknown, so the prediction would be useless.

A slightly more positive statement can be formulated according to the experience collected from the numerical calculation that the decrease of R/m below 3 is impossible or very difficult when the matter is in the relativistic Fermi regime. Therefore one may guess that the ideal candidate to form such a star would be a *boson* with *repulsive* pair interaction. The author does not have any serious proposal for this. However, the list of the hypothetical "subelementary" particles (preons) is quite long today, therefore the existence of compact final states with lightlike orbits cannot be excluded.

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APPENDIX A: ESTIMATION FOR THE RATIO OF KERR PARAMETERS FOR STARS

Because of the unicity theorem for Kerr solution (as far as the existence of a *regular* horizon is postulated), this solution seems to play an important rôle among singular final states of stellar evolution. One cannot directly determine the Kerr parameters (cf. eq. (2.7)) for final states, since the collapse may include steps of substantial mass ejection. However, at least the initial values can be taken from observation. For first estimation one may restrict himself to luminosity class V (main sequence) because if the star does not belong to a close binary, probably no essential mass loss happens before collapse, and all other luminosity classes have evolved from the main sequence.

The mass versus spectral type relation is well established (from binaries). For angular momentum the task is more complicated, because

1) the angular momentum cannot be *directly* measured except for very small General Relativity effects (for the Sun the light bending has an angular momentum correction in the sixth digit);
2) the rotational velocities can be observed via Doppler widening of spectral lines, but the effect has to be separated from other Doppler effects and $v_{rot}/c \approx 10^{-3}$; finally
3) the angular inertia cannot be measured, only calculated from stellar models.

Therefore the data given below are to be received with maximal caution, but one may hope that they are correct for order of magnitude.

The rotational velocities can be found in the literature. Here we use Landolt & Börnstein (1952). The angular momentum can be written as

$$I = \sigma R^2 M v_{rot}, \qquad (A.1)$$

where σ is a number factor depending on the internal mass distribution. For a homogeneous sphere σ=0.4, therefore now one may expect definitely lower values.

Now, Landolt & Börnstein (1952) gives $\langle v_{rot} \rangle$ for various spectral types and luminosity classes, occasionally mean deviations, and distributions *aggregated* for all luminisity classes. Since R substantially differ for different luminosity classes, the most decent attitude is to use only the averages for luminosity class V. As for the number factor σ, we note that from numerical integration of different stellar models one gets

σ=0.078 for polytropes of index 3

σ=0.069 for the present sun, roughly G2

σ=0.064 for 3 solar mass (roughly A0)

(for the last two models see Novotny (1973)). Thus σ is not too model-dependent; here we will use the polytrope 3 value, which seems to overestimate *a*, but even in the worst case only by some 20%.

After this one may take M and $\langle v_{rot} \rangle$ from literature (Landolt & Börnstein, 1952), and calculate m and $\langle a \rangle$ for various spectral types of the luminosity class V according to the formulae of Sect. 2. The result is displayed on Fig. 5. One can see that for all spectral types candidates for collapse (i.e. M greater than *at least* 1.5 solar mass, roughly F5 and earlier types) $\langle a \rangle$ > m, with a very wide margin.

Now, m is characteristic for the given spectral type as indicated by the narrowness of the main sequence on the Hertzsprung-Russel diagram. However, the rotational velocity does not essentially influence the structure of a main sequence star, therefore v_{rot} may have a wide distribution within a spectral class. However, for cases when the mean deviations are listed at all, they are definitely smaller than $\langle v_{rot} \rangle$, therefore for the majority of stars of spectral type F5 and earlier a>m indeed. More definite statements could be manufactured by observing that the published (mean devi-

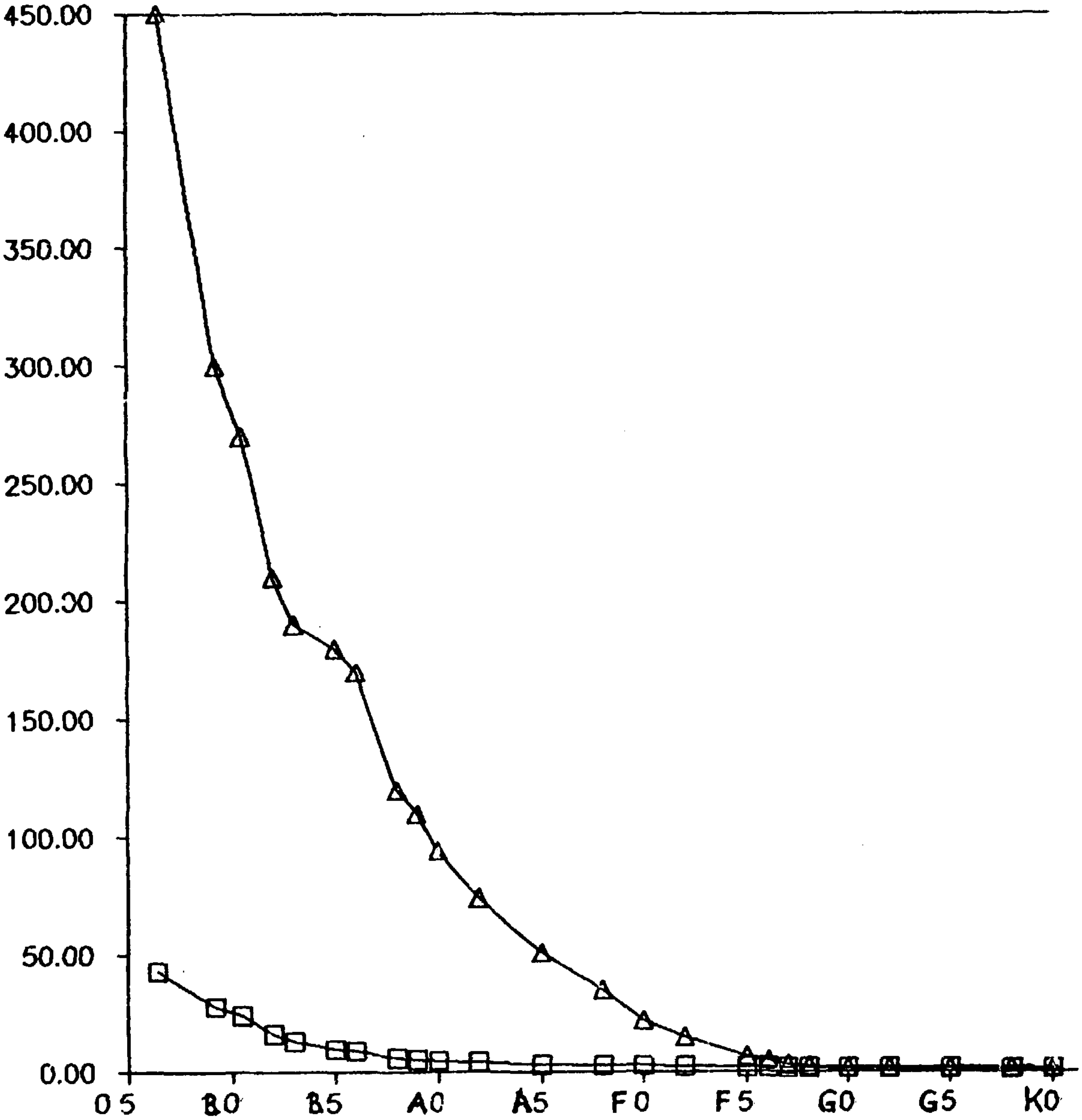

Fig. 5: Average Kerr parameters (in kilometers) versus spectral types for main sequence stars. Symbols: triangles for a (estimated) and squares for m.

ation/average velocity) ratios are roughly conform to Gaussian distributions. (Lukács, 1972) However, for our present purposes a detailed statistics is not needed.

For estimating the decrease of a/m by mass ejection, here we note that for polytropes 3 the calculated a(r)/m(r) does not go below ≈0.6(a/m). (Lukács, 1972) Of course, by magnetic coupling the angular momentum loss may be higher but only if the field does penetrate the deep interior of the star. For any case, according to the above data, we have no evidence that a<m Kerr solutions would be characteristic for singular final states of the collapse.

APPENDIX B: THE ASYMPTOTICALLY EMPTY AND SIMPLF CLASS

According to Hawking & Ellis (1973) a time- and space-orientable space (M, g) is called AES if there exists a strongly causal space (M', g') and an imbedding θ which imbeds M as a manifold with smooth boundary ∂M in M', such that

1) there is a smooth (C^3) function Ω on M' such that on θ(M), Ω is positive and $\theta^* g' = (\theta^* \Omega^2) g$;
2) on ∂M, $\Omega = 0$ and $d\Omega \neq 0$;
3) every null geodesic in M has two endpoints on ∂M;
4) $R_{ik} = 0$ on an open neighbourhood of ∂M in $M \cup \partial M$.

Since for all localized spherical objects the asymptotical region is Schwarzschild (cf. Birkhoff's theorem), which goes to Minkowski sufficiently fast, we concentrate on Cond. 3. Hawking & Ellis (1973) notes that the boundary ∂M can be thought to be at infinity, and for Minkowski space it consists of the two null surfaces I^+ and I^- (future and past), each with topology $R^1 x S^2$. Null geodesics in M must have their past endpoints on I^- and future endpoints on I^+.

Now, this condition does not hold if exterior Schwarzschild solution is still valid at r=3m, because there is a lightlike orbit, i.e. a null geodesic *without* endpoints in the null infinity.

APPENDIX C: FORMULAE FOR EINSTEIN'S DUST

If the particles revolve on circular orbits, with the same probability in each possible directions, then the space-time is static and spherically symmetric, i.e. the form (2.1) remains valid with time-independent a and b. The energy-momentum tensor is a sum of dust ones:

$$T^{ik} = \Sigma_\alpha m_0 n u_\alpha{}^i u_\alpha{}^k \quad (C.1)$$

where m_0 is the particle mass, n is the particle density and α stands for possible directions. Because of the circular shape of orbits

$$u_\alpha{}^1 = 0, \quad (C.2)$$

while from spherical symmetry one obtains that all nondiagonal components of T^{ik} vanish, and

$$\Sigma_\alpha (u_{\alpha 2})^2 = \Sigma_\alpha (u_{\alpha 3})^2 \sin^2\theta \equiv v^2/2. \quad (C.3)$$

Finally, from velocity normalization,

$$u_{\alpha 0} = e^{a/2}(1+v^2/r^2)^{1/2}. \quad (C.4)$$

Therefore the unknown quantities are: a, b, v and n, while for equations we have only the 3 nontrivial components of eq. (2.2). (The geodesic equations for the orbits must be consequences, and indeed they are, as it can be seen by straightforward calculation.) Therefore one function remains free, let it be chosen the density n.

Since $T^{11}=0$, the corresponding component of the Einstein equation connects a and b, namely

$$e^b = 1 + ra' \quad (C.5)$$

where the prime is the r derivative. By introducing again the mass function m(r) as in eq. (4.3), the remaining two components of the Einstein equation read as:

$$m' = 4\pi G c^{-2} m_0 n r^2 (r-2m)/(r-3m) \quad (C.6)$$

$$v^2 = m r^2/(r-3m). \quad (C.7)$$

Hence there is no orbit for $r/m(r)<3$, and the last possible orbit is lightlike.

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